Theoritical Generalized For Indeterminate Coefficients Method

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Abstract :-

As it has been mentioned by D.R.Merk in [1994] that the method of indeterminate coefficients for constructing a Liapunov function is not a suitable for every problem.A new theory given in[2] and in this paper , we generalized this idea in which is applicable for more extended problems. The proof of this idea is given with example.

تعميم نظري لطريقة المعاملات الغير محددة

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الخلاصة :-

في هذا البحث تم تعميم نظرية جديدة لتكوين دوال ليبانوف الخاصة باختبار استقرارية الأنظمة اللاخطية ذات المعاملات الثابت , و كذلك تم عرض برهان لهذه النظرية مع ذكر مثال تطبيقي لها.

Introdction :-

Many papers, it had been seek a Liapunov function in a quadratic form with constant coefficients:

$$V(x)=\frac{1}{2}\sum_{k=1}^{m}\sum_{j=1}^{m}c_{kj}x_k x_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

In which the indeterminate cofficients c_{kj} must be satisfies Sylvester's criterion ,(see[4]), then the function v is positive definite. Since the number of the coefficients c_{kj} equal to $\frac{n(n+1)}{2}$, then we have $\frac{n(n-1)}{2}$ independent coefficients that can be manipulated (see[3]).

In [3] and [5], it had been assumed that the equations of perturbed motion

$$\frac{dx_j}{dt}=x_j(x_1,........,x_n)$$

allow that the integral

$$F(x_1,\ldots\ldots,x_n)=h=\text{const}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

For whih the difference F(x)-F(0) is a positive function of the variables $x_1,\ldots\ldots..,x_n$.

In fact, in view of (2) and by virtue of the equations of perturbed motion the derivative of this function with respect to time is identically equal to zero, and hence this function satisfies all the conditions of Liapunov's theorem of motion stability(see[1]).

The Approach :-

Let us assume that we need to determine the conditions satisfied by the parameters of the system would result in the stability of unperturbed motion. Then we try to choose the rest of the independent coefficents c_{kj} in such a way that the derivative V , obtained by virtue of the equations of the pertubed motion , is either a negative definite function or that is satisfies the conditions of Krasovsky theorem (see[5]and[6]). If such coefficients c_{kj} .Can be found, then the perturbed motion is a symptotically stable. Since this approach is not suitable for every problem,but in some cases it reduces good results.

Therfore, in this paper, we are built a criteria for testing the stability for general extended system based on the folloing theorem:

Theorem :

The system

$$\dot{x}_k = \sum_{j=1}^{m} a_{kj}\, x_j^n x_k^{n-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

where a_{kj} ,(k=1,2,3,…..,m) are any constants and n is any real number such that $m \neq n$, is asymptotically stable if the Sylvester's criterion on $\dot{V}$ is satisfied.

Proof :

Consider the system (1)

$$V = \frac{1}{2}\sum_{k=1}^{m}\sum_{j=1}^{m} c_{kj} x_k x_j , \qquad \text{with} \quad c_{ii}=1 \quad i=2,3,\ldots,m$$

let

$$C = \begin{pmatrix} c_{11} & c_{12} & c_{13} & \ldots.. & c_{1m} \\ c_{21} & 1 & c_{23} & \ldots.. & c_{2m} \\ c_{31} & c_{32} & 1 & \ldots.. & c_{3m} \\ \vdots & \vdots & \vdots & \ldots.. & \vdots \\ c_{m1} & c_{m2} & c_{m3} & \ldots... & 1 \end{pmatrix}$$

Then Sylvester's criterion for the coefficients matrix C has the form :

$$\Delta_1 = c_{11} > 0 \ , \qquad \Delta_2 = \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & 1 \end{vmatrix} = c_{11} - c_{21}c_{12} > 0 \ ,\ldots..,\Delta_m = |c| > 0.$$

Thus

$$\dot{V} = \sum_{k=1}^{m}\sum_{j=1}^{m} c_{kj} x_k \Big(\sum_{j=1}^{m} a_{kj} x_j^n x_k^{n-1}\Big).$$

Now ,if $c_{kj} \neq 0$ and $k \neq j$ (k,j=1,2,…,m), $\forall$ k,j

Then $\dot{V}$ is an indefinite function.

Therefore, we assume $c_{kj}=0$ for $k \neq j$ (k,j=1,2,…,m), and given as:

$$\dot{V}=\sum_{k=1}^{m}\sum_{j=1}^{m} c_{kj}a_{kj}\,x_k^n x_j^n$$

then system (3) is asymptoticall stable if the Sylvester's criterion on $\dot{V}$ is satisfied.That is if the following inequalities are satisfied:

$$\Delta_1^* < 0\,, \qquad \Delta_2^* > 0\,,\ldots$$

i,e, determinants Δ_j^* (j=1,2,…….,m) should alternately change their signs, and the sign of Δ_1^* should be negative.

Example :

Consider the system of equations

$$\left.\begin{aligned}\dot{x}_1 &= -2x_1^3 + x_1x_2^2 + x_1x_3^2\\ \dot{x}_2 &= x_1^2x_2 - 4x_2^3 - x_2x_3^2\\ \dot{x}_3 &= x_1^2x_3 - x_2^2x_3 + x_3^3\end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Let V be a Liapunov function given by:

$$V=\frac{1}{2}(x_1^2+x_2^2+x_3^2)$$

Sylvester's criterion for the coefficients matrix has the form :

$$\Delta_1 = 1 > 0\ \ ,\ \ \Delta_2=\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix}=1>0\,,\quad \Delta_3=\begin{vmatrix}1&0&0\\0&1&0\\0&0&1\end{vmatrix}=1>0$$

We evaluate the derivative $\dot{V}$

$$\dot{V}=x_1\dot{x}_1+x_2\dot{x}_2+x_3\dot{x}_3$$

Substituting for $\dot{x}_1$, $\dot{x}_2$ and $\dot{x}_3$ from equation (4), we get :

$$\Delta_1^* = -2 < 0 \quad , \ \Delta_2^* = \begin{vmatrix} -2 & 2 \\ 2 & -4 \end{vmatrix} = 4 > 0 \quad , \ \Delta_3^* = \begin{vmatrix} -2 & 2 & 2 \\ 2 & -4 & -2 \\ 2 & -2 & 1 \end{vmatrix} = -4 < 0$$

$\therefore$ system (4) is asymptotically stable.

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