

## THE EFFECT OF THE ENVIROMENT CONDITION ON THE UNSTEADY STATE SHELL AND TUBE HEAT EXCHANGER

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### Abstract

The effect of the environment (surrounding) conditions on the effectiveness of the heat exchanger for different working fluids are discussed here. The heat transfer between the heat exchanger external wall and environment are considered taken by radiation and convection. The convection heat transfer coefficient between the heat exchanger wall and the environment is taken as temperature dependence and the heat exchanger external wall is assumed a grey body. We analyzed the effect of the environment conditions on two type of heat exchanger working fluid, low viscid fluid (water) and medium viscid fluid (oil SAE-30). Because of the time is very effected factor on the heat transfer, then the unsteady state of the fluid inside the heat exchanger are assumed and analyzed in the present article. The increase of the emissivity, environment heat coefficient and environment temperature will decrease the hot fluid effectiveness and increase the entropy generation number. The fluid effectiveness is very effected by the environment specially when the working fluid has lower viscosity for same flow rate. The finite difference techniques is used to solve the unsteady state differential equations for the hot and cold fluid.

**Keyword: Heat Exchanger, Environment Fluid, Entropy Generation, Shell and Tube Heat, Finite Difference**

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### الخلاصة

تأثير الظروف البيئية (الظروف الخارجية المحيطة) على فعالية المبادل الحراري لأنواع مختلفة من الموائع تم مناقشتها هنا. انتقال الحرارة بين الجدار الخارجي للمبادل الحراري والبيئة المحيطة يحدث بالإشعاع والحمل. أن معامل انتقال الحرارة بالحمل بين الجدار الخارجي للمبادل الحراري والبيئة المحيطة اعتبر معتمد على درجة الحرارة وجدار المبادل الحراري اعتبر كجسم رمادي. تم دراسة تأثير الظروف البيئية على نوعان من موائع المبادل الحراري كالتالي: سائل منخفض اللزوجة (ماء) و سائل متوسط اللزوجة (زيت SAE-30). بسبب أن عامل الوقت مهم جداً في عملية انتقال الحرارة، لذلك تم فرض الحالة الغير مستقرة للسائل داخل المبادل الحراري.

زيادة انبعاثية الإشعاع ومعامل انتقال الحرارة ودرجة حرارة البيئة المحيطة سيقتصان فعالية السائل الحار ويزيدان العشوائية المتولدة (Entropy Generation). إن فعالية المبادل الحراري تتأثر جداً بالبيئة المحيطة وخصوصاً عندما تكون لزوجة السائل واطئة نفس معامل الجريان. طريقة الفروقات المحدودة تُستعمل لحل المعادلات التفاضلية للحالة الغير مستقرة للسائل الحار والبارد.

**Nomenclature**

A	Area (m <sup>2</sup> )	Pr	Prandtle number
b	Tube thickness (m)	Q	The heat transfer rate (W)
C	Fluid capacitance (W / °C)	R	Thermal resistance (°C/ W)
C <sub>p</sub>	Constant pressure mass heat capacity (KJ/kg.K)	Re	Reynolds number
D	Shell diameter (m)	T	Temperature ( °C)
d	Tube diameter (m)	t	Time (sec)
D <sub>H</sub>	Hydraulic diameter (m)	t <sub>w</sub>	Thickness of the inlet pipe (m)
f	Friction coefficient	U	Overall heat transfer coefficient (W/m <sup>2</sup> .°C)
h	Heat transfer coefficient (W/m <sup>2</sup> .K)	UR	Ratio of the overall heat transfer coefficient
k	Thermal conductivity (W/m.K)	V	Volume (m <sup>3</sup> )
L	Length of the heat exchanger (m)	x	Longitudinal distance in x-direction (m)
n <sub>p</sub>	Number of tube for each shell side	Z	Dimensionless longitudinal distance in x-direction
N <sub>s</sub>	Entropy generation number	$\dot{m}$	Mass flow rate (kg/sec)
P	Perimeter (m)	$\dot{s}$	Entropy generation ( $\dot{s} = dS / dt$ ).

**Greek**

$\rho$	Density (Kg/m <sup>3</sup> )	$\Delta$	Change during element
$\mu$	Dynamic viscosity (Pa.Sec)	$\epsilon$	Heat exchanger external wall emissivity
$\epsilon$	Heat exchanger effectiveness	$\sigma$	Stefan-Bolthzmann constant ( $5.67 \times 10^{-8}$ W/m <sup>2</sup> .K <sup>4</sup> )
$\Phi$	Dimensionless temperature	$\tau$	Dimensionless time

**Subscript**

c	Cold fluid	h	hot fluid
$\infty$	Environment fluid	max	Maximum value
min	Minimum value	r	Ratio
w	Wall	d	Dirty (fouling)
i	Inlet	o	Outlet
$\infty, c$	Environment fluid convection	$\infty, r$	Environment fluid radiation
Un	Universe		

**Superscript**

i	Refer to distance	j	Refer to time
'''	Per unit volume		

**Introduction**

Heat exchangers are widely used and include process, petroleum refining, and refrigeration and air-conditioning industries. Heat exchanger designers work constantly to improve the performance of individual units.

In the analyzing of the heat exchangers system usually are assumed that the heat transfer occurs only between the two fluids inside the heat exchanger with perfectly insulated of the heat exchanger wall with the surroundings (the heat transfer to the surrounding is zero). In some cases the assumption of perfectly insulated of the heat exchanger wall may not be valid and may lead to incorrect sizing of the heat exchanger or to errors in the performance predictions and heat exchanger effectiveness. In cryogenic systems, a considerable temperature difference exists between the fluid streams in heat exchanger and the surrounding environment. Thus, even with highly effective insulation, there can be significant heat transfer to the heat exchanger fluids. In addition, because cryogenic refrigerators typically have small COPs, a large amount of work must be supplied to remove even a small amount of heat transferred to the working fluids from the surroundings. In liquefaction systems, heat exchangers of high effectiveness should be used to achieve a significant temperature drop of the hot fluid stream. External heat transfer from or to heat exchanger results variation in the liquid yield. Due to fabrication limitations, these heat exchangers are typically constructed with outer insulating layers, but in some heat exchanger, the working fluids may be exposed to significant heat transfer with the surroundings. To keep good prediction yield, the transient response of heat exchanger has to be controlled in real time. The transient behavior is considered in the present research and investigated numerically.

The steady state solution are commonly divided in to two methods, “Log Mean Temperature Difference (LMTD)” and “Effectiveness- Number of Transfer Unit ( $\epsilon$  –NTU)”. The steady state for different types of heat exchanger with perfect wall insulated is widely discussed and analyzed in many literatures and textbook,( Kays,1984),( Walker,1982), (et. al,1983), (Holman,,1997), (Incoropera, ,2002).

Ameel and Hewavitharana, (Kreith,, Bohn,2003), developed a mathematical model to study the performance of a steady state counter flow heat exchanger in which both fluid streams are exposed to external heating. Ameel, (Kupprn,2003), studied the performance of a parallel flow heat exchanger in which both fluid streams are interacting thermally with the surroundings. The fluid temperatures are found to be dependent on the magnitude of the ambient temperature relative to fluid inlet temperatures. Manikandanprithiviraj and Malcolm, (Aulds, Barron,1967), presented a comparison for pressure drop of a three dimensional computational fluid dynamics simulation and HEATX for shell-and-tube heat exchangers with experimental data. Bagul, (Barron,,1983), indicate a new steady state formulation for countercurrent double pipe heat exchanger with constant parameter experimentally.

In the present study , we analyzed shell and tube heat exchanger as case study which used to heat a cold fluid (flow inside tube) by a hot fluid (flow inside the shell),we used two cases of the heat exchanger working fluid to analyzed the effect of the environment on the type of working fluids (low viscid fluid (water) and medium viscid fluid (Oil SAE-30)). The finite difference techniques used to solve the unsteady state differential equations for heat exchanger with environment heat interaction by convection and radiation heat transfer. The convection heat transfer coefficient between the heat exchanger wall and the environment are assumed variable with temperature and the heat exchanger wall is assumed a grey body.

Also, our analysis is considered to check the traditional assumption like steady state heat exchanger and perfect insulation of heat exchanger external wall.

### **Common Assumption**

The following assumptions are commonly made in derivative the heat exchanger governing equations are following:

1. The heat transfer from the heat exchanger wall with environment taken by convection and radiation.
2. The case study will used shell and tube heat exchanger, each pass of heat exchanger has the same heat transfer area; that is unsymmetrical pass arrangements are not considered.
3. The specific heat of each fluid is constant and independent temperature.
4. There is no phase change for the hot and cold fluids during the heat exchanger.
5. Hot and cold fluids are incompressible and turbulent in flow.
6. The hot, cold fluids temperatures subject to unsteady state behavior.
7. The convection heat transfer coefficient between the heat exchanger wall and the environment is taken as temperature dependence.
8. The hot fluid flow inside the shell and the cold fluid flow inside the pipes.
9. The heat exchanger wall is assumed a grey body.

Firstly, we derive the governor differential equations for the hot and cold fluids along the heat exchanger, then the heat exchanger is subdivided in to many elements volumes of length ( $\Delta x$ ) as shown in **Fig.(1)**. The hot fluid which flows inside the shell will reject heat to the cold fluid (inside the pipe) and for the environment fluid , these will be resulting reduction in the outlet hot fluid enthalpy and

internal stored thermal energy and in same time, these will increase the cold fluid enthalpy and internal stored thermal energy.

After applying the energy balance (energy, continuity, and momentum equations) of the hot and cold fluid leads to the differential equations in the system, as following<sup>(2),(3)</sup>:

### Continuity Equations

$$\vec{V} = u \mathbf{i} + v \mathbf{j} + w \mathbf{k} , \nabla \cdot \vec{V} = 0 , \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

### Momentum Equations

$$\rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] = - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (2)$$

### Energy Equations

$$\rho \frac{D(C_p T)}{Dt} = \Phi + \nabla \cdot (k \nabla T) \pm Q''' \quad (3)$$

$$\Phi = \mu \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + 2 \left( \frac{\partial w}{\partial z} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 \right] \quad (4)$$

Applying Navier-Stoke equation for the cold fluid as following:

1- Continuity Equations : Where  $v = w = 0$  , Then  $\frac{\partial u_c}{\partial x} = 0 \Rightarrow u_c$  is not depenenton x

2- Momentum Equation : Where  $v = w = 0$  , Then,  $\frac{\partial p_c}{\partial x} = -\rho_c \frac{\partial u_c}{\partial t} = 0 \Rightarrow p_c$  is not depenenton x

3- Energy Equation:  $\rho_c \frac{D(C_{pc} T_c)}{Dt} = \Phi_c + \nabla \cdot (k_c \nabla T_c) + Q_c''' \quad (5)$

$$\rho_c \frac{\partial (C_{pc} T_c)}{\partial t} = \rho_c u_x \frac{\partial (C_{pc} T_c)}{\partial x} + \rho_c u_y \frac{\partial (C_{pc} T_c)}{\partial y} + \rho_c u_z \frac{\partial (C_{pc} T_c)}{\partial z} + \frac{\partial}{\partial x} (k_c \frac{\partial T_c}{\partial x}) + \frac{\partial}{\partial y} (k_c \frac{\partial T_c}{\partial y}) + \frac{\partial}{\partial z} (k_c \frac{\partial T_c}{\partial z}) + \Phi_c + Q_c''' \quad (6)$$

Where  $v = w = 0$  , then  $\Phi_c = 0$  and

$$\rho_c C_{pc} \frac{\partial T_c}{\partial t} = \rho_c u_c C_{pc} \frac{\partial T_c}{\partial x} + k_c \left( \frac{\partial^2 T_c}{\partial x^2} + \frac{\partial^2 T_c}{\partial y^2} \right) + Q_c''' \quad (7)$$

$$\text{Where } L = L_c = L_h , u_c = \frac{\dot{m}_c}{\rho_c A_c} \quad (8)$$

$$\rho_c C_{pc} A_c \frac{\partial T_c}{\partial t} = \dot{m}_c C_{pc} \frac{\partial T_c}{\partial X} + k_c A_c \left( \frac{\partial^2 T_c}{\partial X^2} + \frac{\partial^2 T_c}{\partial Y^2} \right) + \frac{UA_c}{L_c} (T_h - T_c) \quad (9)$$

Finally

$$\rho_c C_{pc} V_c \frac{\partial T_c}{\partial t} = \dot{m}_c C_{pc} \frac{\partial T_c}{\partial X} L_c + k_c V_c \left( \frac{\partial^2 T_c}{\partial X^2} + \frac{\partial^2 T_c}{\partial Y^2} \right) + UA_c (T_h - T_c) \quad (10)$$

Applying Navier-Stoke equation for the hot fluid as following:

1- Continuity Equations : Where  $v = w = 0$  , Then  $\frac{\partial u_h}{\partial X} = 0 \Rightarrow u_h$  is not dependent on  $x$

2- Momentum Equation : Where  $v = w = 0$  , Then,  $\frac{\partial p_h}{\partial X} = -\rho_h \frac{\partial u_h}{\partial t} = 0 \Rightarrow p_h$  is not dependent on  $x$

3- Energy Equation:  $\rho_h \frac{D(C_{ph} T_h)}{Dt} = \Phi_h + \nabla \cdot (k_h \nabla T_h) - Q_h'''$  (11)

$$\rho_h \frac{\partial (C_{ph} T_h)}{\partial t} = \rho_h u_x \frac{\partial (C_{ph} T_h)}{\partial X} + \rho_h v_y \frac{\partial (C_{ph} T_h)}{\partial Y} + \rho_h u_z \frac{\partial (C_{ph} T_h)}{\partial Z} + \frac{\partial}{\partial X} (k_h \frac{\partial T_h}{\partial X}) + \frac{\partial}{\partial Y} (k_h \frac{\partial T_h}{\partial Y}) + \frac{\partial}{\partial Z} (k_h \frac{\partial T_h}{\partial Z}) + \Phi_h - Q_h'''$$
 (12)

Where  $u = w = 0$  , then  $\Phi_h = 0$  and

$$\rho_h C_{ph} \frac{\partial T_h}{\partial t} = \rho_h v_h C_{ph} \frac{\partial T_h}{\partial X} + k_h \left( \frac{\partial^2 T_h}{\partial X^2} + \frac{\partial^2 T_h}{\partial Y^2} \right) - Q_h'''$$
 (13)

Where  $v_h = \frac{\dot{m}_h}{\rho_h A_h}$  (14)

$$\rho_h C_{ph} A_h \frac{\partial T_h}{\partial t} = \dot{m}_h C_{ph} \frac{\partial T_h}{\partial X} + k_h A_h \left( \frac{\partial^2 T_h}{\partial X^2} + \frac{\partial^2 T_h}{\partial Y^2} \right) - \frac{UA_c}{L_h} (T_h - T_c) - \frac{U_\infty A_\infty}{L_h} (T_h - T_\infty)$$
 (15)

Finally

$$\rho_h C_{ph} V_h \frac{\partial T_h}{\partial t} = \dot{m}_h C_{ph} \frac{\partial T_h}{\partial X} L + k_h V_h \left( \frac{\partial^2 T_h}{\partial X^2} + \frac{\partial^2 T_h}{\partial Y^2} \right) L - UA_c (T_h - T_c) - U_\infty A_\infty (T_h - T_\infty)$$
 (16)

In dimensionless form, the final governor equations can be shown as following

For cold fluid

$$\frac{\partial \phi_c}{\partial \tau} = \frac{\partial \phi_c}{\partial Z} + \alpha_c t_c \left( \frac{\partial^2 \phi_c}{\partial Z^2} + \frac{\partial^2 \phi_c}{\partial Y^2} \right) + NTU (\phi_h - \phi_c)$$
 (17)

$$t_r \frac{\partial \phi_h}{\partial \tau} = \frac{\partial \phi_h}{\partial Z} + \alpha_h t_h \left( \frac{\partial^2 \phi_c}{\partial Z^2} + \frac{\partial^2 \phi_c}{\partial Y^2} \right) - NTU C_r (\phi_h - \phi_c) - UR_\infty C_r NTU (\phi_h - \phi_\infty) \quad (18)$$

Where

$$Z = \frac{x}{L}, Y = \frac{y}{L}, \tau = \frac{t}{t_c}, NTU = \frac{UA_c}{C_{\min}}, t_{c,h} = \frac{\rho_{c,h} V_{c,h}}{\dot{m}_{c,h}}, C_r = \frac{C_{\max}}{C_{\min}}, UR_\infty = \frac{U_\infty A_\infty}{UA_c} \quad (19)$$

$$t_r = \frac{t_h}{t_c} = \frac{C_r}{E_r}, E_r = \frac{\rho_c C_{pc} V_c}{\rho_h C_{ph} V_h}, \text{ and } \phi_{c,h,\infty}(x,t) = \frac{T_{c,h,\infty}(x,t) - T_{c,i}}{T_{h,i} - T_{c,i}} \quad (20)$$

When the velocities are large and temperature difference significant, we can omit the horizontal diffusion, (Ameel, Hewavitharana, 1999) then at the equilibrium we can write the governor equations as following:

$$\frac{\partial \phi_c}{\partial \tau} = \frac{\partial \phi_c}{\partial Z} + NTU (\phi_h - \phi_c) \quad (21)$$

$$t_r \frac{\partial \phi_h}{\partial \tau} = \frac{\partial \phi_h}{\partial Z} - NTU C_r (\phi_h - \phi_c) - UR_\infty C_r NTU (\phi_h - \phi_\infty) \quad (22)$$

### **Overall Heat Transfer Coefficient**

The overall heat transfer coefficient of heat transfer from fluid to fluid inside the heat exchanger:

$$UA_c = \frac{1}{R_{hi} + R_{wi} + R_{di} + R_{do} + R_c} \quad (23)$$

$$\text{Because of } R_{hi}, R_c \gg R_{wi}, \text{ then } UA_c \approx \frac{1}{R_{hi} + R_c + R_{di} + R_{do}} \quad (24)$$

Also, the overall heat transfer coefficient of heat transfer from hot fluid to environment fluid:

$$U_\infty A_\infty = \frac{1}{R_{ho} + R_{wo} + R_d + R_\infty} \quad (25)$$

$$\text{Because of } R_{ho}, R_\infty \gg R_{wo}, \text{ then } U_\infty A_\infty \approx \frac{1}{R_{ho} + R_d + R_\infty} \quad (26)$$

$$\text{Where } R_{hi} = \frac{1}{h_{hi} A_{hi}}, R_{ho} = \frac{1}{h_{ho} A_\infty}, R_\infty = \frac{1}{h_\infty A_\infty} \text{ and } R_c = \frac{1}{h_c A_c} \quad (27)$$

$$R_{wi} = \frac{\ln((d + b_i)/d)}{2\pi k_i n_p L} \quad \text{and} \quad R_{wo} = \frac{\ln((D + b_o)/D)}{2\pi k_o L} \quad (28)$$

$$A = A_{hi} = A_c \quad (29)$$

$$A = n_p \pi d L \quad (30)$$

$$A_\infty = \pi D L \quad (31)$$

The convective heat transfer coefficient from the heat exchanger wall to the environment is, (Walker,1982):

$$h_{\infty,c} = 7.2 \left( \frac{\dot{m} R T_{av}}{A_{flow} p} \right)^{0.78} = a_o T_{av}^{0.78} \quad (32)$$

and the radiation heat exchanger from the heat exchanger wall to the environment is,( Kawamura,1973):

$$h_{\infty,r} = \sigma \varepsilon F A_\infty (T_h^2 + T_\infty^2)(T_h + T_\infty) \quad (33)$$

$$h_\infty = h_{\infty,r} + h_{\infty,c} \quad (34)$$

For a known flow condition, Re and Pr numbers are calculated. The estimation of the heat transfer coefficient for forced convection of fluid within pipes can be estimated from the Dittus-Boelter equation,<sup>(2)</sup>:

$$Nu = 0.023 Re^{0.8} Pr^m \quad (35)$$

Where (m=0.3) for cooling and (m= 0.4) for heating.

The heat transfer coefficient correlation in the annulus area between two tubes is developed by Gnielinski <sup>(5)</sup>:

$$Nu = \frac{(f/8) Re Pr}{1.07 + 12.7(f/8)(Pr^{\frac{2}{3}} - 1)} \left[ 1 + \left( \frac{D_H}{L} \right)^{\frac{2}{3}} \right] \quad (36)$$

Where the friction factor is determined from the correlation developed by Filonenko,( Holman, 1997).

$$f = (1.82 \log_{10} Re - 1.64)^{-2} \quad (37)$$

$$Re = \frac{4\dot{m}}{\pi D_H \mu} \quad \text{and} \quad Pr = \frac{\mu C_p}{k} \quad (38)$$

About the fouling factor resistance, Andrews,( Aulds, Barron,1967), shows the fouling factor for the heat exchanger can be calculated from the properties of the fouling deposit , the following values

have been taken for fouling deposit:  $k_{ff} = 0.9$  W/mK at a temperature  $T = 273$  K and  $k_{ff} = 1$  W/mK at a temperature  $T = 373$  K. A constant value of  $1,000$  kg/m<sup>3</sup> has been taken as the fouling deposit density and the maximum thickness values considered for the conical fouling distribution were 1 mm on the tube's outer surface and 0.5 mm on the tube's inner, (kawamura, 1973)..

### Solution Analysis

Finite difference method is used to solve the above differential Eqs. (3) and(4):

$$\frac{\phi_c^{i+1,j} - \phi_c^{i,j}}{\Delta Z} + NTU(\phi_h^{i,j} - \phi_c^{i,j}) = \frac{\phi_c^{i,j+1} - \phi_c^{i,j}}{\Delta \tau} \quad (39)$$

$$\frac{\phi_h^{i+1,j} - \phi_h^{i,j}}{\Delta Z} - NTU C_r(\phi_h^{i,j} - \phi_c^{i,j}) - UR_\infty^{i,j} C_r NTU(\phi_h^{i,j} - \phi_\infty) = t_r \frac{\phi_h^{i,j+1} - \phi_h^{i,j}}{\Delta \tau} \quad (40)$$

After arrangement

$$\phi_c^{i,j+1} = \frac{\Delta \tau}{\Delta Z} \phi_c^{i+1,j} + \Delta \tau NTU \phi_h^{i,j} + (1 - \frac{\Delta \tau}{\Delta Z} - \Delta \tau NTU) \phi_c^{i,j} \quad (41)$$

$$\phi_h^{i,j+1} = \frac{\Delta \tau}{t_r \Delta Z} \phi_h^{i+1,j} + NTU E_r \Delta \tau \phi_c^{i,j} + \Delta \tau UR_\infty^{i,j} E_r NTU \phi_\infty + (1 - \frac{\Delta \tau}{t_r \Delta Z} - NTU E_r \Delta \tau - \Delta \tau UR_\infty^{i,j} E_r NTU) \phi_h^{i,j} \quad (42)$$

Where the values of the time increment ( $\Delta \tau$ ) that satisfied the above equations:

$$\Delta \tau = \frac{1}{(1/\Delta Z) + NTU} \text{ for the cold fluid} \quad (43)$$

$$\Delta \tau = \frac{t_r}{(1/t_r \Delta Z) + UR_\infty^{i,j} E_r NTU + E_r NTU} \text{ for the hot fluid} \quad (44)$$

In our analysis, we will select the lower value of the time increment ( $\Delta \tau$ ) in above Eqs. (43) and (44) to avoid the negative terms (fluctuation) in Eqs. (41) and (42).

To complete the above solution, we will be applying of appropriate boundary and initial conditions, which are expressed as :

$$\text{Initial Condition: } \phi_h = 1 \quad \& \quad \phi_c = 0 \quad \text{at } t = 0 \quad (0 \leq x \leq L) \quad (45)$$

$$\text{Boundary conditions: } \phi_h = 1 \quad \text{at } z = 0, \quad \phi_c = 0 \quad \text{at } x = 0 \quad (46)$$

### The Effectiveness ( $\epsilon$ )

The effectiveness heat exchanger ( $\epsilon$ ) is commonly knowing as the as the ratio of the actual heat transfer rate ( $Q_{\text{actual}}$ ) that reject or receive from fluid to the maximum possible heat transfer rate (ideal)



$(Q_{\max})$  accruing between the heat exchanger fluid; the value of the heat exchanger effectiveness varies between 0 and 1, (3,4,5,6).

$$\varepsilon = \frac{Q_{\text{actual}}}{Q_{\max}} \quad (47)$$

The maximum possible heat transfer rate can be occur when the exit temperature of the cold fluid equal the inlet temperature of the hot fluid and the exit temperature of the hot fluid equal the inlet temperature of the cold fluid.

The main object of our study to investigate the effect of the environment conditions on the heat exchanger effectiveness. The effectiveness of the cold, hot fluid as following:  
for the hot fluid :

$$\varepsilon_h = \frac{Q_h}{Q_{\max,h}} \quad (48)$$

$$Q_h = \dot{m}_h C_{ph} \sum_{j=1}^{nx} (T_{h,j} - T_{h,j+1}) \quad (49)$$

$$Q_{\max,h} = \dot{m}_h C_{ph} (T_{h,i} - T_{c,i}) \quad (50)$$

$$\text{Then } \varepsilon_h = \frac{\sum_{j=1}^{nx} (T_{h,j} - T_{h,j+1})}{(T_{h,i} - T_{c,i})} \quad (51)$$

for the cold fluid :

$$\varepsilon_c = \frac{Q_c}{Q_{\max,c}} \quad (52)$$

$$Q_c = \dot{m}_c C_{pc} \sum_{j=1}^{nx} (T_{c,j+1} - T_{c,j}) \quad (53)$$

$$Q_{\max,c} = \dot{m}_c C_{pc} (T_{h,i} - T_{c,i}) \quad (54)$$

$$\text{Then } \varepsilon_c = \frac{\sum_{j=1}^{nx} (T_{c,j+1} - T_{c,j})}{(T_{h,i} - T_{c,i})} \quad (55)$$

The heat removed from the hot fluid to the environment at steady state is

$$Q_h = Q_c + Q_{\infty} \quad (56)$$

$$Q_{\infty} = \dot{m}_h C_{ph} \sum_{j=1}^{nx} (T_{h,j} - T_{h,j+1}) - \dot{m}_c C_{pc} \sum_{j=1}^{nx} (T_{c,j+1} - T_{c,j}) \quad (57)$$

In same manner that used for the cold and hot fluid, the steady state effectiveness of the environment fluid can expressed as:

$$\varepsilon_{\infty} = \frac{Q_{\infty}}{Q_{\max}} = \frac{Q_h - Q_c}{Q_{\max}} = \frac{Q_h}{Q_{\max}} - \frac{Q_c}{Q_{\max}} = \varepsilon_h - \varepsilon_c \quad (58)$$

### **The Entropy Generation Number (N<sub>S</sub>)**

The entropy increase of the universe as the result of a process is the sum of the entropy changes of all elements that are involved in that process. There are two sources of entropy generation in heat exchangers. One is due to heat exchange between two fluid streams of different temperature, and the second one due to viscosity (frictional pressure drop) of moving fluids. The entropy generation rate due to viscosity is usually much smaller than the generation due to heat exchange, so it can be neglected, (Bejan,1982). These losses refer to irreversibility quantity, and some methods have been devised for minimizing these losses, (Hesselgreaves,2000).

The rate of entropy production of the universe,

$$\dot{S}_{Un} = \dot{S}_{Un,h} + \dot{S}_{Un,c} + \dot{S}_{Un,\infty} + \dot{S}_{Un,w} \quad (59)$$

Where  $\dot{S}_{Unw} = 0$ , since these term is constant with time.

Since the entropy generation can expressed as following:

$$\frac{dS_{rev}}{dt} = \dot{S}_{rev} = \frac{Q_{rev}}{T_{rev}} \quad (60)$$

Then, in the analyzing of the heat exchangers must be determined the heat transfer for each term are:

$$Q_h = - \int_{A_c} U(T_h - T_c) dA - \int_{A_{\infty}} U_{\infty} (T_h - T_{\infty}) dA = -2\pi dU \sum_{j=1}^{nx} (T_{h,j} - T_{c,j}) \Delta x - 2\pi DU_{\infty} \sum_{j=1}^{nx} (T_{h,j} - T_{\infty}) \Delta x \quad (61)$$

$$Q_c = \int_{A_c} U(T_h - T_c) dA = 2\pi dU \sum_{j=1}^{nx} (T_{h,j} - T_{c,j}) \Delta x \quad (62)$$

$$Q_{\infty} = \int_{A_{\infty}} U_{\infty} (T_h - T_{\infty}) dA = 2\pi DU_{\infty} \sum_{j=1}^{nx} (T_{h,j} - T_{\infty}) \Delta x \quad (63)$$

The entropy generation rate due to heat transfer between the heat exchanger working fluid in Eq.(60) can defined as, (Hesselgreaves,2000).

$$\dot{S}_{Un} = - \int_{A_h} \frac{Q_h}{T_h} dA + \int_{A_c} \frac{Q_c}{T_c} dA + \int_{A_{\infty}} \frac{Q_{\infty}}{T_{\infty}} dA = - \int_{A_h} \frac{Q_c + Q_{\infty}}{T_h} dA + \int_{A_c} \frac{Q_c}{T_c} dA + \int_{A_{\infty}} \frac{Q_{\infty}}{T_{\infty}} dA \quad (64)$$

$$\dot{S}_{Un} = - \int_{A_{hi}} \frac{Q_c}{T_h} dA - \int_{A_{ho}} \frac{Q_{\infty}}{T_h} dA + \int_{A_c} \frac{Q_c}{T_c} dA + \int_{A_{\infty}} \frac{Q_{\infty}}{T_{\infty}} dA \quad (65)$$

Since  $A_{hi}=A_{\infty}$  and  $A_{ho}=A_c$ , then the Eq.(65) can be written as:

$$\dot{S}_{Un} = - \int_{A_{\infty}} \frac{Q_c}{T_h} dA - \int_{A_c} \frac{Q_{\infty}}{T_h} dA + \int_{A_c} \frac{Q_c}{T_c} dA + \int_{A_{\infty}} \frac{Q_{\infty}}{T_{\infty}} dA = \int_{A_{\infty}} Q_{\infty} \left( \frac{1}{T_{\infty}} - \frac{1}{T_h} \right) dA + \int_{A_c} Q_c \left( \frac{1}{T_c} - \frac{1}{T_h} \right) dA \quad (66)$$

$$\dot{S}_{Un} = \int_{A_{\infty}} Q_{\infty} \left( \frac{1}{T_{\infty}} - \frac{1}{T_h} \right) dA + \int_{A_c} Q_c \left( \frac{1}{T_c} - \frac{1}{T_h} \right) dA = 2\pi D U_{\infty} \sum_{j=1}^{nx} \frac{(T_{h,j} - T_{\infty})^2}{T_{\infty} T_{h,j}} \Delta x + 2\pi d U \sum_{j=1}^{nx} \frac{(T_{h,j} - T_{c,j})^2}{T_{c,j} T_{h,j}} \Delta x \quad (67)$$

Because of the  $T_h > T_c$  and  $T_{\infty}$ , the value of entropy generation in Eq.(67) is always greater than zero (positive).

Bejan, used a parameter called entropy generation number for minimizing both losses, and described this parameter as the ratio between entropy generation rate and the heat capacity rate. The entropy generation number limit  $N_s \rightarrow 0$  implies that these losses approach zero, and that these losses increase when  $N_s$  has higher values. Entropy generation can be written as, (Hesselgreaves,2000).

$$N_s = \frac{\dot{S}_{Un}}{C_{max}} = \frac{2\pi D U_{\infty}}{C_{max}} \sum_{j=1}^{nx} \frac{(T_{h,j} - T_{\infty})^2}{T_{\infty} T_{h,j}} \Delta x + \frac{2\pi d U}{C_{max}} \sum_{j=1}^{nx} \frac{(T_{h,j} - T_{c,j})^2}{T_{c,j} T_{h,j}} \Delta x \quad (68)$$

### Case Study

In our case study, The information of the countercurrent shell and tube heat exchanger, (Bagui,2002), as mentioned below in **Table (1), (2) and (3)**:

### Results and Discussion

Firstly before the results discussion, we must check the validity of our technique (computer program) by compared the results of the present program with solved example for the fully insulated steady state shell and tube heat exchanger, (Holman,1997), from **Fig.(2)**, the results of the present program is very acceptable with the initial and final values of the example.

Our techniques is used to evaluate the heat exchanger effectiveness under conditions of heat transfer between hot fluid (annulus fluid) and the environment. In the present study, we will study this effect.

**Fig.(3)**, shows the effect of the heat exchanger external wall emissivity on the effectiveness of the hot, cold and environment fluid. We can notes the increase of the emissivity value will decrease the hot fluid effectiveness, because the increase of the emissivity will increase the heat transfer from the hot fluid to the environment and then decrease the hot fluid temperature values and effectiveness.

Also, we can notes the increase of the emissivity will decrease the effectiveness of the cold fluid because the increase of emissivity will decrease the heat transfer from the hot fluid to cold fluid due to decrease of the hot fluid temperature and then decrease the cold fluid effectiveness (see **Fig.(4)**). The effectiveness of the environment fluid is plotted in **Fig.(5)**, the effect of the environment fluid temperature and emissivity depend on the hot fluid temperature, then the increase of the emissivity will decrease the environment effectiveness as hot fluid temperature decrease.

Also from the previous figures we can notes the time of the reach steady state value is increased as the emissivity increase because the increasing of the emissivity will increase the heat transfer to the environment and also the environment heat transfer coefficient (radiation and convection) is variable with the temperature, then the increase of these factors will increase the time for reaching the steady state value.

The effect of the heat transfer coefficient on the effectiveness are plotted in **Figs. (6 and 7)** for both cold and hot fluid. The convection coefficient effect is low compared with the radiation coefficient effect because the radiation coefficient is proportional with third power of the temperature summation while convection coefficient is proportional with temperature difference to power (0.78). Because of the convection resistance is added with the radiation resistance to generate the environment fluid resistance, then the increase of the convection resistance is low compared with the radiation resistance and also because we used air as environment fluid, then convection resistance for air is low (heat transfer coefficient for the air is low) compared with other fluid convection resistance. The increase of the convection coefficient is decrease the hot fluid effectiveness relatively low and also relatively small time increase to reach the steady state for the same reasons mentioned above.

The effect of environment temperature, on the hot fluid effectiveness and the cold fluid temperature effectiveness is relatively minor with the present of radiation and high hot fluid convection coefficient inside the shell, as shown in **Figs. (8 and 9)**.

The increase of the environment temperature is reduce the convection effect and increase the radiation effect by increase the radiation resistance and also decrease the temperature difference between the hot and environment fluids. The combination of the previous effect will produce small reduction in the hot and cold fluid effectiveness.

The effect of present and non-present of the environment resistances is plotted in **Fig. (10)**, from this figures, the effect of the radiation is very noticeable compared with convection effect, also we can notes the reducing of the hot fluid effectiveness will increase the time for reaching to steady state because increasing of the heat removed from the hot fluid will need more time to steady state.

For the same properties of **Figs (3 and 4)** is plotted for different working fluid (oil) (case 2 in **Table 2**) in **Figs. (11 and 12)**. The effect of the environment resistance here become low compared with case-1 (**Table 2**), because the oil has heat transfer coefficient lower than for water in case-1 (because the oil has greater viscosity from air, then the oil has lower Reynolds number for same properties) and because inverse of the hot fluid resistance is added with the inverse of the environment heat coefficient to generate overall heat transfer coefficient with environment, then the inverse of the hot fluid resistance become too large compared with the inverse of the environment resistance.

The effect of the entropy generation number is discussed here, from **Fig.(13)**, we can notes the increase of the emissivity will increase the entropy generation because this factor is increase the heat transfer from the hot fluid to environment fluid and then increase the entropy. The effect of the NTU on the entropy generation number is plotted in **Fig. (14)** for different values of  $C_r$ , the increase of NTU will reduce  $N_s$ , because increase of the NTU will decrease the temperature of the hot and cold fluid and then decrease the ( $N_s$ ) because the entropy generation number is proportional inversely with the fluid temperatures, also increase of the  $C_r$  will decrease the  $N_s$  curve, due to increase of the  $C_r$  will effect to increase the fluid temperature drop and because  $C_r$  in the denominator of the  $N_s$ , then the increase of  $C_r$  will decrease  $N_s$ . Also, the effect of the of the  $C_r$  on the  $N_s$  for different values of NTU are plotted in **Fig.(15)**, the increase of the  $C_r$  will decrease the  $N_s$  for the same reasons mentioned previously.

## **Conclusions**

Heat transfer between the environment fluid and heat exchanger fluid may be unavoidable in some situations, failure to account for the effects of this typically neglected heat transfer will result in an incorrectly sized heat exchanger and erroneous performance predictions specially in cryogenics. Then our research is considered to study the effect of the environment fluid (air) on the heat exchanger effectiveness. From the results and discussion, we can notes the following issues:

- 1- The environment conditions is very important for the case of the hot fluid is subject to environment fluid and the radiation effect must considered in the environment effect.

- 2- The environment effect may be neglected if we used a medium or high fluid viscosity and this effect is appear when we used a low fluid viscosity (like water and air).
- 3- Also, in order to reduce the environment effect, the designer must taken in account increasing of the insulation thickness of heat exchanger external wall especially when the heat exchanger used in extremely environment and the working fluid is low viscosity.
- 4- The effect of the heat exchanger emissivity, environment heat transfer coefficient, environment temperature, present and non-present environment resistance are discussed.
- 5- The increasing of the emissivity, environment heat coefficient and environment temperature will decrease the hot fluid effectiveness and increase the entropy generation number.
- 6- Also, we can notes the assumption of the perfectly insulated of the heat exchanger wall become correct as the viscosity of the hot fluid increase, but for large heat exchanger with low viscosity, the heat exchange with environment must be taken in account.
- 7- The assumption of the steady state is not valid for the critical design application.

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**Table1 : Physical geometry of the heat exchanger**

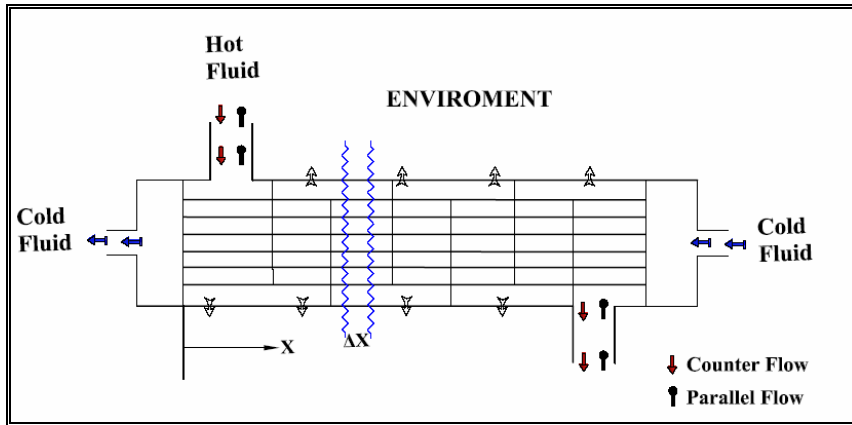
<b>Tube material</b>	Admiralty	<b>Number of tubes</b>	38	<b>Shell diameter (m)</b>	0.20
<b>Tube arrangement</b>	$\Delta$ pitch 60°	<b>Baffles shell clearance</b>	0.005	<b>Baffles cut (%)</b>	15
<b>Tube passes</b>	2	<b>Tube conductivity (W/mK)</b>	112.7	<b>Extreme baffles spacing (m)</b>	0.25
<b>Tube diameter (m)</b>	0.019	<b>Central baffles spacing (m)</b>	0.24	<b>Number of baffles</b>	18
<b>Tube spacing (m)</b>	0.024	<b>Baffles tube clearance</b>	0.000005	<b>Tube length (m)</b>	4.56

**Table 2 : Properties of different fluids that used in the present analysis**

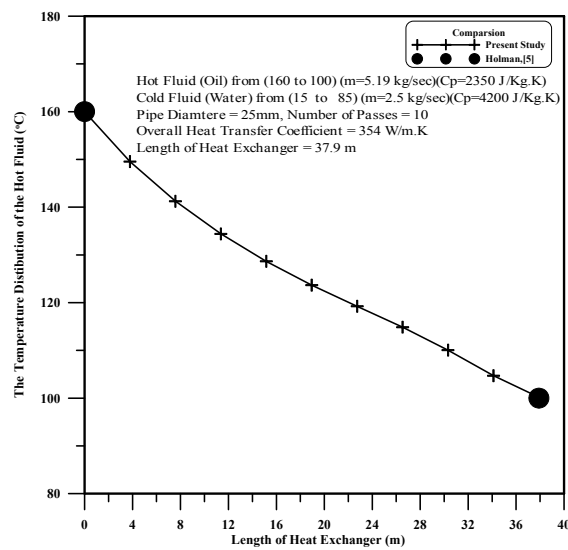
<b>Parameter</b>	<b>Water Properties (Low Viscosity Fluid)</b>			<b>SAE-30 Oil Properties (Medium Viscosity Fluid)</b>	
	<b>303 K</b>	<b>373 K</b>	<b>400 K</b>	<b>303 K</b>	<b>373 K</b>
<b><math>\mu</math> (Pa.S)</b>	$8.5 \cdot 10^{-4}$	$3 \cdot 10^{-4}$	$3 \cdot 10^{-4}$	$5 \cdot 10^{-2}$	$10^{-2}$
<b><math>\rho</math> (Kg/m<sup>3</sup>)</b>	993	927	902	885	851
<b>Cp (J/Kg.°C)</b>	4185	4220	4220	1998	2206
<b>k (W/m.°C)</b>	0.623	0.689	0.689	0.161	0.155

**Table 3 : Operation conditions for the two cases study**

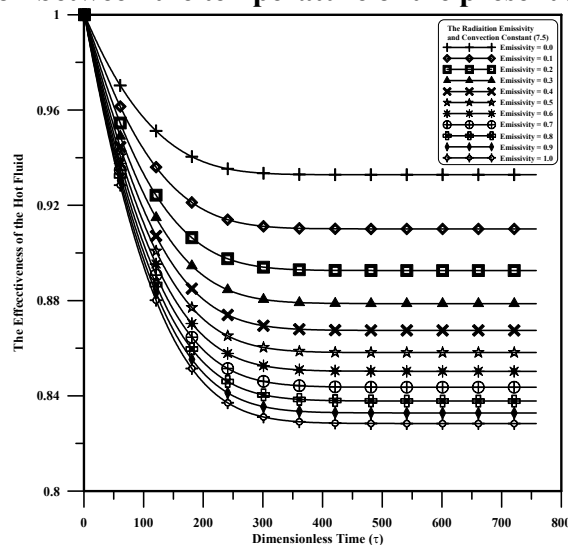
<b>Parameters</b>	<b>Case 1</b>		<b>Case 2</b>	
	<b>Shell Side</b>	<b>Tube Side</b>	<b>Shell Side</b>	<b>Tube Side</b>
<b>Fluid</b>	<b>Water</b>	<b>Water</b>	<b>SAE-30</b>	<b>Water</b>
<b>Flow (Kg/S)</b>	18.2	9.1	18.2	9.1
<b>Inlet Temperature (K)</b>	370	305	370	305



**Fig. (1): Schematic of shell and tube heat exchanger**



**Fig.(2): The comparison between the temperature of the present study and reference (5)**



**Fig.(3): The effect of the emissivity on the hot fluid effectiveness**

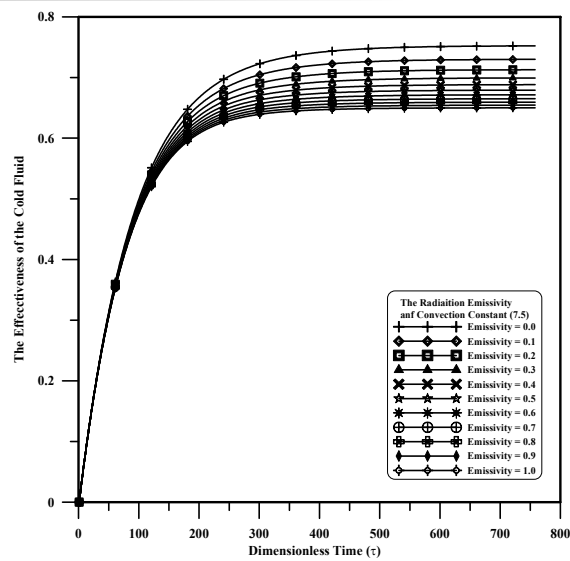


Fig.(4): The effect of the emissivity on the cold fluid effectiveness

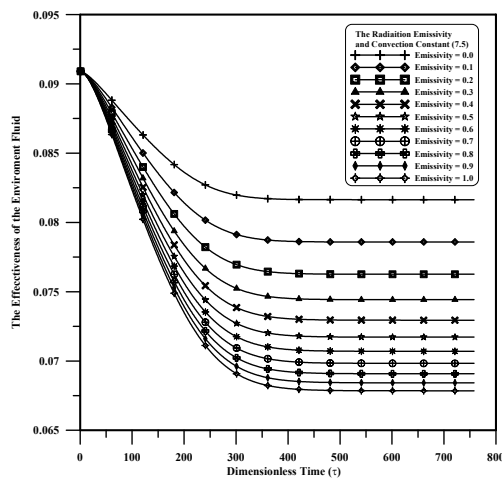


Fig.(5): The effect of the emissivity on the environment fluid effectiveness

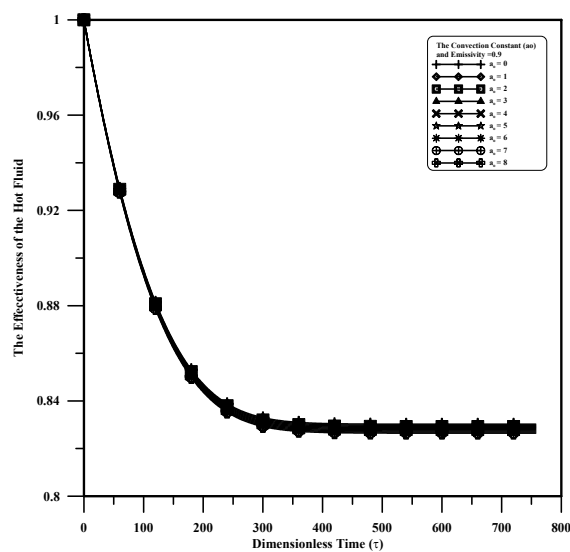


Fig.(6): The effect of the environment convection coefficient on the hot fluid effectiveness



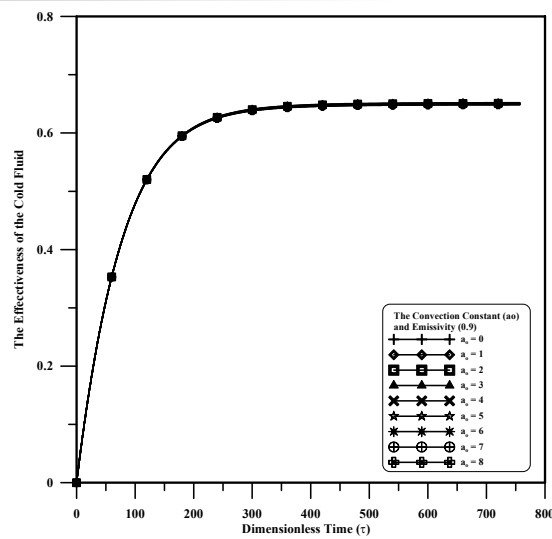


Fig.(7): The effect of the environment convection coefficient on the cold fluid effectiveness

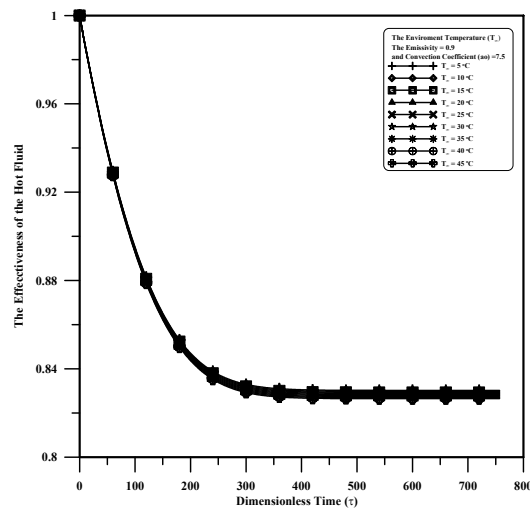


Fig.(8): The effect of the environment temperature on the hot fluid effectiveness

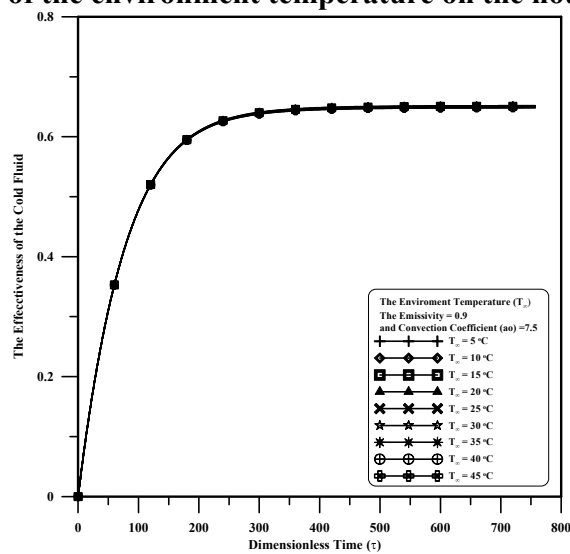


Fig.(9): The effect of the environment temperature on the cold fluid effectiveness

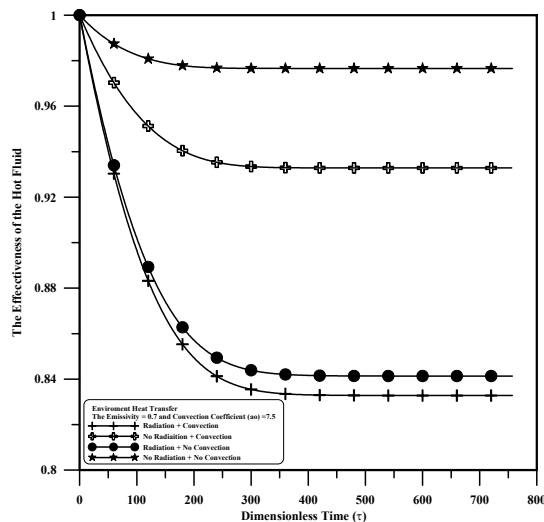


Fig.(10): The effect of the present and non-present radiation and convection environment resistance on the hot fluid effectiveness

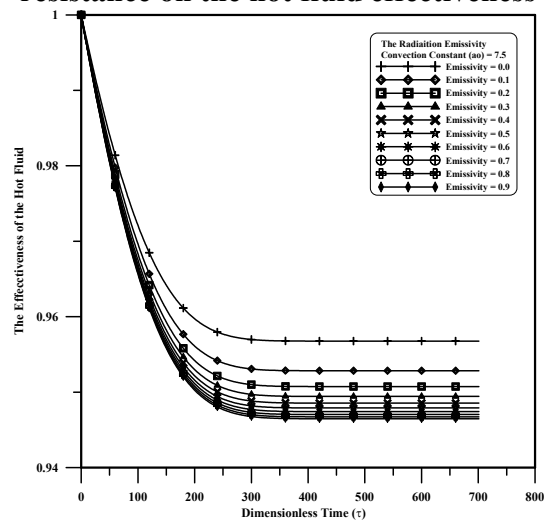


Fig.(11): The effect of the emissivity on the hot fluid effectiveness (Oil)

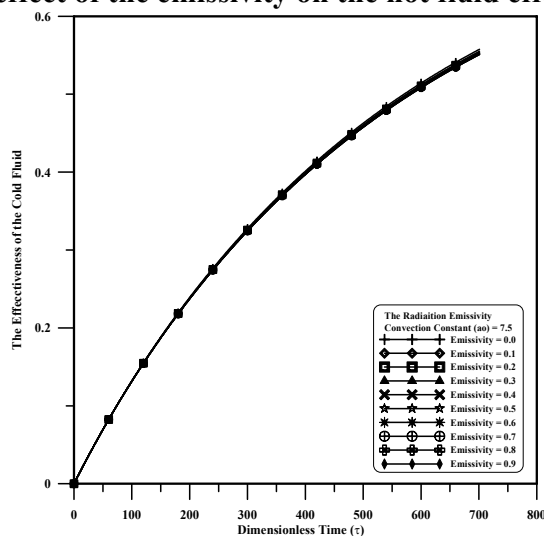


Fig.(12): The effect of the emissivity on the cold fluid effectiveness (Oil)

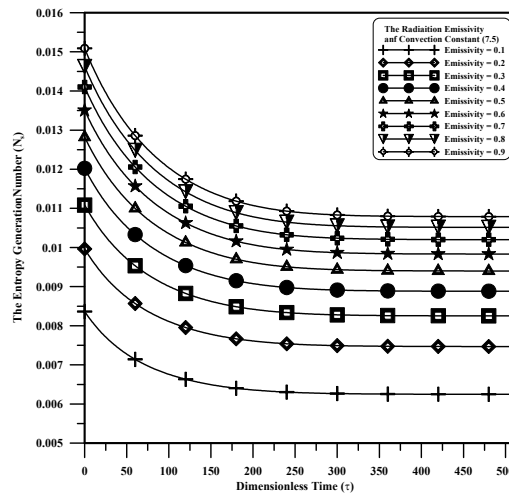


Fig.(13): The effect of the emissivity on the entropy generation number for (NTU = 0.1009 and  $C_r = 0.5011933$ )

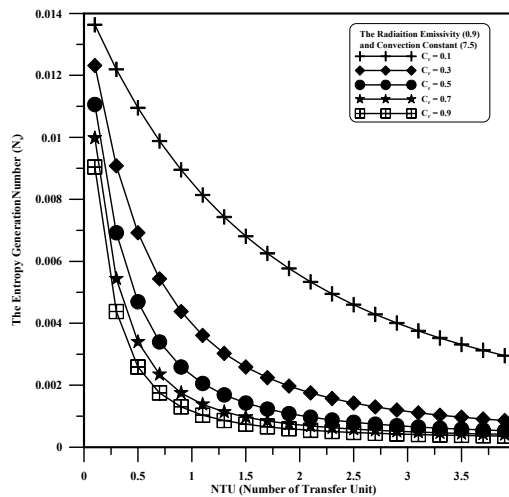


Fig.(14): The effect of the number of transfer unit on the entropy generation number for different  $C_r$

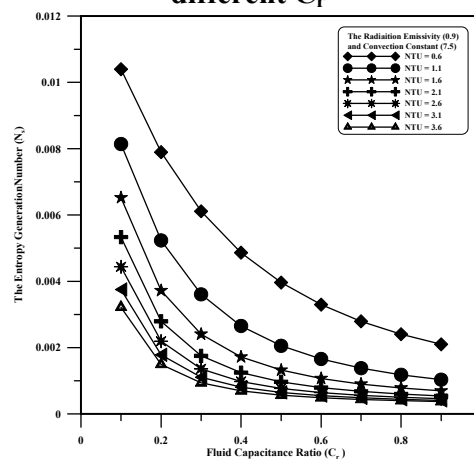


Fig.(15): The effect of the fluid capacity ratio ( $C_r$ ) on the entropy generation number for (NTU = 0.1009)