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δ −derived and δ −Scattered Sets

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Abstract

In this paper, we introduce new class of Sets called δ −Scattered and investigate the Properties of this Set. we use the concept of δ-limit point and δ −drived Set to construct the definition of this class. we give the relation between types of scattered Sets and types of limit points .

Key words:: $\boldsymbol{\delta}$ −Sets, $\boldsymbol{\delta}$ −limit point, $\boldsymbol{\delta}$ −drived Set , $\boldsymbol{\delta}$ −isolated point , $\boldsymbol{\delta}$ −Scattered Set ,Scattered Sets

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1-Introduction

Many Mathematican wrote papers about Sets, points and Spaces in mathematics where these Spaces defined at the Sets like semi-open, α-Sets, preopen Sets,….. etc. ,also the others defined points on this Sets like limit ,isolated, α-limit, α-isolated ,semi-isolated,……….. etc. .

In 1998 [3] J. Dontchev and D. Rose studied anew types of Sets called nowhere dense Scattered ,and later they wrote about α-Scattered Sets that depend on the definition of T^{α}-Space and Scattered Space, many researchers in many papers studies scattered Sets [3], [4], [6] ,[9] ,[13]and others ,these Sets deals with isolated points as base to reach to define these Sets, in 1998 [12] T.M.NOUR study more properties about semi-open Sets and define anew Sets called semi-scattered .

In 2007 [6] Melvin Henrikseon and others define anew point called Sp-points to define Sp-Scattered Sets.

We summarized the concept of δ −drived Sets and some properties in section 2, we introduce the concept of $\delta -$ isolated points and some properties of the Sets that contain this points to define anew Sets called δ −Scattered and study the relationships between this Set and with the other Sets in section 4 .

Throughout this paper (X, T_X) (or simply X) represent topological Space.
Sub Set A of Space X is said to be semi-open [12] (resp. α−open [8], nowhere dense [3],regular open[10] ,regular closed [10], pre-open [14],regular clopen [16]) if $A \subseteq cl(int(A))$ (resp. $A \subseteq \mathrm{int}(cl(int(A)))$,

$int(cl(A))$ =Ø, $int(cl(A)) = A$, $cl(int(A)) = A$, $A \subseteq$ int(cl(A)) ,if it is regular open and regular closed) . Apoint $x \in X$ is called limit [2] (resp. δ −adherent [9]) point of $A \subseteq X$ if $U \cap (A - \{x\}) \neq \emptyset$
(resp.$A \cap u \neq \emptyset$) where $x \in U$ for every U is open(resp. regular open)Set .The Set of all limit
(resp. δ −adherent) points of A is called the derived [11] (resp. δ −closure[10]) of A and this denoted by *D(A)* (resp. $cl_\delta(A)$ or δ −cl(A)), point $x \in A$ is called isolated point [7] of A if $x \notin D(A)$.
Sub Set $A \subseteq X$ is called Crowded or dense in itself [4] (resp. δ −closed [1], δ −open [1],perfect [4], δ −clopen [16]) if it does not have any isolated point (resp. $cl_\delta(A) = A$, $A = \bigcup_{i \in I} U_i$ where U_i is regular open $\forall i$, closed and crowded , if it is δ −open and $\delta -$ closed), the union (resp. intersection) of all δ −open (resp. $\delta -$ closed) Sets in X contained in A (resp. containing A) is called $\delta - interior$ [10] (resp. δ −closure [10]) of A and is denoted by $\delta int(A)$ or $int_\delta(A)$ (resp. $\delta cl(A)$ or $cl_\delta(A)$) .Also
$\delta int(X - A)$ is called δ −Exterior [10] of A and is denoted by $Ext_\delta(A)$. Sub Set A of Space X is called
Scattered [13] if it have an isolated point .The collection of all δ −open Sets is topological Space (X, T_X)
forms topology T_δ on X is called the semi generalization topology of T, T_δ is weaker than T and the class of all regular open Sets in T forms an open basis for T. the complement of δ −open (resp. semi−open, α−open , regular open ,Per−open) Sets is δ −closed (resp. semi−closed, α−closed, regular closed ,Pre−closed) .

2- Some properties of δ –derived Set

In this section we introduce the concept of δ −derived Set which depend on the concept of δ −limit Points and some properties of this Set .

Definitions 2.1

Let $A \subseteq X$, point $x \in X$ is said to be α−limit[4] (resp. Semi−limit [12] ,Pre−limit [14]) Point of Set A if $U \cap (A - \{x\}) \neq \emptyset$ for every α-open (resp. Semi−open ,Pre−open) Sub Set U of X containing x .

The Set of all $\alpha -$ limit(resp. Semi−limit, Pre−limit) points of A is called α −derived(resp. Semi−derived
, Pre−derived) and is denoted by $D_\alpha(A)$ (resp. $D_S(A)$, $D_P(A)$).

Definitions 2.2 [10]

Let $A \subseteq X$, point $x \in X$ is said to be $\delta -$ limit Point of A if $U \cap (A - \{x\}) \neq \emptyset$ for every δ−open Sub Set U of X containing x .

. The Set of all δ−limit points of A is called δ –derived Set of A and is denoted by $D_\delta(A)$

Proposition 2.3 [15] [16]

For Sub Set A of Space X ,then:
(1) If A is δ −closed ,then A is closed(resp. $\alpha -$ closed ,semi-closed).
(2) If A is regular open (resp. regular closed) ,then A is δ −open (resp. δ −closed) .
(3) If A is $\alpha -$ open ,then A is semi−open (resp. Pre−open) .
(4) If A is δ −closed ,then A is Pre−closed .
(5) If A is closed ,then A is $\alpha -$ closed (resp. Pre−closed) .

Proposition 2.4 [10]

For Sub Sets A, B of Space X ,the following statements hold:-
(1) $D_\delta(A) \cup A = cl_\delta(A)$.
(2) $int_\delta(A) \subseteq A$ and $A \subseteq cl_\delta(A)$.
(3) If $A \subseteq B$, then $cl_\delta(A) \subseteq cl_\delta(B)$.
(4) If $A \subseteq B$,then $int_\delta(A) \subseteq int_\delta(B)$.
(5) $Ext_\delta(A) = int_\delta(A^c) = X - cl_\delta(A)$.
(6) $int_\delta(A) \subseteq cl_\delta(A)$.

Proposition 2.5

For Sub Set A of Space X ,the following Statements hold:-

(1) Suppose that p$\notin$ A in Space X. Then p is not δ −limit point of A if and only if There exist an

δ −open Set U with p $\in$ U *and* $U \cap A = \emptyset$.
(2) $D_\delta(A) \subseteq cl_\delta(A)$.
(3) If A singleton δ −closed not regular closed, then $D_\delta(A) = \emptyset$.
(4) A is $\delta -$ closed if and only if the $D_\delta(A) \subseteq A$.
(5) A is $\delta -$ open if and only if the $D_\delta(A^c) \cap A = \emptyset$.
(6) if $cl_\delta(D_\delta(A))$ is nowhere dense ,then $D_\delta(A)$ is nowhere dense .
(7) $A \subseteq A \cup D_\delta(A)$.

Proof

(1) Clearly .

(2) let $x \in cl_\delta(A)$,then for every regular open Sub Set U of X containing x such that

$U \cap A \neq \emptyset$, by Proposition 2.3 Part (2) U is δ −open ,thus $U \cap (A - \{x\}) = \emptyset$ for every δ −open
Sub Set containing x ,then $x \notin D_\delta(A)$.

(3) Suppose that A is singleton δ −closed not regular closed, and let $x \in D_\delta(A)$ then there exist $\delta -$open Set U containing x such that $U \cap (A - \{x\}) \neq \emptyset$ this means that there is point p such that $p \in U \cap (A - \{x\})$, and different from x ,so $p \in D_\delta(A)$ and A is not singleton δ −closed .but this is contract that A is singleton $\delta -$ closed .Therefore $D_\delta(A)$ must be empty Set .

(4) ⇒: Suppose A be δ −closed,let x is a$\delta -$ limit point of A , if $x \notin A$,then $x \in A^c$,since A^c δ −open and it is not contain any point from A implies the existence of an δ −open Set U containing x such that
$U \subset A^c$,hence $U \cap A = \emptyset$ so $x \notin D_\delta(A)$, this contradicted the fact that x is a$\delta -$ limit of A ,therefore $x \in A$ and A contains all its $\delta -$ limit points .

⇐ : assume that A contains all its $\delta -$ limit points , then no point of A^c can be δ −limit point of A , that is for each point of A^c there must exist δ −open sub Set U containing x such that $U \subseteq A^c$,thus
$U \cap A = \emptyset$ it follows from this A^c is δ −open .Therefore A is δ −closed .

(5) ⇒: Suppose A be δ −open and $x \notin D_\delta(A)$, if $x \notin A$,so $x \in A^c$,since A^c is $\delta -$ closed ,by Part (4)
$D_\delta(A^c) \subseteq A^c$,thus $D_\delta(A^c) \cap A^c \neq \emptyset$,so $D_\delta(A^c) \cap A = \emptyset$ that is all the $\delta -$ limit point of A^c is not
$\delta -$ limit point of A .
⇐: Let $x \in D_\delta(A^c)$ and $x \notin A$,then $x \in A^c$, $D_\delta(A^c) \cap A^c \neq \emptyset$, by Part (4) since $D_\delta(A^c) \subseteq A^c$,then A^c
is δ −closed ,therefore A is δ −open .

(6) let $cl_\delta(D_\delta(A))$ is nowhere dense so $int_\delta(cl_\delta(cl_\delta(D_\delta(A)))) = \emptyset$,by Part (2) $D_\delta(A) \subseteq cl_\delta(D_\delta(A))$,so by Proposition2.4 Part (3) and (4) $int_\delta(cl_\delta(D_\delta(A))) \subseteq int_\delta(cl_\delta(cl_\delta(D_\delta(A)))) = \emptyset$,therefore $D_\delta(A)$ must be nowhere dense Set .

(7) clearly .

Proposition 2.6 [14]

For Sub Set A of Space X , $D_P(A) \subseteq D_\alpha(A)$.

Proposition 2.7

For Sub Set A of Space X ,the following Statements hold:-

(1) $D(A) \subseteq D_\delta(A)$.
(2) $D_\alpha(A) \subseteq D_\delta(A)$.
(3) $D_S(A) \subseteq D_\delta(A)$.
(4) $D_S(A) \subseteq D_\alpha(A)$.
(5) $D_P(A) \subseteq D_\delta(A)$.

Proof

Clearly by proposition 2.3 .

Definition 2.8

Sub Set A of Space X is δ −dense if $cl_\delta(A)$ contains all δ −adherent points of X .or equivalently if every δ −open sub Set of X contains point of A

Proposition 2.9

For Sub Sets A, B of Space X ,the following properties are equivalent:-

(1) A is $\delta-$dense in X .
(2) $cl_\delta(A) = X$.
(3) if B is any $\delta-$closed Sub Set of X ,and $A \subseteq B$,then $B = X$.
(4) for $x \in X$ and $U \subseteq X$,for every $\delta-$open Sub Set U containing x , $U \cap A \neq \emptyset$.
(5) $int_\delta(A^c) = \emptyset$.

Proof

1⇒2 :since $cl_\delta(A) = \{x \in X / U \cap A \neq \emptyset, for\ every\ U\ is\ regular\ open\ and\ x \in U\}$ is the Set of all $\delta-$adherent points of A in X and since A is $\delta-$dense in X ,so by Definition 2.8 $cl_\delta(A) = X$.

2⇒3 : since $A \subseteq B$, then $cl_\delta(A) \subseteq cl_\delta(B)$ Proposition 2.4 part(3),from part(2) $cl_\delta(A) = X$. Thus
$X \subseteq cl_\delta(B)$ and since B is δ-closed so $X \subseteq cl_\delta(B) = B$, $X \subseteq B$…………..(a)
Since $cl_\delta(B) \subseteq X$ so $B = cl_\delta(B) \subseteq X$ thus $B \subseteq X$ ………(b) ,from (a) and (b) we have $B = X$.

3⇒4 : Let U is $\delta-$open and $U \neq \emptyset$,so $U \cap A = \emptyset$,thus $A \cap U^c \neq \emptyset$, hence $U^c \neq \emptyset$ and $A \subseteq U^c$
,but this contradiction that part(3) since U^c is $\delta-$closed ,so $U \cap A \neq \emptyset$.

4⇒5 : Let $int_\delta(A^c) \neq \emptyset$,since $int_\delta(A^c)$ is $\delta-$open and non-empty, then there is an regular open sub Set U containing x such that U is $\delta-$open by Proposition 2.3 Part(2) and $U \subset int_\delta(A^c)$,since $int_\delta(A^c) = A^c$ so $U \subset A^c$ that is U has empty intersection with A . But this contradiction part(4) .Thus $int_\delta(A^c) = \emptyset$.

5⇒1 : By proposition 2.4 part(5), $int_\delta(A^c) = X - cl_\delta(A)$, since $int_\delta(A^c) = \emptyset$,

So $\emptyset = X - cl_\delta(A) = X \cap (cl_\delta(A))^c = X \cap int_\delta(A^c) = int_\delta(A^c)$,therefore $(int_\delta(A^c))^c = \emptyset^c$

,so $cl_\delta(A) = X$ .Thus by definition 2.8 A is $\delta-$dense Set in X .

Proposition 2.10

For Sub Sets A,B of Space X ,the following properties are true:-

(1) $int_\delta(A) \cup int_\delta(B) \subseteq int_\delta(A \cup B)$
(2) $int_\delta(A \cap B) = int_\delta(A) \cap int_\delta(B)$
(3) $cl_\delta(A \cap B) \subseteq cl_\delta(A) \cap cl_\delta(B)$.
(4) $cl_\delta(A \cup B) = cl_\delta(A) \cup cl_\delta(B)$.
(5) $cl_\delta(cl_\delta(A)) = cl_\delta(A)$.

Proof

(1) clearly .
(2) clearly .
(3) clearly .
(4) clearly .
(5) since$A \subseteq cl_\delta(A)$,so by Proposition 2.4 Part (3) we have $cl_\delta(A) \subseteq cl_\delta(cl_\delta(A))$……….(a)
Let $x \in cl_\delta\,(cl_\delta(A))$, so there is regular open Sub Set U of Xcontaining x such that $U \cap cl_\delta(A) \neq \emptyset$, $x \in cl_\delta\,(A)$, there is regular open Sub Set $V = U$ containing x such that
$x \in U \cap A \neq \emptyset$Therefore $cl_\delta(cl_\delta(A)) \subseteq cl_\delta(A)$……….(b),from (a) and (b) we have $cl_\delta\big(cl_\delta(A)\big) = cl_\delta(A)$.

Proposition 2.11

For Sub Sets A, B of Space X ,the following properties are true:-

(1) $D_\delta(A) \cup D_\delta(B) \subseteq D_\delta(A \cup B)$.

(2) $D_\delta(A \cap B) \subseteq D_\delta(A) \cap D_\delta(B)$.
(3) $cl_\delta(D_\delta(A)) \subseteq cl_\delta(A)$
(4) $D_\delta\big(D_\delta(A)\big)/A \subseteq D_\delta(A)$.
(5) If $A \subseteq B$, then $D_\delta(A) \subseteq D_\delta(B)$.

Proof

(1) clearly by definition 2.2 .
(2) clearly by definition 2.2 .
(3) By Proposition 2.5 part(2) $D_\delta(A) \subseteq cl_\delta(A)$, $cl_\delta(D_\delta(A) \subseteq cl_\delta\big(cl_\delta(A)\big) = cl_\delta(A)$this by
Proposition 2.10 Part (5) .Thus $cl_\delta(D_\delta(A) \subseteq cl_\delta(A)$.

(4) Le $x \in D_\delta(D_\delta(A))/A$,and let U be δ $-$open Sub Set containing x such that $U \cap (D_\delta(A) - \{x\}) \neq \emptyset$
,let $y \in U \cap (D_\delta(A) - \{x\}) \neq \emptyset$,since $y \in D_\delta(A)$ and $y \in U$ so $U \cap (A - \{y\}) \neq \emptyset$, let
$z \in U \cap (A - \{y\})$, then $z \neq x$ and $U \cap (A - \{x\} \neq \emptyset$. Therefore $x \in D_\delta(A)$.

(5) clearly .

Example 2.12
Let $X = \{a, b, c, d\}$, $T_X = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\ \ \}$ be topology defined on X .Let $A = \{c, d\} \subset X$

$B = \{b\} \subset X$ note that $D_\delta(A \cup B) = D_\delta\{b, c, d\} = \{b, c, d\} \not\subset D_\delta(A) = \{b, d\} \cup D_\delta(B) = \{b, d\} = \{b, d\}$,

Also if $C = \{b, c\} \subset X, D = \{a\} \subset X$ note that $D_\delta(C) = \{b, c, d\} \cap D_\delta(D) = \{d\} \not\subset$
$D_\delta(A \cap B) = D_\delta(\emptyset) = \emptyset$.

3- δ –isolated points and some relations

In this section we introduce the concept of δ −isolated point ,also we gives some results and some relations about this Points with the other Points as we will shown in diagram (1) .

Definition 3.1
Let A be Sub Set of Space X , point $x \in A$. is called δ −isolated point of A if $x \notin D_\delta(A)$.or equivalently

Apoint $x \in A$ is an δ −isolated point of A if there is δ −open sub Set of X containing x intersect A only in $\{x\}$. The Set of all δ −isolated points will denoted by $\delta I(A)$.

Example 3.2
Let $X = \{a, b, c\}$, $T_X = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ } be topology defined on X ,note that $x = a$ is

δ −isolated point of $A = \{a, c\} \subseteq X$, since $x = a \in A - D_\delta(A) = \{a\}$,that is $\delta I(A) = \{a\}$.

Proposition 3.3
For Sub Set A of Space X ,the following properties are true:-
(1) No δ −isolated point is δ −limit point of any Set A .

(2) If A is open or dense then $x \in$ A is an $\delta -$ isolated point of A if and only if $\{x\}$ is δ −open sub

Set in A .

Proof

(1) If x is δ – isolated point, then the Set $\{x\}$ is δ −open sub Set containing x ,that contains no point other then x ,so $\{x\} \cap (\{x\} - \{x\}) = \{x\} \cap \emptyset = \emptyset$, thus x is not δ −limit point .

(2) ⇒: $x \in$ A is an $\delta -$ isolated point of A, by definition 3.1 there is δ −open sub Set U containing x such

That $U \cap A = \{x\}$,since U is δ −open in A and A is open or dense in X ,so by Proposition 3.12 [16] $U \cap A$ is δ −open in A .Thus $\{x\}$ is δ −open sub Set in A .

⇐:Let $\{x\}$ is δ −open sub Set in A ,meaning $U \cap A = \{x\}$ is δ −open in A by Proposition 3.12 [16]

$U \cap A = \{x\}$ is δ −open sub Set in A where U is δ −open in A and A is open or dense in X, by definition3.1 $x \in$ A is an $\delta -$ isolated point of A .

Proposition 3.4

For Sub Set A of Space X ,the following properties are true:-

(1) $\delta I(A) \subseteq A$
(2) $int_{\delta}(\delta I(A)) \subseteq A$
(3) $\delta I(A) \subseteq cl_{\delta}(A)$
(4) $\delta I(A) \subseteq cl_{\delta}(\delta I(A)) \subseteq cl_{\delta}(A)$
(5) $D_{\delta}(A) \cap \delta I(A) = \emptyset$
(6) $cl_{\delta}(A) \cap \delta I(A) \subseteq A$
(7) $int_{\delta}(A) \subseteq A \cup D_{\delta}(A)$
(8) $A \cup \delta I(A) = A$
(9) $D_{\delta}(A \cup D_{\delta}(A)) \subseteq A \cup D_{\delta}(A)$
(10) $cl_{\delta}(A) \cup \delta I(A) = cl_{\delta}(A)$

Proof

(1) Let $x \in \delta I(A)$,so $x \notin D_{\delta}(A)$, $x \in A - D_{\delta}(A)$,therefore $x \in A$ .Thus $\delta I(A) \subseteq A$.

(2) From part(1) $\delta I(A) \subseteq A$,so by proposition 2.4 part(4) $int_{\delta}(\delta I(A)) \subseteq int_{\delta}(A)$
Thus from Part (1) from this proposition $int_{\delta}(\delta I(A)) \subseteq A$.

(3) By part (1) $\delta I(A) \subseteq A$,also by Proposition 2.4 part (2) $A \subseteq cl_{\delta}(A)$ .Therefore $\delta I(A) \subseteq cl_{\delta}(A)$.

(4) By proposition 2.4 part(2) $\delta I(A) \subseteq cl_{\delta}(\delta I(A))$……..(a) From part(1) $\delta I(A) \subseteq A$,by
proposition 2.4 part(3) we have $cl_{\delta}(\delta I(A)) \subseteq cl_{\delta}(A)$ ……..(b),also by part(3) $\delta I(A) \subseteq cl_{\delta}(A)$ ……(c)
from (a),(b) and (c) we have $\delta I(A) \subseteq cl_{\delta}(\delta I(A)) \subseteq cl_{\delta}(A)$.

(5) For $x \in \delta I(A)$ so for every $\delta -$open Sub Set U of X containing x we have $U \cap (A - \{x\} = \emptyset$, thus $x \notin D_{\delta}(A)$,so $D_{\delta}(A) \cap \delta I(A) = \emptyset$ or directly by proposition 3.3 .

(6) from part(3) $\delta I(A) \subseteq cl_{\delta}(A)$, so $\delta I(A) \cap cl_{\delta}(A) = \delta I(A)$ and from part(1)
We have $cl_{\delta}(A) \cap \delta I(A) \subseteq A$.

(7) from proposition 2.4 part(6) $int_{\delta}(A) \subseteq cl_{\delta}(A)$
and from part(1) from this proposition $D_{\delta}(A) \cup A = cl_{\delta}(A)$ so we have $int_{\delta}(A) \subseteq A \cup D_{\delta}(A)$.

(8) By part(1) $\delta I(A) \subseteq A$,so $A \cup \delta I(A) \subseteq A \cup A = A$, $A \cup \delta I(A) \subseteq A$ ……(a) ,if for every $x \in A$ then $x \in \delta I(A)$,thus $A \subseteq \delta I(A)$ therefore $A = A \cup A \subseteq A \cup \delta I(A)$ ……..(b) from (a) and (b) we get $A \cup \delta I(A) = A$.

(9) Let x be point , either $x \in A$ and $x \notin D_{\delta}(A)$ or $x \notin A$ and $x \in D_{\delta}(A)$, let $x \in D_{\delta}(A \cup D_{\delta}(A))$,if

$x \in A$ and $x \notin D_\delta(A)$,so $x \in A \cup D_\delta(A)$ and $D_\delta(A \cup D_\delta(A)) \subseteq A \cup D_\delta(A)$. or $x \notin A$ and
$x \in D_\delta(A)$, so there is δ −open Set U containing x such that $U \cap ((A \cup D_\delta(A)) - \{x\}) \neq \emptyset$ so
$U \cap (A - \{x\}) \neq \emptyset$ or $U \cap ((D_\delta(A)) - \{x\}) \neq \emptyset$ thus $x \in D_\delta(A)$ or $x \in D_\delta(D_\delta(A))$ and $x \notin A$.
So $x \in D_\delta\big(D_\delta(A)\big)/A \subseteq D_\delta(A)$ by Proposition 2.11 Part (4), so $x \in A \cup D_\delta(A)$.
Therefore $D_\delta(A \cup D_\delta(A)) \subseteq A \cup D_\delta(A)$

(10) By part (3) $\delta I(A) \subseteq cl_\delta(A)$,since $cl_\delta(A) \cup \delta I(A) \subseteq cl_\delta(A) \cup cl_\delta(A) = cl_\delta(A)$……………(a)
Let $x \in cl_\delta(A)$,then there is regular open Set U containing x such that $U \cap A \neq \emptyset$,so either
$U \cap (A - \{x\}) \neq \emptyset$ or $U \cap (A - \{x\}) = \emptyset$ where U is δ −open by Proposition 2.3 part (2) ,either $x \in D_\delta(A)$ or $x \notin D_\delta(A)$, if $x \in D_\delta(A)$,then
$x \in cl_\delta(A)$ by Proposition 2.5 Part (2) or $x \in A - D_\delta(A)$,thus either $x \in cl_\delta(A)$ or $x \in \delta I(A)$,
, $x \in cl_\delta(A) \cup \delta I(A)$, therefore $cl_\delta(A) \subseteq cl_\delta(A) \cup \delta I(A)$ ………………..(b)
From (a) and (b) we get $cl_\delta(A) \cup \delta I(A) = cl_\delta(A)$.

Proposition 3.5

For Sub Sets A,B of Space X ,the following properties are true:-
(1) $\delta I(A \cup B) \subseteq \delta I(A) \cup \delta I(B)$.
(2) $\delta I(A) \cap \delta I(B) \subseteq \delta I(A \cap B)$.

Proof

(1) $x \in \delta I(A \cup B)$,$x \notin D_\delta(A \cup B)$ by proposition 2.11 part(1) ,so $x \notin D_\delta(A)$ or $x \notin D_\delta(B)$
Therefore $x \in \delta I(A)$ or $x \in \delta I(B)$ so $x \in \delta I(A) \cup \delta I(B)$, $\delta I(A \cup B) \subseteq \delta I(A) \cup \delta I(B)$.

(2) similarly the proof of part(1) .

Examples 3.6

(1) Let $X = \{a, b, c, d\}$, $T_X = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$ let $A = \{b\} \subset X$ and
$B = \{d\} \subset X$, note that $\delta I(A) \cup \delta I(B) = \{b, d\} \not\subset \delta I(A \cup B) = \{b\}$ since $\delta I(A) = \{b\}$ and
$\delta I(B) = \{d\}$.
(2) Let $X = \{a, b, c, d\}$, $T_X = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$ let $A = \{a, b, c\} \subseteq X$ and $B = \{a, b, d\} \subseteq X$,
note that $\delta I(A \cap B) = \{a, b\} \not\subset \delta I(A) \cap \delta I(B) = \{a\}$ since $\delta I(A) = \{a\}$ and $\delta I(B) = \{a, b\}$.

Proposition 3.7

For Sub Set A of Space X ,the following properties are true:-

(1) $cl_\delta(A) = D_\delta(A)) \cup \delta I(A)$.

(2) $X = D_\delta(A) \cup \delta I(A) \cup Ext_\delta(A)$.

Proof

(1) $x \in cl_\delta(A)$,so $U \cap A \neq \emptyset$,for every regular open Set U containing x, so if $x \notin A$ And $U \cap (A - \{x\}) \neq \emptyset$,or if $x \in A$, and $U \cap (A - \{x\}) = \emptyset$ where U is δ −open by Proposition 2.3 part (2) ,thus $x \in D_\delta(A)$ or $x \in \delta I(A)$,so $x \in D_\delta(A) \cup \delta I(A)$, therefore

$cl_\delta(A) \subseteq D_\delta(A) \cup \delta I(A)$ ……(a)

Since $D_\delta(A) \cap \delta I(A) = \emptyset$ so $D_\delta(A) \cup \delta I(A) \neq \emptyset$,since by proposition 2.5 part (2) $D_\delta(A) \subseteq cl_\delta(A)$, $D_\delta(A) \cup \delta I(A) \subseteq cl_\delta(A) \cup \delta I(A) = cl_\delta(A)$ this by proposition 3.4 part(10)

So $D_\delta(A) \cup \delta I(A) \subseteq cl_\delta(A)$ ………..(b) ,from (a) and (b) we get $cl_\delta(A) = D_\delta(A) \cup \delta I(A)$.

(2) let $p \in D_\delta(A) \cup \delta I(A) \cup Ext_\delta(A)$, $p \in (D_\delta(A) \cup \delta I(A))$ or $p \in Ext_\delta(A)$, by part(1)

$p \in cl_\delta(A)$ or $p \in Ext_\delta(A)$,then either $U \cap A \neq \emptyset$ for every regular open Set U containing p

Or $p \in int_\delta(X - A) = X - cl_\delta(A)$ by proposition 2.4 part(5) , thus either $p \in U \cap A \subseteq X$

Or $p \in X - cl_\delta(A)$,so either $p \in cl_\delta(A) \subseteq X$ or $p \in X - cl_\delta(A) \subseteq X$, since $cl_\delta(A) = D_\delta(A) \cup \delta I(A)$

So $p \in (D_\delta(A) \cup \delta I(A)) \subseteq X$ or $p \in Ext_\delta(A) \subseteq X$,Thus $D_\delta(A) \cup \delta I(A) \cup Ext_\delta(A) \subseteq X$ ……….(a)

Let $p \in X$, and $A \subseteq X$,either if $p \in A$ thus $p \in cl_\delta(A)$ or $p \notin A$ and $p \notin cl_\delta(A)$ thus $p \in X - cl_\delta(A)$,so either $p \in cl_\delta(A)$ or by proposition 2.4 part (5) $p \in Ext_\delta(A)$, thus $p \in cl_\delta(A) \cup Ext_\delta(A)$,

therefore $X \subseteq cl_\delta(A) \cup Ext_\delta(A)$,by part (1)

$X \subseteq D_\delta(A) \cup \delta I(A) \cup Ext_\delta(A)$ ………(b),from (a) and (b) we get $X = D_\delta(A) \cup \delta I(A) \cup Ext_\delta(A)$.

Definitions 3.8

Sub Set A of Space X ,is called :

(1) $\delta -$ perfect if it is δ −closed and $\delta -$ crowded .

(2) δ −nowhere dense if $int_\delta\big(cl_\delta(A)\big) = \emptyset$.

Proposition 3.9

Every δ −dense has an $\delta -$ isolated point .

Proof

Let $x \in A$,since A δ −dense Sub Set so by definition 2.8 for every δ −open Sub Set U of X contain

point of A ,that is $U \cap A = \{x\} \neq \emptyset$,so $U \cap (A - \{x\}) = \emptyset$,thus $x \notin D_\delta(A)$ .Therefore $x \in \delta I(A)$.

Proposition 3.10

If Sub Set A of Space X is δ −closed ,then A is δ −nowhere dense if and only if it is nowhere dense .

Proof

⇒: Let A is δ-closed ,so by Proposition 2.3 Part (1) A is closed, then $A = cl(A) = cl_\delta(A)$, since
$int\big(cl(A)\big) = int_\delta\big(cl_\delta(A)\big) = \emptyset$,therefore $int\big(cl(A)\big) = \emptyset$ .Thus A is nowhere dense .

⇐: Let A is δ-closed and A is nowhere dense ,also by Proposition 2.3 Part (1) A is closed, then
$A = cl(A) = cl_\delta(A)$, since $\emptyset = int\big(cl(A)\big) = int_\delta\big(cl_\delta(A)\big)$,therefore
$int_\delta\big(cl_\delta(A)\big) = \emptyset$.
Thus A is $\delta -$ nowhere dense .

Remark 3.11

In the above proposition if A is closed and nowhere dense, then it is not necessarily that A is δ −nowhere dense ,see the following Example .

Example 3.12

Let $X = \{a, b, c\}$, $T_X = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ let $A = \{c\} \subset X$ is closed, note that A is nowhere dense ,that is $int\big(cl(A)\big) = \emptyset$,but $int_\delta\big(cl_\delta(A)\big) = int_\delta(cl_\delta\{c\} = \{a, c\}) = \{a, c\}$
$\emptyset \subseteq int_\delta\big(cl_\delta(A)\big)$,but $int_\delta\big(cl_\delta(A)\big) \neq \emptyset$.

Proposition 3.13

If $A \subseteq X$ is δ −closed and has no δ −isolated points, then the following statements are hold:-

(1) $A \subseteq D_\delta(A)$.
(2) $cl_\delta(A)$ is δ −perfect .

Proof

(1) clearly ,since $\delta I(A) = \emptyset$.
(2) to prove that $cl_\delta(A)$ is δ −perfect must prove that $cl_\delta(A)$ is δ −closed and δ −crowded , Let $x \in A$ since $\delta I(A) = \emptyset$, so $x \in D_\delta(A)$,since A is δ −closed by proposition 2.5 part(2) we get

$x \in D_\delta(A) \subseteq cl_\delta(A)$, that is $x \in cl_\delta(A)$ and x is δ −limit point not δ −isolated point , so $cl_\delta(A)$

has no δ −isolated points ,since A is δ −closed ,so $cl_\delta(A) = A$,thus $cl_\delta(A)$ is $\delta -$ closed and δ −crowded since it does not have any δ −isolated point .

Proposition 3.14

If A is δ −closed and has no δ −isolated point, then $int_\delta(A)$ has no δ −isolated point .

Proof

By proposition 2.4 part(6) $int_{\delta}(A) \subseteq cl_{\delta}(A)$,since A is δ −closed so $int_{\delta}(A) \subseteq cl_{\delta}(A) = A$, since A has no δ −isolated point , so $int_{\delta}(A)$ has no δ −isolated point .

Definition 3.15

Let A be Sub Set of Space X , point $x \in A$ is called α −isolated(resp. Semi− isolated ,Pre− isolated) point of A if $x \notin D_{\alpha}(A)$ (resp. $x \notin D_S(A)$, $x \notin D_P(A)$).The Set of all α −isolated ,Semi−isolated ,
Pre−isolated points Will denoted by $\alpha I(A)$, $SI(A), PI(A)$ respectively .

Proposition 3.16

Every δ −isolated point is isolated(resp. α −isolated, Semi−isolated ,Pre−isolated) point .

Proof

By definition 3.1 and proposition 2.3 .

Remark 3.17

The converse of the above proposition is not true in general, see the following Example .

Example 3.18

Let $X = \{a, b, c\}$, $T_X = \{\emptyset, X, \{b\}, \{b, c\}\}$ let $A = \{b, c\} \subseteq X$ is semi-open, α −open and open note that $D(A) = D_{\alpha}(A) = D_S(A) = D_P(A) = \{a, c\}$ and I(A)=$\alpha I(A) = SI(A) = PI(A) = \{b\}$ but $x = b$ is not
$\delta -$ isolated of A ,since $\delta I(A) = \emptyset$.

Proposition 3.19

Every α −isolated point is Semi-isolated (resp. Pre−isolated) .

Proof

By definition 3.15 and proposition 2.3part(3) .

Remark 3.20

The converse of the above proposition is not true in general, see the following Example .

Examples 3.21

(1) Let $X = \{a, b, c, d\}$, $T_X = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ let $A = \{a, c, d\} \subseteq X$, note that $\alpha I(A) = \{a\}$, $SI(A) = \{a, c, d\}$ so $x = c$ is Semi-isolated point but not α −isolated.

(2) $X = \{a, b, c, d\}$, $T_X = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$ let $A = \{b, d\} \subset X$, note that $PI(A) = \{b, d\}$, $\alpha I(A) = \{b\}$ so $x = d$ is Pre−isolated point but not α −isolated of A.

Proposition 3.22

Every isolated point is Semi-isolated [12] (resp. α −isolated , Pre−isolated) point .

Proof

By definition 3.15 and proposition 2.3 part(5) .

Remark 3.23

The converse of the above proposition is not true in general, see the following Examples .

Examples 3.24

(1) Note that in Example (2) in [4] the Sub Set $A = [0,1] \times \{0\} \in R^2$ have α − isolated but not isolated point .

(2) in Example 3.21( ) if $A = \{b, c\} \subset X$ note that $x = c$ is Semi-isolated but not isolated point .

(3) in Example 3.21(2) if $A = \{a, b, c\} \subseteq X$ note that $PI(A) = \{a, b, c\}$ and $I(A) = \{a\}$,so $x = c$

Is Pre−isolated but not isolated point of A.

The following diagram shows the relations among these type of Points .

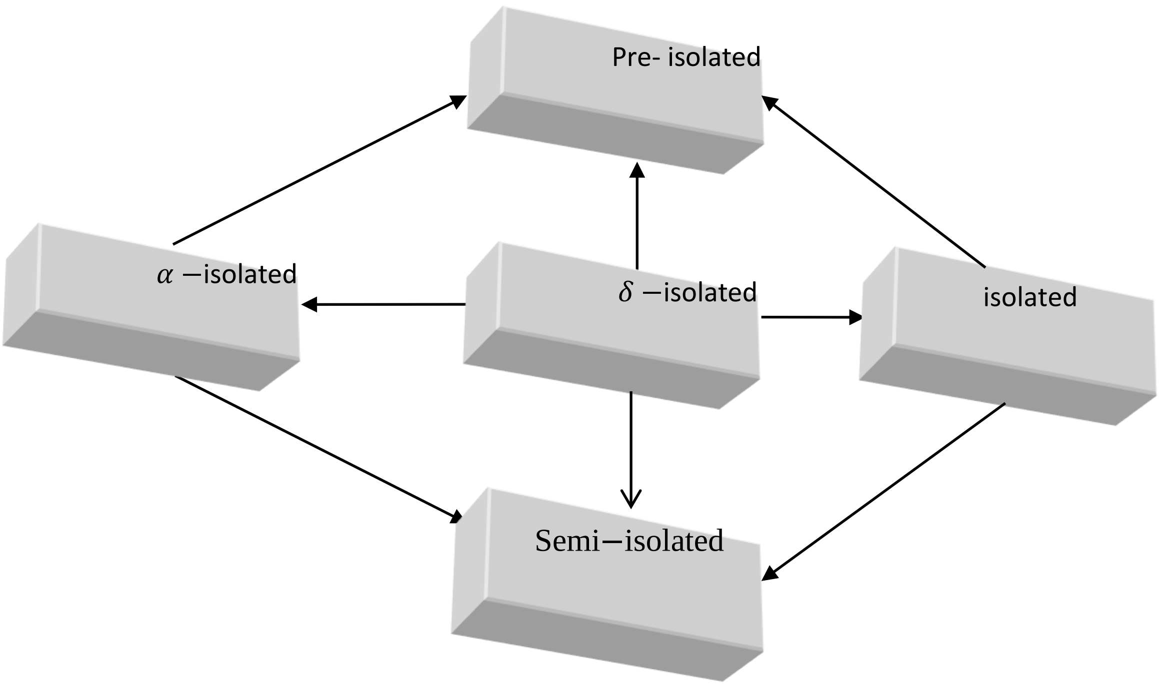

Diagram (1)

4- Some properties of δ −Scattered Sets

In this section we introduce the concept of δ −Scattered Set and some properties of this Sets ,as well as we study the relation between this Set with the other Scattered Sets as we will shown in diagram (2).

Definition 4.1

Sub Set A of Space X is called δ −Scattered if it have at least one δ −isolated point .

Example 4.2

Let $X = \{a, b, c, d\}$, $T_X = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$ let $A = \{a, d\} \subset X$, note that A is δ −Scattered ,

,since $D_\delta(A) = \{d\}$ and $\delta I(A) = \{a\}$.

Remarks 4.3

(1) The intersection of two δ −Scattered Sets is not necessarily δ −Scattered .

(2) The union of two δ −Scattered Sets is also δ −Scattered .

(3) δ −Scattered Set is an δ −open hereditarily property .

Examples 4.4

(1) Let $X = \{a, b, c, d\}$ and $T_X = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}\}$ be topology defined on X ,note that $A = \{a, b, d\} \subseteq X$ is δ −Scattered Set, since $\delta I(A) = \{a\}$,let $B = \{b, c, d\} \subseteq X$ is also δ −Scattered Set ,since $\delta I(B) = \{c\}$, but $A \cap B = \{b, d\} \subset X$ is not δ− Scattered, since $\delta I(A \cap B) = \delta I\{b, d\} = \emptyset$.

(2) Let $X = \{a, b, c\}$, $T_X = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ let be topology defined on X ,note that

$A = \{b, c\} \subseteq X$ is δ −closed and δ −Scattered Set, $T_A = \{\emptyset, A, \{b\}\}$,but $A = \{b, c\}$ is not

δ −Scattered in T_A .

Proposition 4.5

For Sub Set A of Space X , then the following statements are hold:-

(1) If A is nonempty δ −open and $\delta I(A) = A$,then A is δ − Scattered Set if and only if $A \cap D_\delta(A) = \emptyset$

(2) If A is singleton regular open(resp. singleton regular clopen) then it is $\delta-$ Scattered .

Proof

(1) $\Rightarrow$: Let A be $\delta-$ Scattered and let $x \in A$,since $\delta I(A)=A$, thus $x \notin D_{\delta}(A)$,therefore $D_{\delta}(A)=\emptyset$,

This means that every point of A is not $\delta-$limit point, by Proposition 3.4 Part (5) $\delta I(A) \cap D_{\delta}(A)=A \cap D_{\delta}(A)=\emptyset$ .Therefore $A \cap D_{\delta}(A)=\emptyset$.

$\Leftarrow$:Let $A \cap D_{\delta}(A)=\emptyset$ and since A is nonempty $\delta-$open ,so $A \neq \emptyset$,since $\delta I(A)=A$,thus

$\delta I(A) \neq \emptyset$ therefore $D_{\delta}(A)=\emptyset$,so every point of A is not $\delta-$limit point, therefore A has $\delta-$isolated points .Thus A is $\delta-$ Scattered .

(2) By Proposition 2.3 Part (2) A is Singleton $\delta-$ open (resp. singleton $\delta-$open) ,let $x \in A$ so $A \cap (A-\{x\})=A \cap \emptyset=\emptyset$,therefore $x \notin D_{\delta}(A)$ that is $x \in \delta I(A)$ .Thus A is $\delta-$ Scattered .

Proposition 4.6

Let A, B are two $\delta-$Scattered Sets , if $A \subseteq B$ and A is not singleton $\delta-$closed then $\delta I(A) \subseteq \delta I(B)$.

Proof

Clearly .

Proposition 4.7

For Sub Set A of Space X ,the following statements are true :

(1) If A is Singleton $\delta-$closed and not singleton regular open, then it is $\delta-$nowhere dense and $\delta-$Scattered .

(2) A is $\delta-$nowhere dense and $D_{\delta}(A)=\emptyset$,then A is Singleton $\delta-$closed .

Proof

(1) Let A be Singleton $\delta-$closed , so by Proposition 2.3 Part (2) Singleton closed and not Singleton open

, Since $A=cl_{\delta}(A)$, therefore $int_{\delta}\left(cl_{\delta}(A)\right)=int_{\delta}(A)=\emptyset$, thus A is $\delta-$nowhere dense, since A is Singleton $\delta-$closed so by proposition 2.5 part(3) $D_{\delta}(A)=\emptyset$ .Therefore $\delta I(A)=A$ is $\delta-$Scattered .

(2) Let A is $\delta -$nowhere dense and $D_\delta(A) = \emptyset$,suppose that A is Singleton $\delta -$open, let $x \in A$ thus

$\delta I(A) = \{x\}$ means that there is $\delta -$open Sub Set U containing x only such that $U \cap (A - \{x\}) = \emptyset$,so

$U \cap A = \{x\}$,thus $U \subseteq A$, by Proposition 2.3 Part (2) U = {x} is singleton regular open and since A is $\delta -$open so $int(cl(U)) \subseteq int_\delta(cl_\delta(A))$, Since A is $\delta -$nowhere dense so $int(cl\{x\}) = \{x\} \subseteq \emptyset$ but

this contract that U is Singleton $\delta -$ open. Thus A is must be singleton $\delta -$ closed.

Proposition 4.8

For Sub Set A of Space X ,then $A \cup D_\delta(A) = cl_\delta(A)$ and $A \cap D_\delta(A) = \{a \in A : a\ is\ not\ \delta -$isolated point of A }
,the following statements are true :

(1) A is $\delta -$closed if and only if $D_\delta(A) \subseteq A$

(2) A has no $\delta -$isolated points if and only if A $\subseteq D_\delta(A)$) .
(3) A is $\delta -$open and δ-Scattered if and only if A $\cap D_\delta(A) = \emptyset$.
(4) if A is $\delta -$closed (not regular closed) and $\delta -$ Scattered if and only if $D_\delta(A) = \emptyset$.

Proof

Since by Proposition 2.5 part(2) $D_\delta(A) \subseteq cl_\delta(A)$ and by Proposition 2.4 part(1) we have

$A \cup D_\delta(A) \subseteq cl_\delta(A)$ and so all the points in $cl_\delta(A)$ are not in A are the $\delta -$limit points

by $\mathrm{cl}_\delta(\mathrm{A}) - \mathrm{A} = \{\mathrm{x} \in \mathrm{X} - \mathrm{A}: \mathrm{U} \cap \mathrm{A} \neq \emptyset$ for every regular open Set U containing x} Proposition 2.3

Part (2) $\mathrm{cl}_\delta(\mathrm{A}) - \mathrm{A} = \{\mathrm{x} \in \mathrm{X} - \mathrm{A}: \mathrm{U} \cap \mathrm{A} \neq \emptyset$ for every $\delta -$ open sub Set U containing x} $\subseteq$

$D_\delta(A) \subseteq cl_\delta$,so that $cl_\delta(A) = A \cup (cl_\delta(A) - A) \subseteq A \cup D_\delta(A)$,Thus $A \cup D_\delta(A) = cl_\delta(A)$,

from above that the Set of $\delta -$ isolated points in A , $A \cap D_\delta(A) = A - (A - D_\delta(A))$ is the Set of all non $\delta -$isolated points of A .Thus A has no $\delta -$isolated points .

If A $\cap D_\delta(A) = \emptyset$,then all the points of A are $\delta -$ isolated ,so by proposition 4.5 part (1) A is $\delta -$open and
$\delta -$ Scattered , and if $D_\delta(A) = \emptyset$, so $\delta I(A) = A$ by Proposition 3.4 Part (5) $\delta I(A) \cap D_\delta(A) = \emptyset$,thus A $\cap D_\delta(A) = \emptyset$ .Therefore by proposition 3.3 part (1) thus A is $\delta -$ Scattered .

Remark 4.9

Any regular clopen and Crowded Set not Singleton is not $\delta-$ Scattered .see Examples 4.4(1),where $A=\{b,d\}\subset X$ is regular clopen and Crowded not Singleton ,thus it is not Singleton $\delta-$clopen Sub Set and its $\delta-$ Crowded not $\delta-$Scattered,since $D_{\delta}(A)=A$ and $\delta I(A)=\emptyset$.

Definition 4.10

Sub Set A of Space X is called:

(1) $\alpha-$ Scattered if it has an $\alpha-$isolated points .[4]

(2) Semi$-$ Scattered if it has Semi$-$isolated points .[12]

(3) Pre$-$Scattered if it has Pre$-$isolated points .

Proposition 4.11

Every $\delta-$Scattered Set is scattered(resp. $\alpha-$ Scattered, Semi$-$ Scattered , Pre$-$Scattered) Set .

Proof

By definitions 4.1 and 4.10 and proposition 3.16 .

Remark 4.12

The converse of the above proposition is not true in general ,see the following Example .

Examples 4.13

(1) Let $X=\{a,b,c\}$, $T_X=\{\emptyset,X,\{a\},\{a,b\}\}$ be topology defined on X ,note that $A=\{a,b\}\subseteq X$ is

$\alpha-$ Scattered, Semi$-$ Scattered , Pre$-$Scattered and Scattered but not $\delta-$Scattered Set, since $I(A)=\alpha I(A)=SI(A)=\{b\}$ and $x=b$ is not $\delta-$ isolated point of A .

(2) Let $X=\{a,b,c\}$, $T_X=\{\emptyset,X,\{a\}\}$ be topology defined on X ,note that $A=\{a,c\}\subseteq X$ is

Scattered but not $\delta-$Scattered Set .

Proposition 4.14

Every Scattered Set is $\alpha-$ Scattered (resp. Semi$-$ Scattered , Pre$-$Scattered) Set .

Proof

By definition 4.10 and proposition 3.22 .

Remark 4.15

The converse of the above proposition is not true in general ,see Examples 4.13(2),note that

$A = \{b, c\} \subseteq X$ is $\alpha -$ Scattered , Semi$-$ Scattered, Pre$-$ Scattered but not Scattered Set .

Proposition 4.16

Every $\alpha -$ Scattered Set is Semi$-$ Scattered(resp. Pre$-$ Scattered) Set .

Proof

By definition 4.10 and proposition 3.19 .

Remark 4.17

The converse of the above proposition is not true in general ,see the following Example .

Example 4.18

Consider the usual topology on R, let $A = [0,1] \subset R$,Sub Set $B = [1,2)$ of R is Semi-open but not $\alpha -$open and $A \cap B = \{1\}$ note that $1 \in A$ is Semi$-$isolated but not $\alpha -$ isolated .

Proposition 4.19

Every $\delta -$open (resp. $\delta -$closed) Sub Set not $\delta -$perfect is $\delta -$ Scattered Set .

Proof

Clearly .

Definition 4.20

Let A be Sub Set of Space X :

(1) $\delta -$kernel of A is denoted by $Ker_\delta(A) = \cap \{O \in T_\delta : A \subseteq O\}$.

(2) A is $\delta -$Crowded if it is contain no $\delta -$isolated point .

(3) The perfect $\delta -$kernel of A denoted by $PK_\delta(A)$ which is largest possible $\delta -$Crowded Sub Set

Contained in A .

(4) The Scattered $\delta -$ kernel of A is the Set $SK_\delta(A) = A - PK_\delta(A)$.

Example 4.21

Let $X = \{a, b, c, d\}$, $T_X = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$ be topology defined on X :

(1) let $A = \{b, c\} \subset X$ its δ −open and δ −Crowded since $\delta I(A) = \emptyset$,note that $Ker_\delta(A) = X \cap \{a, b, c\} = \{a, b, c\}$, $PK_\delta(A) = \{b, c\}$,so $SK_\delta(A) = \emptyset$.

(2) let $B = \{b, c, d\} \subseteq X$ δ −closed Set and $\delta I(B) = \emptyset$, $PK_\delta(B) = \{b, c\}$ so $SK_\delta(B) = B - PK_\delta(B) = \{d\}$.

Proposition 4.22

For Sub Set A of Space X ,the following are hold:

(1) $PK_\delta(A) \subseteq D_\delta(A)$ and $SK_\delta(A) \subseteq A$.

(2) $SK_\delta(A)$ is δ −Scattered Sub Set .

(3) $PK_\delta(A) \cap SK_\delta(A) = \emptyset$.

(4) If A is δ −closed,then $A = PK_\delta(A) \cup SK_\delta(A)$.

(5) If A is δ −closed and $\delta -$ Crowded , then $PK_\delta(A)$ is δ −perfect Sub Set of A

Proof

(1) Let $x \in PK_\delta(A)$ since $PK_\delta(A)$ is the largest possible δ −Crowded Set in A ,so $PK_\delta(A) \subseteq A$,

Let $x \in PK_\delta(A)$is $\delta-$Crowded and $x \in A$ then $x \notin \delta I(A)$,thus $x \in D_\delta(A)$. Therefore

$PK_\delta(A) \subseteq D_\delta(A)$.

(2) Clearly by definition (4.20) .

(3) Clearly by definition (4.20) .

(4) from part(3) $PK_\delta(A) \cap SK_\delta(A) = \emptyset$,so we get $PK_\delta(A) \subseteq D_\delta(A)$,by Preposition 2.5 Part (4) we

have $D_\delta(A) \subseteq A$ by part(1) we get $PK_\delta(A) \subseteq A$ and $SK_\delta(A) \subseteq A$ so $PK_\delta(A) \cup SK_\delta(A) \subseteq A$.….(a)

Suppose that $x \in A$,so either $x \in \delta I(A)$ or $x \in D_\delta(A)$,if $x \in \delta I(A)$ then $x \notin PK_\delta(A)$ and $x \in A - Pk_\delta(A) = Sk_\delta(A)$,thus $x \in SK_\delta(A)$ or $x \in D_\delta(A)$ $x \in PK_\delta(A)$ since A is δ −closed contain all $\delta -$ limit point, so $x \in PK_\delta(A) \cup SK_\delta(A)$ therefore $A \subseteq PK_\delta(A) \cup SK_\delta(A)$ ……….(b) ,

from (a) and (b) we get $A = PK_\delta(A) \cup SK_\delta(A)$.

(5) clearly By definition 4.20 .

Proposition 4.23

Let A be Sub Set of Space X ,then $\delta I(A) \subseteq SK_\delta(A) \subseteq cl_\delta(A)$.

Proof

Since $\delta I(A) = A \cap (D_\delta(A))^C \subseteq A \cap (PK_\delta(A))^C = SK_\delta(A)$ and $A \cap (cl_\delta(\delta I(A)))^C \subseteq A \cap (cl_\delta(A))^C$ is largest $\delta -$Crowded Set, then $SK_\delta(A) = A \cap (PK_\delta(A))^C$,and $A \cap (A \cap (cl_\delta(\delta I(A)))^C)^C \subseteq cl_\delta\big(\delta I(A)\big)$ .Thus $\delta I(A) \subseteq SK_\delta(A) \subseteq cl_\delta(A)$.

Proposition 4.24

For Sub Set A of Space X ,if A is dense not $\delta -$perfect then $SK_\delta(A)$ is $\delta -$open in A .

Proof

Since $SK_\delta(A) = A - PK_\delta(A) = A \cap (PK_\delta(A))^C$,since A is dense and not $\delta -$perfect , by proposition 4.22 part (5) $(PK_\delta(A))^C$ is $\delta -$open not $\delta -$perfect in X , since by proposition 3.12 [16]

$A \cap (PK_\delta(A))^C$ is $\delta -$open in A .Thus $SK_\delta(A)$ is $\delta -$open .

The following diagram shows the relations among these types of Sets .

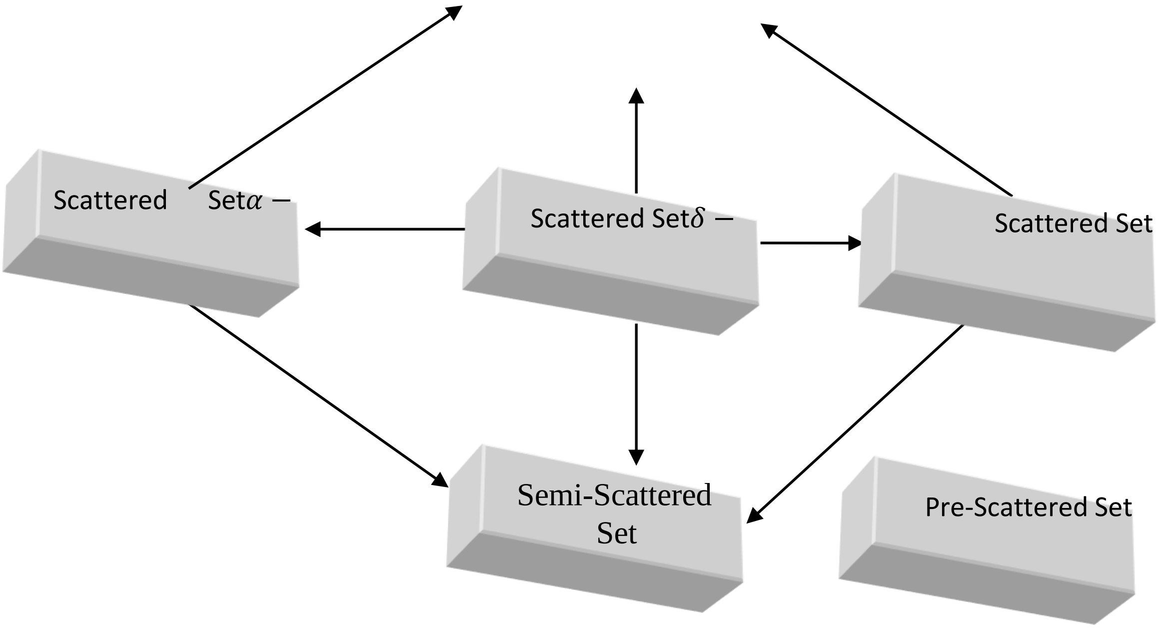

Diagram (2)

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الخلاصة

في هذا البحث استخدمنا مفهوم نقاط الغاية [2] ، المجموعة المشتقة [11]، المجموعة الموزعة [13]،المجموعة المفتوحة– δ لتعريف نوع جديد من النقاط هي نقطة العزل– δ وقدمنا بعض النتائج حول تلك النقطة ثم عرفنا المجموعة المشتقة– δ وقدمنا بعض النتائج عنها كتمهيد لأيجاد نوع جديد من المجموعات الموزعة هي المجموعة الموزعة– δ والتي تعتمد على امتلاك المجموعة للنقاط المعزولة– δ والتي بدورها تعتمد على المجموعة المشتقة– δ من حيث كون النقطة هي أحدى نقاط المجموعة المشتقة– δ ام ليست كذلك . درسنا العلاقة بين نقطة العزل– δ ونقاط العزل الاخرى كذلك العلاقة بين المجموعة الموزعة– δ والمجموعات الموزعة الاخرى

الكلمات المفتاحية: المجموعات – δ ، نقطة الغاية – δ ، المجموعة المشتقة– δ، نقطة العزل– δ، المجموعة الموزعة– δ، والمجموعات الموزعة .

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