SYNTHESIS AND OPTIMIZATION OF EPICYCLIC-TYPE AUTOMATIC TRANSMISSIONS BASED ON NOMOGRAPHS

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ABSTRACT

A new methodology is developed to design and optimize epicyclic-type automatic transmission gear trains using kinematic nomographs. From such nomographs, the kinematic characteristics of an epicyclic gear mechanism can be expressed in terms of the gear ratios of its gear pairs. The main properties of this methodology are that; from a single nomograph, the angular velocities for all of the coaxial links can be estimated and compared directly without specifying the exact size of each gear, the angular velocities can be arranged in a descending sequence without using complicated techniques, and all of the feasible clutching sequences can be enumerated directly. The optimization procedure to find the optimum gear ratios is applicable to any transmission mechanism composed of two or more fundamental gear entities (FGEs) depending on kinematic and geometric constraints.
The reliability of the methodology is established by applying it to the conventionally available three-velocity Simpson gear train for which optimal gear ratios are fully available. The theoretical results are in complete agreement with the practical applications.

Keywords: **Automatic transmission, epicyclic gear train, gear ratio, nomograph, and optimization**

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الموجز

في هذا البحث تم تطوير طريقه جديده للتصميم الامثل للمسلسلات الترسيه الكوكبيه لاليات نقل الحركه الاليه باستخدام المخططات النوموغرافيه الحركيه. ومن هكذا مخططات نوموغرافيه يمكن التعبير عن السمات الحركيه للاليات كوكبيه التروس بدلاله نسبة اسنان ازواج التروس المكونه لها. ان الخصائص الاساسيه للطريقه هي : انه يمكن تخمين ومقارنة السرع الدورانيه لجميع الاجزاء الدواره مباشرة من مخطط نوموغرافي واحد ودون تحديد عدد اسنان التروس ابتداءَ ، كما يمكن ترتيب تتابع تعشيق التروس من حيث السرعه الدورانيه الخارجه تنازليا دون استخدام تقنيات حل معقده وتعداد جميع تتابعات تعشيق التروس الممكنه عمليا مباشرة. ان الاجراءات المثلى للحصول على امثل النسب السرعيه قابله للتطبيق على اي اليه لنقل الحركه مكونه من وحدتين ترسيتين بنائيتين او اكثر واعتمادا على المحددات الحركيه والهندسيه.

وقد تم التحقق من الطريقه بتطبيقها على مجموعه سمبسن الترسيه ثلاثيه السرع والمتوفره صناعيا بكثره والمعروفه نسبها السرعيه المثلى وقد كانت النتائج النظريه مطابقه للتطبيقات العمليه .

1. INTRODUCTION

Most automatic transmission mechanisms employ epicyclic gear trains (EGTs) to achieve a set of desired velocity ratios. Figure 1 shows an EGT employing the Simpson gear set as the ratio-change gear train. This gear train with the above clutching sequence is the most popular gear train. It has been developed by nearly all automotive manufacturers as three-velocity automatic transmissions [**Tasi 2001**]. It can be found in Ford C3, Ford C5, Mercedes Benz, Toyota A40 and Nissan, to name a few.

In an EGT, the velocity ratio is defined as the ratio of the input shaft velocity to the output shaft velocity. Various velocity ratios are obtained by using clutches to connect various links to the input power source and to the casing of a transmission gearbox, respectively.

Typically, a rotating clutch is used for connecting two rotating links and a band clutch is used to fix a link to the casing. In **Figure 1** rotating and band clutches are denoted by C and B, respectively. Also it is always possible to achieve a direct drive by locking all the links in the EGT together such that they rotate as a single link. The velocity ratios selected for a transmission are tailored for vehicle performance and fuel economy. Typically, they include a first gear for starting, a second and/or third gear for passing, an overdrive for fuel economy at road speeds and a reverse. A table depicting a set of velocity ratios and their clutching conditions is called a clutching sequence. **Table 1** shows the clutching sequence of the transmission shown in **Figure 1**, where an X*i* indicates that the corresponding clutch is activated on the *i*th link for that gear.

For example, when the mechanism is in the first gear, the rotating clutch C1 and the band clutch B1 are activated. Hence, link 4 is connected to the input power and link 1 is fixed to the casing. In arranging a clutching sequence, it is highly desirable to achieve a single-shift transition, i.e., only one clutch is turned on while another is simultaneously turned off between two successive velocity ratios.

The gear ratios for the Ford C4 transmission as an example of a three-velocity automatic transmission that uses a Simpson gear train, are: 2.46:1 1st gear, 1.46:1 2nd gear, 1.00:1 3rd gear, and 2.20:1 reverse [11]. Ford Motor Company has five basic transmissions that use the Simpson gear train and are quite similar to each other: the C-3, C-4, C-5, C-6, and A4LD. The A4LD is a 4-speed, whereas the others are 3-speeds.

Most General Motors' transmission and transaxles are based on the Simpson gear train. The turbo hydramatic (THM) 375, 400, and 475 transmissions are heavy-duty transmissions generally used in full-sized rare wheel drive (RWD) cars and trucks. The 425 is based on the same design as the others but had been modified for use in forward wheel drive (FWD) vehicles with a longitudinally placed engine. The 3L80 (THM 400) was introduced in 1964 and has been used by all divisions of General Motors, as well as by many different manufacturers, such as Jaguar and Rolls-Royce.

The THM 250, 250C, 350, 350C, and 375B transmissions are 3-speed units. The 250 and 250C are light-duty, the 350 and 350C are medium-duty, and the 375B is a heavy-duty version of the 350 [12].

[**Mogalapalli 1993**] developed an interactive design system for the design of automatic automotive transmission gear trains. Mogalapalli's system can optimize the gear ratios using the Augmented Lagrangian Multiplier Method. In what follows an overview to the optimization theory is given [**Arora 2004**].

2. OPTIMIZATION THEORY OVERVIEW

Optimization techniques are used to find a set of design parameters, $x = \{x_1, x_2, ..., x_n\}$, that can in some way be defined as optimal [**Mogalapalli 1993**]. In a simple case this might be the minimization or maximization of some system characteristic that is dependent on *x*. In a more advanced formulation the objective function, *f*(*x*), to be minimized or maximized, might be subject

to constraints in the form of equality constraints, $G_i(x) = 0\ (i = 1, ..., m_e)$; inequality constraints, $G_i(x) \le 0\ (i = m_e + 1, ..., m)$; and/or parameter bounds, x_l, x_u.
A General Problem (GP) description is stated as

$$\min_x \quad f(x),$$

subject to

$G_i(x) = 0 \qquad (i = 1, ..., m_e)$

$G_i(x) \le 0 \qquad (i = m_e + 1, ..., m)$

where *x* is the vector of length *n* design parameters, *f*(*x*) is the objective function, which returns a scalar value, and the vector function *G*(*x*) returns a vector of length *m* containing the values of the equality and inequality constraints evaluated at *x*.
An efficient and accurate solution to this problem depends not only on the size of the problem in terms of the number of constraints and design variables but also on characteristics of the objective function and constraints. When both the objective function and the constraints are linear functions of the design variable, the problem is known as a Linear Programming (LP) problem.
Quadratic Programming (QP) concerns the minimization or maximization of a quadratic objective function that is linearly constrained. For both the LP and QP problems, reliable solution procedures are readily available. More difficult to solve is the Nonlinear Programming (NP) problem in which the objective function and constraints can be nonlinear functions of the design variables.
A solution of the NP problem generally requires an iterative procedure to establish a direction of search at each major iteration. This is usually achieved by the solution of an LP, a QP, or an unconstrained sub-problem.
In transmission design, optimization based on desired velocity ratios and mechanism kinematic and geometric constraints generates the best possible gear ratios. Optimization uses an objective function, based on a set of design variables and constraints, assigns a numerical value to the desired or required velocity ratios. The design variables stem from the physical constraints. Along with the numerical algorithm, the objective function plays a big role in the effectiveness of the optimization problem [**Arora 2004**].

3. NOMOGRAPHS

Nomograph is defined as three or more axes, or scales, arranged such that problems of three or more variables can be solved using a straightedge. In the particular case of EGTs, a nomograph can be constructed using three or more vertical parallel axes [**Esmail 2007, 2008**].
A basic EGT consists of a sun gear, a ring gear, a planet, and a carrier as shown in **Fig. 2**.
Figure 3 shows the basic form of the graph to be created for a basic EGT.
The term "gear ratio" is used in this paper to denote the ratio of a meshing gear pair. It is defined by a planet gear *p* with respect to a sun or ring gear *x*

$$N_{p,x} = \mp Z_p / Z_x \qquad (1)$$

Where Z*p* and Z*x* denote the numbers of teeth on the planet and the sun or ring gear, respectively, and the positive or negative sign depends on whether *x* is a ring or sun gear.
Considering the kinematics of a fundamental circuit, the fundamental circuit equation can be written as [**Buchsbaum and Freudenstein 1996**]:

$$(\omega_x - \omega_c)/(\omega_p - \omega_c) = N_{p,x} \qquad (2)$$

Equation (2) can be re-written for the links of the basic EGT as follows

$$(\omega_r - \omega_c)/(\omega_p - \omega_c) = N_{p,r} \tag{3}$$

$$(\omega_s - \omega_c)/(\omega_p - \omega_c) = N_{p,s} \tag{4}$$

$$(\omega_p - \omega_c)/(\omega_p - \omega_c) = N_{p,p} = 1 \tag{5}$$

and

$$(\omega_c - \omega_c)/(\omega_p - \omega_c) = N_{p,c} = 0 \tag{6}$$

These values have been used to place the axes of the nomograph shown in Fig. 3. The ω_c axis passes at the origin, and the ω_p axis is one unit apart from it.

Let the symbol $R^{z}_{x,y}$ denote the velocity ratio between links *x* and *y* with reference to link *z*, where *x*, *y* and *z* are any three links in the gear train. Applying Eq. (2) twice for links *y* and *z* and simplifying it yields:

$$R^{z}_{x,y} = \frac{\omega_x - \omega_z}{\omega_y - \omega_z} = \frac{N_{p,x} - N_{p,z}}{N_{p,y} - N_{p,z}} \tag{7}$$

From the nomograph shown in **Fig. 3**, we can find directly that

$$R^{c}_{s,r} = \frac{\omega_s - \omega_c}{\omega_r - \omega_c} = \frac{N_{p,s}}{N_{p,r}} \tag{8}$$

$$R^{c}_{r,s} = \frac{N_{p,s}}{N_{p,r}} = \frac{1}{R^{c}_{s,r}} \tag{9}$$

$$R^{r}_{s,c} = \frac{N_{p,s} - N_{p,r}}{-N_{p,r}} = 1 - \frac{N_{p,s}}{N_{p,r}} = 1 - R^{c}_{s,r} \tag{10}$$

and

$$R^{s}_{c,r} = \frac{1}{R^{s}_{r,c}} = \frac{1}{1 - R^{c}_{r,s}} = \frac{1}{1 - (1/R^{c}_{s,r})} = \frac{R^{c}_{s,r}}{R^{c}_{s,r} - 1} \tag{11}$$

3.1 Clutching Sequence Nomograph

Due to the fact that an EGT can be decomposed into several FGTs [**Chatterjee and Tsai, 1995, 1996**] the kinematics of an EGT is closely related to the kinematics of each individual FGE.

In order to derive a clutching sequence, it is important to unify the nomographs of the FGEs in one nomograph called the system nomograph and develop a methodology for arranging the output velocities in a sequence.

We first unify FGEs that are connected to each other by two common links i.e. having two links with the same labeling, as shown in **Fig. 4**.

Let *b*1 and b2 be the common links, $P^{(1)}$ be the first planet of the first FGE or subsystem to be unified, y be any link from the second FGE (not the basic one) other than *b*1 and *b*2, then by using link *b*1 as a bridge, we express the velocity ratio $R^{b2}_{y,p^{(1)}}$ of link *y* with respect to the basic FGE in terms of two of the velocity ratios of the FGEs or subsystems as

$$R^{b2}_{y,p(1)} = R^{b2}_{y,b1} \cdot R^{b2}_{b1,p(1)} \tag{12}$$

Where $R^{b2}_{b1,p(1)}$ is associated with one of the FGEs or subsystems (basic one) and $R^{b2}_{y,b1}$ is associated with the other FGE or subsystem.

Now, we shall deal with link *y* as if it were a link that belongs to the basic FGE. From the unified nomograph, and making use of Eq. (7), we can write

$$R^{b2}_{y,p(1)} = \frac{N_{p(1),y} - N_{p(1),b2}}{N_{p(1),p(1)} - N_{p(1),b2}} \tag{13}$$

where $N_{p(1),p(1)}$ is equal to one and $N_{p(1),y}$ is the virtual gear ratio of link y in terms of the planet of the basic FGE or the distance between the first FGE carrier and link *y* axes on the unified nomograph .

For a particular EGT, two nomographs are drawn in **Fig. 5**. Since *b*2 is the carrier in this train, then making use of Eq. (6), $N_{p(1),b2}$ is zero.

Applying Eq. (13) yields

$$R^{b2}_{y,p(1)} = N_{p(1),y} \tag{14}$$

Combining equations (12) and (14) yields

$$N_{p(1),y} = R^{b2}_{y,b1} \cdot R^{b2}_{b1,p(1)} \tag{15}$$

For a system having more than two FGEs, the unification process continues between FGEs and/or unified subsystems until the highest-level system becomes the required mechanism. This way, a system nomograph can be obtained in terms of the gear ratios of its gear pairs.

The Simpson gear train which is composed of two single-planet FGEs will be used as a design example to illustrate the present methodology.

Figure 6 shows the unification process and the system nomograph for the Simpson gear train.

The gear ratios for this train are

$$N_{5,2^2} = Z_5 / Z_{2^2} \tag{16}$$

$$N_{5,3^2} = -Z_5 / Z_{3^2} \tag{17}$$

$$N_{6,4} = Z_6 / Z_4 \tag{18}$$

and

$$N_{6,3^1} = -Z_6 / Z_{3^1} \tag{19}$$

It can be shown that

$$0 \le N_{5,2^2} \le 1 \tag{20}$$

$$-\infty \le N_{5,3^2} \le 0 \tag{21}$$

$$0 \le N_{6,4} \le 1 \tag{22}$$

and

$$-\infty \le N_{6,3^1} \le 0 \tag{23}$$

From Eq. (12)

$$R^2_{1,6} = R^2_{1,3} \cdot R^2_{3,6} \tag{24}$$

From the first FGE nomograph

$$R^2_{3,6} = N_{6,3^1} \tag{25}$$

And from the second FGE nomograph

$$R^2_{1,3} = -\frac{N_{5,2^2}}{N_{5,3^2} - N_{5,2^2}} \tag{26}$$

Substituting Eqs (25) and (26) into Eq. (24), yields

$$R^2_{1,6} = -\frac{N_{5,2^2} . N_{6,3^1}}{N_{5,3^2} - N_{5,2^2}} \tag{27}$$

But from the unified nomograph

$$R^2_{1,6} = \mathrm{N}_{6,1} \tag{28}$$

Therefore

$$\mathrm{N}_{6,1} = -\frac{N_{5,2^2} . N_{6,3^1}}{N_{5,3^2} - N_{5,2^2}} \tag{29}$$

Figure 7 shows the basic form of the unified system nomograph for the Simpson gear train without the planet axis. Here it is called the basic form of clutching sequence nomograph.

As shown in **Fig.7**, the ranges of output velocities can be classified into three kinds: drive (D), overdrive (OD), and reverse (N) according to whether the velocity is between zero and the input velocity, greater than the input velocity, or less than zero. Note that a positive velocity is divided into D and OD because there is usually a "direct drive" (DD) between them [**Hsieh and Tsai, 1996**].

If the output velocity ranges of an EGT can be identified without specifying the gear sizes, a descending sequence of clutching conditions can be roughly configured. This will effectively facilitate the enumeration of all the feasible clutching sequences of an EGT.

3.2 Procedure for Enumeration of Clutching Sequences

An EGT can provide several velocities depending on the assignment of the input, output, and fixed links. These various velocities need to be estimated and arranged in a descending order to arrive at a proper clutching sequence.

In a transmission mechanism, usually the output link is permanently attached to a final reduction unit, and the change of velocity is accomplished by switching either the fixed link or the input link [**Hsieh,1996**].

- To a predetermined output link, o, choose two coaxial links (x, y), from the EGT at a time to construct the clutching sequence nomograph. For an EGM with m coaxial links, there is (m-1)! /2 possible sets of two coaxial links. By taking one of the two coaxial links as the common input link the other as the fixed link and by drawing a line through them, two output velocities are obtained at a time: a total of (m-1)! velocities. The ends of each line are labeled with B and C to denote a band clutch on the line of zeros and a rotating clutch on the line of the input velocity.

- Perform this operation as many times as possible until all possible arrangements of (x, y, o) are exhausted. In the present methodology the velocities are arranged automatically in a descending order into three kinds: D, OD and N.

Returning to our design example, the Simpson gear train, because there are four coaxial links in this mechanism and link two is pre-assigned as the output link, this mechanism can provide six clutching conditions. **Figure 8** shows the clutching sequence nomograph for this mechanism.
In arranging a clutching sequence, it is highly desirable to achieve a single-shift transition [**Hsieh, and Tsai, 1998**]. In order to achieve single-shift transitions, the D-velocities can be further classified into two sets as shown in **Fig. 9**.
A direct drive is obtained by simultaneously clutching two coaxial links of an EGT to the input power source. A reverse drive can be obtained by applying one or two of the clutches designed for the forward drives for the reverse drive.
As a result, we obtain two descending sequences of velocities as shown in **Figures 9(a) and (b)**, which result in two three-velocity and two four-velocity clutching sequences as shown in **Tables 2(a) through (d)**.
Table 2 (c) shows one feasible clutching sequence with rotating clutches attached to links 3 and 4, and band clutches attached to links 1 and 3. This clutching sequence has been applied in most three-velocity automatic transmission. **Figure 1** shows the functional representation of this three-velocity clutching sequence for the second reduction set. From **Fig. 9 (b)**, the velocity ratios can be written for three- or four-velocity automatic transmission as

$$R_{D1} = R^{1}_{4,2^1} = \frac{N_{5,2^2} \cdot N_{6,3^1} + N_{6,4}\left(N_{5,3^2} - N_{5,2^2}\right)}{N_{5,2^2} \cdot N_{6,3^1}} \tag{30}$$

$$R_{D2} = R^{3^1}_{4,2^1} = 1 - \frac{N_{6,4}}{N_{6,3^1}} \tag{31}$$

$$R_{N} = R^{1}_{3^1,2^1} = \frac{N_{5,3^2}}{N_{5,2^2}} \tag{32}$$

$$R_{OD} = R^{3^1}_{1,2^1} = \frac{N_{5,3^2}}{N_{5,3^2} - N_{5,2^2}} \tag{33}$$

Under certain clutch combinations the gear train may act as a rigid body, in which case the velocity ratio will equal to one ($R_{DD} = 1$), and the designer will not affect it by his selection of gear ratios.
The other clutching sequences obtained are in agreement with those reported in the literature [**Chatterjee and Tsai,1995,1996**], except that there is no need to any information containing the approximate gear sizes arranged in a descending order that is required in other methods.

4. OPTIMIZATION TECHNIQUE AND PROBLEM FORMATION

In the systematic creation of mechanisms the functional evaluation stage deals with the selection of acceptable mechanisms from the enumerated list of mechanisms, in order to satisfy the functional requirements.

The next stage in the creation of mechanisms is its design and analysis. In this stage dimensional synthesis techniques are used to determine the mechanism's proportions. The designer has to choose a gear train with a set of clutches that are to be operated in a chosen sequence, and a set of gear ratios that will provide a set of velocity ratios. The classical approach of finding proper gear ratios has been to choose a gear train and a corresponding clutching sequence, and then to vary the gear ratios by trial and error until the best possible velocity ratios are obtained. To eliminate trail and error, an objective function F is defined in terms of the velocity ratios R_k for the gear train, the minimization of which ensures that the best possible gear ratios are achieved, subjected to certain constraints.

The objective function can be written as

$$F(R) = \sum_{i=1}^{k} \left(\frac{R_i}{R_{di}} - 1\right)^2 \qquad (34)$$

Where R_i **;** is the i^{th} velocity ratio parameter obtained by the minimization process and R_{di} is the i^{th} desired velocity ratio. The velocity ratios $R_{1,}$ R_2......... R_k depend on the clutch selection and the topology of the gear train.

It should be realized that for an arbitrary combination of R_{di} there may not be a set of solutions for the gear ratios $N_{j,i}$ that satisfy the velocity ratio equations. In other words, the ideal minimum value of the objective function F(R) → 0 may or may not be achievable. The constraints under which the function F(R) is minimized are now determined.

4.1. Geometric Constraints

Let d_i be the diameter of a gear element *i*. If the diametral pitch *P* of all the gears is the same, then $d_i = P.Z_i$ and we can get the geometric constraints for the Simpson gear train from **Fig. 1** as $d_4 - d_{3^1} = 2d_6$ or $Z_4 - Z_{3^1} = 2Z_6$ and therefore, $Z_4/Z_6 - Z_{3^1}/Z_6 = 2$ thus

$$\frac{1}{N_{6,4}} + \frac{1}{N_{6,3^1}} = 2 \qquad (35)$$

And in a similar manner

$$\frac{1}{N_{5,2}} + \frac{1}{N_{5,3^2}} = 2 \qquad (36)$$

Note that both $N_{6,3^1}$ and $N_{5,3^2}$ are negative numbers since they correspond to external gear meshes.

4.2. Kinematic Constraints

The kinematic constraints are determined from the system nomograph shown in **Fig.9**. They are rewritten here as

$$0 \le N_{5,2^2} \le 1 \tag{37}$$

$$-\infty \le N_{5,3^2} \le 0 \tag{38}$$

$$0 \le N_{6,4} \le 1 \tag{39}$$

$$-\infty \le N_{6,3^1} \le 0 \tag{40}$$

5. CONSTRAINED OPTIMIZATION USING MATLAB'S fmincon

Matlab provides the command (*fmincon*) for constrained minimization of an objective function *f(x)*. *Fmincon* finds a vector *x* that is a local minimum to a scalar function *f*(*x*) of several variables subject to constraints on the allowable *x* starting at an initial estimate[**Brian, 2007**]. This is generally referred to as *constrained nonlinear optimization.*

It finds a minimum of a constrained nonlinear multivariable function $\min_x \; f(x)$, subjected to one or more of the following constraints: $c(x) \le 0$, $ceq(x) = 0$, $A \cdot x \le b$, $Aeq \cdot x = beq$, $l \le x \le u$. where *x, b, beq, lb,* and *ub* are vectors, *A* and *Aeq* are matrices, *c(x)* and *ceq(x)* are functions that return vectors, and *f(x)* is a function that returns a scalar. *f(x), c(x),* and *ceq(x)* can be nonlinear functions.

The objective function must be coded in a function file. In this work this file is called *Simpson* and saved as *Simpson.m* in the working directory.

Without any extra options, *fmincon* with linear inequality and equality constraints, lower and upper bounds, and nonlinear inequality and equality constraints is called as follows:

[x,fval]=fmincon(@Simpson,x0,A,b,Aeq,beq,lb,ub,'constraint')

The last input argument in this call is the name of a function file (denoted constraint in this work and saved as Simpson_confun.m in the working directory), in which the nonlinear constraints are coded.

5.1. Constraint function file:

Simpson_confun.m is a function file (any name can be chosen) in which both the inequality functions c(x) and the equality constraints ceq(x) are coded and provided in the form of column vectors. The function call

[c,ceq]=constraint(x)

must retrieve c(x) and ceq(x) for given input vector x. If only equality constraints are given, define c = [ ].

5.2. Interpretation:

The retrieved ceq (x) is interpreted by fmincon as equality constraint ceq(x) = 0. The inequalities associated with c(x) are interpreted as c(x) ≤ 0. Thus, if a constraint of the form c(x) ≥ 0 is given, rewrite this as -c(x) ≤ 0 and code -c(x) in the constraint function file.

5.3. Placeholders:

As shown above, the constraints have to be passed to fmincon in the following order:

1. linear inequality constraints,
2. linear equality constraints,
3. Lower bounds,
4. Upper bounds, and
5. Nonlinear constraints
6. If a certain constraint is required, all other constraints appearing before it have to be inputted as well, even if they are not required in the problem. If this is the case, their input argument is replaced by the placeholder [ ] (empty input).

6. OPTIMIZATION RESULTS

The designer initially specifies the gear train, enumerates the clutching sequence and then specifies the reduction ratios to be achieved. The optimization problem starts at an arbitrary design point and at the end of its execution displays the optimized gear ratios. Results of sample inputs for the three-velocity Simpson gear train are shown in **Table 3**. The trial and error approach of finding the gear ratios for an automotive gear train to obtain a set of velocity ratios has been eliminated by formulating the task as a constrained nonlinear optimization problem. The optimization procedure was successfully applied to find those gear ratios for which the velocity ratios are closest to the original specification. The theoretical results are in complete agreement with the practical applications as shown in columns 2 and 3 in **Table 3**. The Optimized gear ratios shown in the last column of **Table 3** satisfy the geometric and kinematic constraints given by Eqs. (35) Through (40). **Figure 10** shows a MATLAB plot to the optimization results given in Appendix B for Ford C4, C5 and C6 Simpson gear train given in the first row of **Table 3**.

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[11] http://transmissions101.com/automatic-transmissions.php

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[13]Mogalapalli, S.N., Magrab, E.B. And Tsai, L-W, 1993, "A CAD System for the Optimization of Gear Ratios for Automotive Automatic Transmissions", SAE paper, no. 930675. (http://papers.sae.org/930675)

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Table 1 Clutching sequence of the mechanism shown in **Figure 1**.

	Activated clutches			
Range	C1	C2	B1	B2
First	X4		X1	
Second	X4			X3
Third	X4	X3		
Reverse		X3	X1	

Table 2 (a) Three-velocity clutching sequence for the first reduction set.

	Activated clutches			
Range	C1	C2	B1	B2
First	X3		X4	
Second		X1	X4	
Third	X3	X1		
Reverse	X3			X1

Table 2 (b) Four-velocity clutching sequence for the first reduction set.

	Activated clutches				
Range	C1	C2	B1	B2	B3
First	X3		X4		
Second		X1	X4		
Third	X3	X1			
Fourth		X1		X3	
Reverse	X3				X1

Table 2 (c) Three-velocity clutching sequence for the second reduction set.

	Activated clutches			
Range	C1	C2	B1	B2
First	X4		X1	
Second	X4			X3
Third	X4	X3		
Reverse		X3	X1	

Table 2 (d) Four-velocity clutching sequence for the second reduction set.

	Activated clutches				
Range	C1	C2	C3	B1	B2
First	X4			X1	
Second	X4				X3
Third	X4	X1			
Fourth		X1			X3
Reverse			X3	X1	

Table 3 Table of optimized gear ratios for sample inputs for Simpson gear train.

Transmission	Desired overall velocity ratio		Achieved overall velocity ratio	F(R)	Optimized gear ratios	
Ford C4 C5 And C6	$R_1 = R^{1}_{4,2^1}$	2.8401	2.8401	5.819850e-010	$x_1 = N_{6,3^1}$	-0.3333
	$R_2 = R^{3^1}_{4,2^1}$	1.6000	1.6000		$x_2 = N_{5,2^1}$	0.2581
	R_3	1.0000	1.0000		$x_3 = N_{5,3^2}$	-0.5333
	$R_4 = R^{1}_{3^1,2^1}$	-2.0666	-2.0666		$x_4 = N_{6,4}$	0.2000
General motors Super Turbine 400 / THM400 / 3L80 / 3L80HD	$R_1 = R^{1}_{4,2^1}$	2.4800	2.4786	9.166488e-008	$x_1 = N_{6,3^1}$	-0.5383
	$R_2 = R^{3^1}_{4,2^1}$	1.4800	1.4815		$x_2 = N_{5,2^1}$	0.2585
	R_3	1.0000	1.0000		$x_3 = N_{5,3^2}$	-0.5352
	$R_4 = R^{1}_{3^1,2^1}$	-2.0700	-2.0705		$x_4 = N_{6,4}$	0.2592
General motors Turbo Hydra-matic THM350, 250,250-C	$R_1 = R^{1}_{4,2^1}$	2.5200	2.5366	1.998745e-004	$x_1 = N_{6,3^1}$	-0.4971
	$R_2 = R^{3^1}_{4,2^1}$	1.5200	1.5015		$x_2 = N_{5,2^1}$	0.2578
	R_3	1	1.0000		$x_3 = N_{5,3^2}$	-0.5322
	$R_4 = R^{1}_{3^1,2^1}$	-2.0700	-2.0644		$x_4 = N_{6,4}$	0.2493
General motors THM200	$R_1 = R^{1}_{4,2^1}$	2.7400	2.7423	2.790624e-006	$x_1 = N_{6,3^1}$	-0.3808
	$R_2 = R^{3^1}_{4,2^1}$	1.5700	1.5677		$x_2 = N_{5,2^1}$	0.2584
	R_3	1.0000	1.0000		$x_3 = N_{5,3^2}$	-0.5346
	$R_4 = R^{1}_{3^1,2^1}$	-2.0700	-2.0693		$x_4 = N_{6,4}$	0.2162
Daimler Chrysler 30RH and 32RH	$R_1 = R^{1}_{4,2^1}$	2.7400	2.7585	2.062212e-004	$x_1 = N_{6,3^1}$	-0.4080
	$R_2 = R^{3^1}_{4,2^1}$	1.54000	1.5506		$x_2 = N_{5,2^1}$	0.2720
	R_3	1.0000	1.0000		$x_3 = N_{5,3^2}$	-0.5967
	$R_4 = R^{1}_{3^1,2^1}$	-2.2000	-2.1934		$x_4 = N_{6,4}$	0.2247
Nissan 3N71B	$R_1 = R^{1}_{4,2^1}$	2.7400	2.7327	2.900969e-005	$x_1 = N_{6,3^1}$	-0.3663
	$R_2 = R^{3^1}_{4,2^1}$	1.5700	1.5772		$x_2 = N_{5,2^1}$	0.2503
	R_3	1.0000	1.0000		$x_3 = N_{5,3^2}$	-0.5011
	$R_4 = R^{1}_{3^1,2^1}$	-2.0000	-2.0022		$x_4 = N_{6,4}$	0.2114

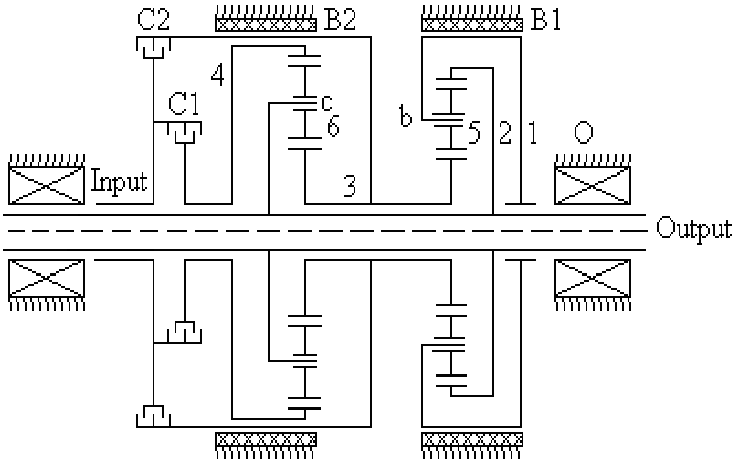


Figure 1: A Typical transmission mechanism.

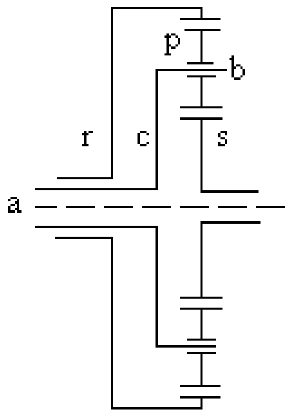


Figure 2: A basic epicyclic gear train.

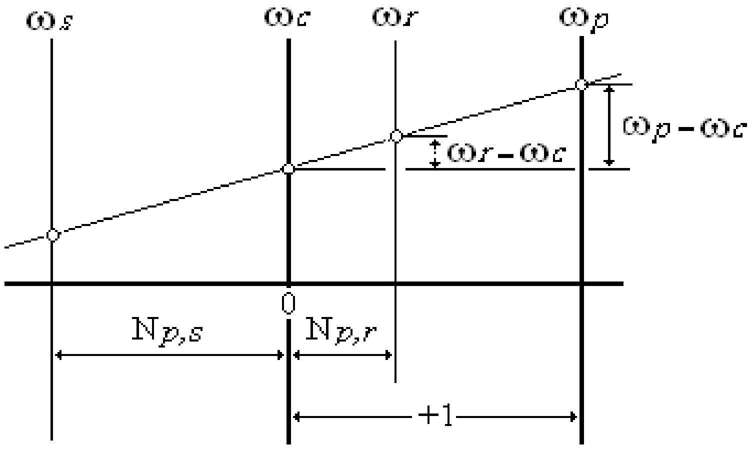


Figure 3: Nomograph for the basic epicyclic gear train.

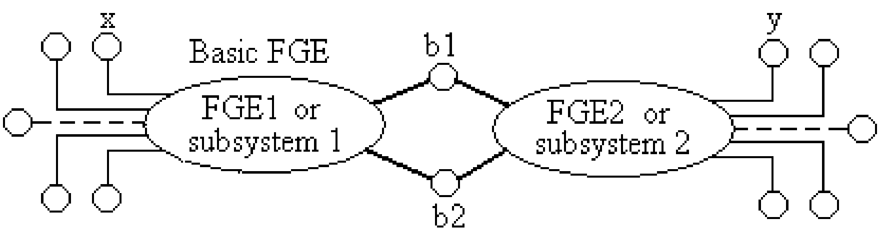


Figure 4: Unification of FGEs and/or subsystems.

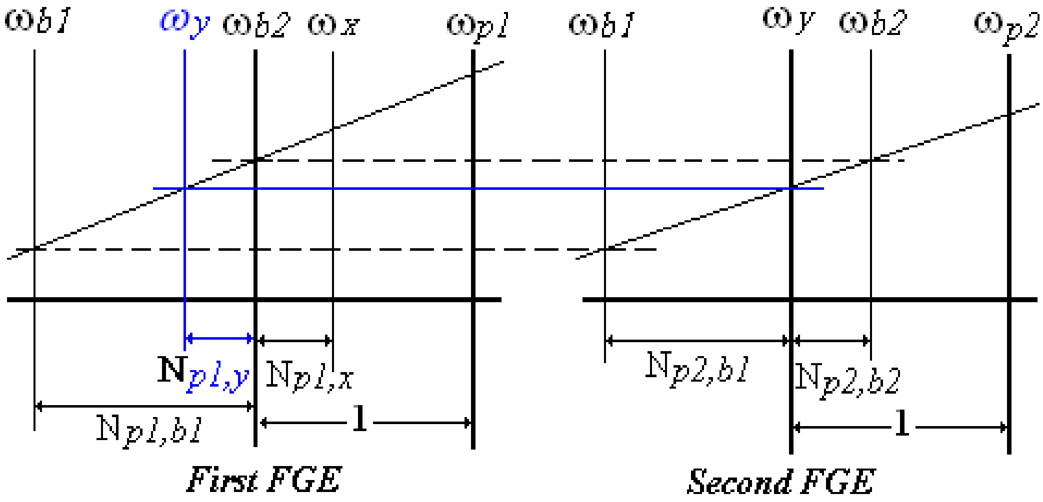
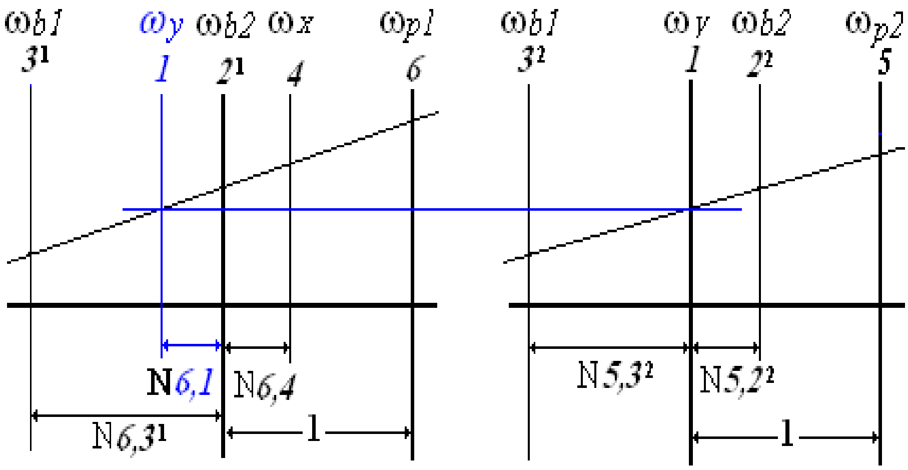
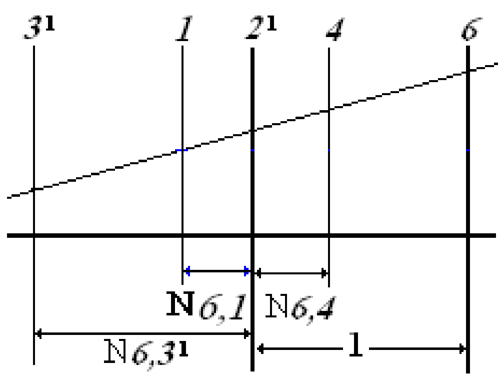


Figure 5: Unification of two FGE nomographs into one nomograph.



(a) Nomograph unification of the two FGEs of the Simpson gear train shown in **Fig. 1**.



(b) Unified system nomograph

Figure 6: System nomograph of the Simpson gear train shown in **Fig. 1**.

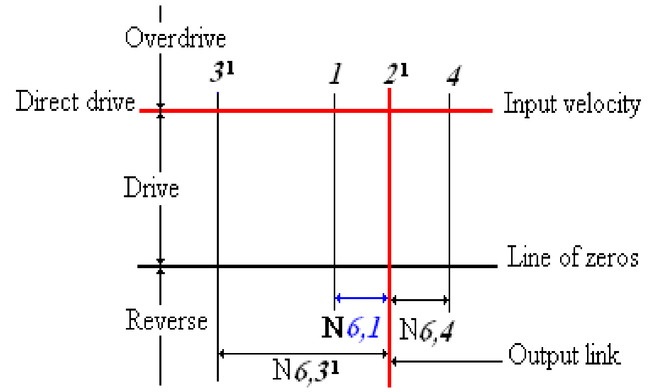


Figure 7: Basic form of the clutching sequence nomograph for the Simpson gear train.

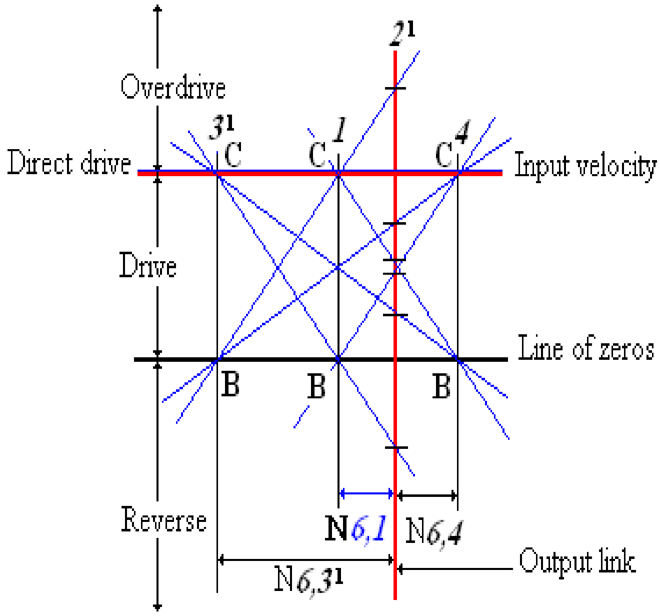
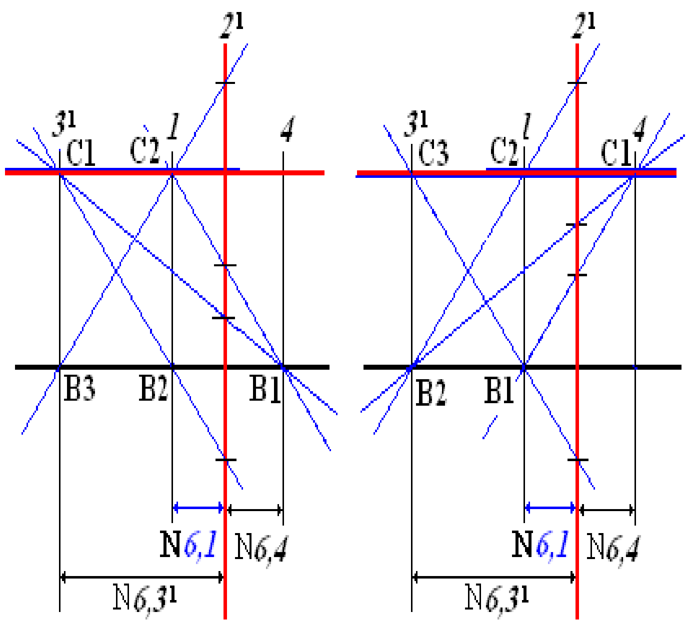


Figure 8: Clutching sequence nomograph for the Simpson gear train**.**



(a) First reduction set. (b) Second reduction set.

Figure 9: Clutching sequence nomographs for the two reduction sets of Simpson gear train**.**

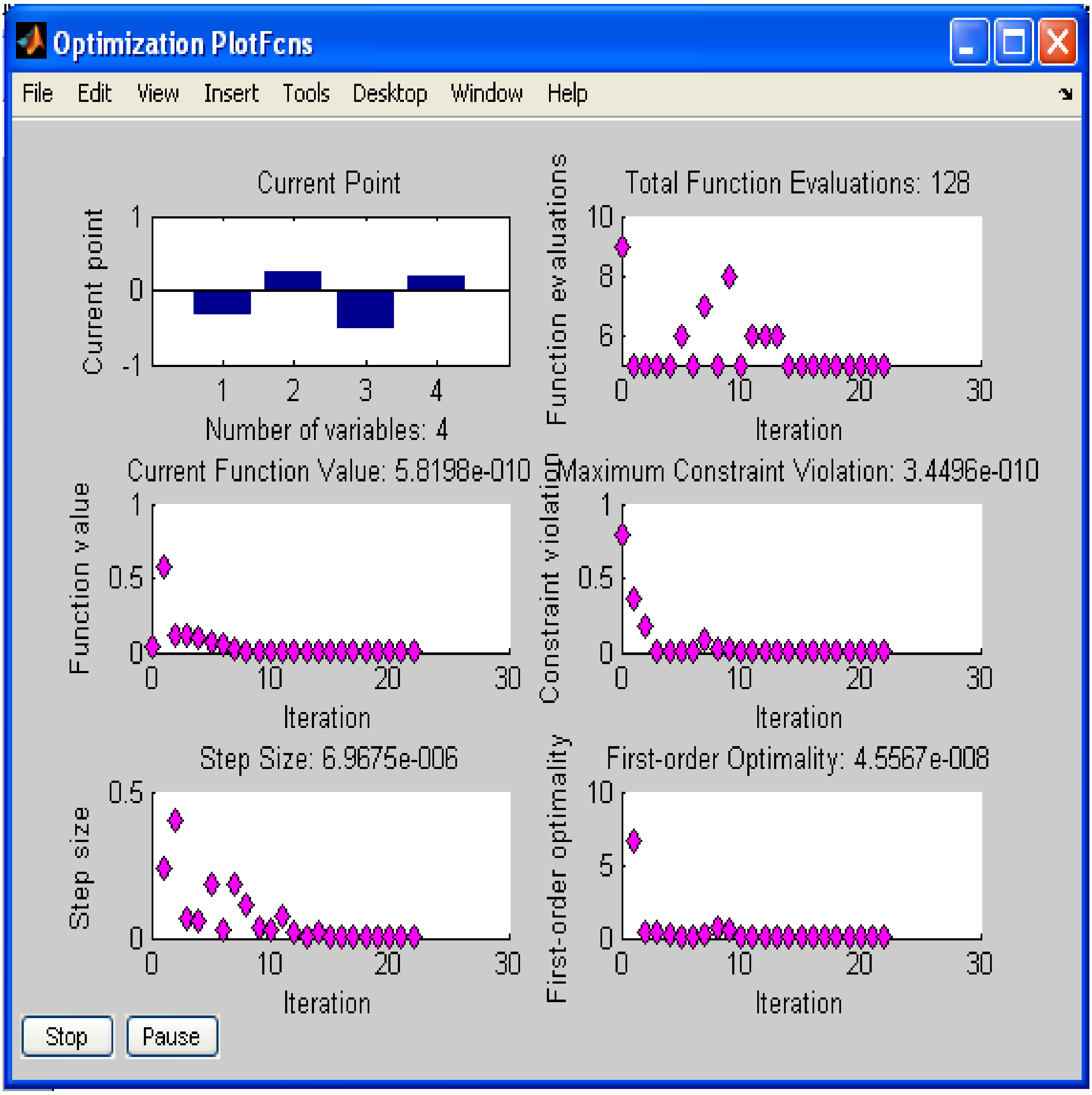


Figure 10: Example of MATLAB plot to the optimization results given in Appendix B.

APPENDIX A:

(1) MATLAB objective function m.file (Simpson.m)

```
function f = Simpson(x)
R1=(x(1)*x(2)+(x(3)-x(2))*x(4))/(x(1)*x(2));R2=1-(x(4)/x(1));R3 = 1;R4=(x(3)/x(2));
k1 = 2.8401; k2 = 1.6; k3 = 1.0; k4 = -2.0666; % Ford C4, C5, And C6
f=((R1/k1)-1)^2+((R2/k2)-1)^2+((R3/k3)-1)^2+((R4/k4)-1)^2;
```

(2) MATLAB Nonlinear constraints m.file (Simpson_confun.m)

```
function [c, ceq] = Simpson_confun(x)
c=[];
ceq = [(1/x(2))+(1/x(3))-2;
     (1/x(1))+(1/x(4))-2];
```

(3) MATLAB optimization m.file (simpson_main_file.m)

```
function [x,fval,exitflag,output,lambda,grad,hessian] = simpson_main_file(x0,lb,ub)
x0 = [0,0,0,0]
lb = [-Inf,0,-Inf,0]
ub = [0,1,0,1]
k1 = 2.8401; k2 = 1.6; k3 = 1.0; k4 = -2.0666; % Ford C4, C5, And C6
options = optimset;
options = optimset(options,'Display' ,'iter');
options = optimset(options,'PlotFcns' ,{ @optimplotx @optimplotfunccount @optimplotfval
@optimplotconstrviolation @optimplotstepsize @optimplotfirstorderopt });
options = optimset(options,'Algorithm' ,'interior-point');
options = optimset(options,'Diagnostics' ,'on');
[x,fval,exitflag,output,lambda,grad,hessian] = ...
fmincon(@Simpson,x0,[],[],[],[],lb,ub,@Simpson_confun,options);
R1=(x(1)*x(2)+(x(3)-x(2))*x(4))/(x(1)*x(2));R2=1-(x(4)/x(1));R3 = 1;R4=(x(3)/x(2));
Desired_Velocity_Ratios_k=[k1 k2 k3 k4]
Optimal_Velocity_Ratios_R=[R1 R2 R3 R4]
Gear_Ratios_r=[x(1) x(2) x(3) x(4)]
```

APPENDIX B:

MATLAB optimization results sample

```
x0 =     0     0     0     0
lb =  -Inf     0  -Inf     0
ub =     0     1     0     1
Diagnostic Information
Number of variables:      4
Functions
 Objective                          Simpson
 Gradient:                          finite-differencing
 Hessian:                           finite-differencing (or Quasi-Newton)
 Nonlinear constraints:             Simpson_confun
 Gradient of nonlinear constraints: finite-differencing
Constraints
 Number of nonlinear inequality constraints: 0
 Number of nonlinear equality constraints:   2
 Number of linear inequality constraints:    0
 Number of linear equality constraints:      0
 Number of lower bound constraints:          2
 Number of upper bound constraints:        4
```

Algorithm selected
interior-point
End diagnostic information

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
0	9	3.086034e-002	7.879e-001	4.395e-002	
1	14	5.703720e-001	3.569e-001	6.575e+000	2.412e-001
2	19	1.143033e-001	1.785e-001	3.634e-001	4.008e-001
3	24	1.087254e-001	1.280e-002	4.185e-001	6.795e-002
4	29	9.720484e-002	2.355e-003	1.889e-001	5.592e-002
5	35	6.278506e-002	3.704e-003	1.538e-001	1.814e-001
6	40	5.691928e-002	1.242e-003	1.000e-001	2.678e-002
7	47	1.933844e-002	8.514e-002	3.016e-001	1.861e-001
8	52	6.567615e-003	2.740e-002	7.239e-001	1.112e-001
9	60	5.065902e-003	2.492e-002	5.317e-001	3.185e-002
10	65	3.090380e-003	2.568e-003	2.023e-002	2.931e-002
11	71	9.384946e-005	2.126e-003	5.481e-002	7.282e-002
12	77	2.879866e-004	2.381e-006	3.371e-002	1.913e-002
13	83	1.735759e-004	1.001e-007	4.000e-003	5.889e-003
14	88	7.257730e-007	1.938e-003	8.343e-003	1.697e-002
15	93	7.319344e-006	4.473e-005	8.000e-004	2.669e-003
16	98	2.539741e-008	9.698e-005	1.908e-003	3.645e-003
17	103	1.993598e-009	6.574e-008	1.033e-004	1.585e-004
18	108	1.243654e-009	1.522e-009	8.189e-006	2.435e-005
19	113	1.243537e-009	2.531e-014	8.000e-006	8.416e-008
20	118	6.036931e-010	5.857e-009	1.981e-006	2.867e-005
21	123	6.074269e-010	2.042e-012	1.600e-006	5.229e-007
22	128	5.819850e-010	3.450e-010	4.557e-008	6.968e-006

Optimization terminated: first-order optimality relative error less than options. TolFun, and relative constraint violation less
than options.TolCon.
Desired_Velocity_Ratios_k = 2.8401 1.6000 1.0000 -2.0666
Optimal_Velocity_Ratios_R = 2.8401 1.6000 1.0000 -2.0666
Gear_Ratios_r = -0.3333 0.2581 -0.5333 0.2000