The primal element in integral domain

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Abstract

An element x of an integral domain R is called primal if whenever x divides a product a_1a_2 with a_1 , $a_2 \in$ R, x can be written as $x = x_1x_2$ such that x_i divides a_i ,i =1,2.We study whenin X^2 primal in A + X B[X] or A + X B[[X]], when $A \subseteq B$ be an extension of domains. Also we show that if A is an integral domain and $S \subseteq A$ a splitting multiplicative system, then $A+XA_S[X]$ is a semirigid GCD-domain if and only if A is a semirigid GCD-domain and for each two elements of S, one of them divides the other.

Keyword: Integral Domain, Primal element, Principle ideal, Semirigid GCD-Domains.

Introduction:

Let $A \subseteq B$ be an extension of integral domain and X an indeterminate. In this paper, we study some arithmetic properties of the subring A +X B[X] (resp.A + X B[[X]]) of B [X] (resp. B [[X]]). According to (P. M. Cohn)[1] ,an element x of an integral domain R is called primal if whenever x divides a product a_1a_2 with a_1 , $a_2 \in$ R, x can be written as $x = x_1x_2$ such that x_i divides a_i ,i =1,2 (an element whose divisors are primal elements is called completely primal). A domain R is called GCD-Domain if every pair of elements of R has a greatest common divisor. Let A be a domain and $S \subseteq A$ a saturated multiplicative system of A. A nonzero element $a \in A$ is said to be LCM-prime to S[2], if $aA \cap tA = atA$ (equivalently tA : a = tA) for each $t \in S$. S is said to be a splitting multiplicative system [3], if each nonzero element x of A can be written as $x = as$, where a is LCM-prime to S and $s \in S$. As in[3], an extension of rings $A \subseteq B$ is called inert if whenever $xy \in A$ for nonzero $x,y \in B$, then xu, $yu^{-1} \in A$ for some $u \in U(B)$. An element x of an integral domain R is called an extractor[4], if $xR \cap yR$ is a principal ideal for each $y \in R$.

In Section 1, we prove that X is primal in $A + X B[X]$ or $A + X B[[X]]$ if and only if $B = A_S$ and S is good, where $S = U(B) \cap A$ (we say that S is good if whenever $s \in S$, $a,b \in A \setminus \{0\}$ and $s \mid_A ab$, there exists $t \in S$ such that $t \mid_A a$ and $s \mid_A tb$). If $n \geq 2$ and $S = U(B) \cap A$, we prove that X^n is primal in $A + X B[X]$ or $A + X B[[X]]$ if and only if S is good, $A_S = B \cap Q(A)$ and for each $b \in B$ there exists $c \in U(B)$ such that $bc \in A$. We also include some remarks about the goodness of a multiplicative system.

In Section 2 we study when $A + X B[X]$ is a semirigid GCD-domain. We recall that, according to (M. Zafrullah 1975,1987,1988)[5,6,7], an element x of integral domain R is called rigid if whenever $r,s \in R$ and $r,s \mid x$, we have $s \mid r$ or $r \mid s$. Also R is called semirigid if every nonzero element of R can be expressed as a product of a finite number of rigid elements. We show that if A is an integral domain and $S \subseteq A$ a splitting (saturated) multiplicative system, then $A + X A_S[X]$ is a semirigid GCD-domain if and only if A is a semirigid GCD-domain and for each two elements of S, one of them divides the other.

Throughout, all rings are commutative with unit element and subrings have the same unit element. If A is a domain, then $U(A)$ denotes the set of invertible elements of A and A_S denotes the quotient ring of A with respect to the multiplicative S. Any unexplained notation or terminology is standard as in (R. Gilmer)[8,9].

1. Primal elements

In this section we study the primality of X^n in domain of type $A + X B[X]$ or $A + X B[[X]]$. When $n = 1$, this primality forces B to be a fraction ring of A, that is:

Remark 1.1. If $A \subseteq B$ is an extension of domains and X is primal in $A + X B[X]$ or $A + X B[[X]]$, then $B=A_S$ where $S = U(B) \cap A$. Indeed, if R denotes $A + X B[X]$ or $A + X B[[X]]$ and $0 \neq b \in B$, then X divides $(bX)^2$ in R. So, there exist $f, g, u, v \in R$, $f(0) \neq 0$, such that $X=fg$ and $bX=fu=gv$. If g' denotes the (formal) derivative of g, then $1=f(0)g'(0)$, so $f(0) \in S$ and $b=g'(0)v(0)= v(0) / f(0) \in A_S$.

The next result describes the primality of X in $A + X B[X]$ or $A + X B[[X]]$. When f is a nonzero power series (polynomial), the order of f is denoted by ord(f).

Theorem 1.2: Let A be a domain and $S \subseteq A$ a saturated multiplicative system. The following assertions are equivalent:

(a) . X is primal in $A + X A_S [X]$,

(b) .X is primal in $A + X A_S[[X]]$,

(c) .If $s \in S$, $a,b \in A$ and $s \mid ab$, there exists $t \in S$ such that $t \mid a$ and $s \mid tb$

(let us agree to say that S is good if it satisfies property (c)).

Proof**:** Set $R = A + X A_S[X]$ or $A + X A_S[[X]]$. First, we prove that ((a) or (b)) implies (c). Let $s \in S$ and $a,b \in A$ such that $s \mid_A ab$. Then $X \mid_R a(bX/s)$, so there exists $t \in S$ such that $X = t(X/t)$, $t \mid_R a$ and $(X/t) \mid_R (bX/s)$. So $t \mid_A a$ and $s \mid_A bt$. Conversely, we prove, that (c) implies ((a) and (b)). Assume that $X \mid_R fg$ with $f,g \in R\setminus \{0\}$ and $\text{ord}(f) \leq \text{ord}(g)$. If $\text{ord}(g) \geq 2$, then $X \mid_R g$. If $\text{ord}(f) = \text{ord}(g) = 1$, then $0 \neq g'(0) = b/s$ with $b \in A$, $s \in S$, hence $X = s(X/s)$ and $s \mid_R f$, $(X/s) \mid_R g$ (again, g' denotes the formal derivative of g). If $\text{ord}(f) = 0$ and $\text{ord}(g) = 1$, then $0 \neq f(0) = a \in A$, $0 \neq g'(0) = b/s$ with $b \in A$, $s \in S$ and $ab/s \in A$. Since S is good, there exists $t \in S$ such that a/t, $bt/s \in A$. Hence $X = t(X/t)$ and $t \mid_R f$, $(X/t) \mid_R g$.

Remark 1.3 : Let A be a domain and $S \subseteq A$ a saturated multiplicative system of A.

(a). If S is a splitting multiplicative system, then S is good. Indeed, assume that $s \mid ab$ with $s \in S$ and $a,b \in A$. We can write $a = a't$ with $t \in S$ and a' LCM-prime to S , so $s \mid bt$.

(b). If S is consisting of (completely) primal elements, then S is good. Indeed, If $s \mid ab$ with $s \in S$ and $a,b \in A$, then $s = tu$ (so $t,u \in S$), such that $t \mid a$ and $u \mid b$, hence $s \mid bt$.

(c). If A is atomic and S is good, then S is splitting. Indeed, it suffices to show that each atom a not belonging to S is LCM-prime to S. If $b \in A$, $s \in S$ and $s \mid ab$, then $s \mid b$, because S is good and a has only unit divisors in S. As a specific example, we can consider the atomic domain $A = k[X^2, XY, Y^2]$, where k is a field, X,Y are indeterminate and S is the saturated multiplicative system generated by X^2. Here the atom XY is not LCM-prime to S, because both X^2, XY divide $(XY)^2$ but their product does not.

(d). S is good if and only if for each $a \in A$, $aA_S \cap A = \cup_{t \in S,\, t \mid a} (a/t)A$. Indeed, to see that the condition is necessary, let $x \in aA_S \cap A$. Then, there exists $s \in S$, $b \in A$ such that $sx = ab$, hence $x = ab/s$, so there exists $t \in S$ such that a/t, $tb/s \in A$, therefore $x = (a/t)(bt/s) \in (a/t)A$. Conversely, if $a, b \in A$, $s \in S$ such that $x = ab/s \in A$, then $sx = ab \in aA_S \cap A$, so there exist $t \in S$ with $t \mid a$ and $c \in A$ such that $x = (a/t)c$, whence $(ab)/s = (a/t)c$, therefore $bt/s = c \in A$.

(e). Consequently, S is splitting if and only if S is good and every nonzero $a \in A$ has a divisor $t \in S$, such that any other divisor $w \in S$ of a divides t.

(f). Let T be the m-complement of S [(4)]. Then , cf. [(10)] each nonzero prime ideal P disjoint of S contains an LCM-prime to S element if and only if the saturation of ST is $A \setminus \{0\}$ (that is, for each nonzero x, there exists x' in A, $s \in S$, $t \in T$ such that $xx' = st$). If S is good and every prime ideal P disjoint of S contains an LCM-prime to S element, then S is splitting. Indeed, if x is a nonzero element of A, then $xx' = st$ for some $x' \in A$, $s \in S$, $t \in T$ .So $s \mid xx'$ and, since S is good, $w \mid x$ and $s \mid wx'$ for some $w \in S$. Now since T is saturated and $t = (x/w)(wx'/s)$, $x/w \in T$, so $x = w(x/w)$. That is S is splitting.

Corollary 1.4: Let A be a domain and S a saturated multiplicative system. Then X is completely primal in $A + X A_S[X]$ or $A + X A_S[[X]]$ if and only if S is consisting of (completely) primal elements of A.

Proof: Let R denote $A + X A_S[X]$ or $A + X A_S[[X]]$. Notice that, if $s \in S$, s is primal in A if and only if s is primal in R . Indeed, if s is primal in A and $s \mid_R fg$, $f,g \in R$, then $s \mid_A f(0)g(0)$, hence s can be written as $s = tu$ with $t,u \in S$ such that $t \mid_A f(0)$, $u \mid_A g(0)$, thus $t \mid_R f$ and $u \mid_R g$. Since the divisors of X in R are of type s or X/s with $s \in S$ and since R has an automorphism sending X/s, the assertion of Corollary 1.4 follows from Remark 1.3 (b).

The next result describes the primality of X^n in $A + X B[X]$ or $A + X B[[X]]$, when $n \geq 2$. Here, if B is a domain and $b \in B$, $b\,U(B) = \{ bw; w \in U(B)\}$.

Theorem 1.5: Let $A \subseteq B$ be an extension of domains, $S = U(B) \cap A$ and K the quotient field of A. Let R denote $A + X B[X]$ or $A + X B[[X]]$. The following statements are equivalent:

(a). X^2 is primal in R,

(b). X^n is primal in R for some $n \geq 2$,

(c). X^n is primal in R for each $n \geq 2$,

(d). $b\,U(B) \cap A \neq \emptyset$ for each $b \in B$ is good and $A_S = B \cap K$,

(e). $b\,U(B) \cap A \neq \emptyset$ for each $b \in B$ and whenever $a \in A$, $b \in B$ are nonzero elements such that $ab \in A$, there exists $t \in S$ such that at^{-1}, $bt \in A$.

Remark 1.6: Let $A \subseteq B$ be an extension of domain. If condition (d) of Theorem 1.5 holds, then $A \subseteq B$ is an inert extenstion. Indeed, let $b_1, b_2 \in B \setminus \{0\}$ such that $b_1 b_2 = a \in A$. Since b_1 $U(B) \cap A = \emptyset$, there exists $u \in U(B)$ such that $a' = b_1 u^{-1} \in A$. Therefore $a = b_1 b_2 = (b_1 u^{-1})(b_2 u) = a'(b_2 u)$. Since (d) holds, there exists $t \in S$ such that $b_1 u^{-1} t^{-1} = b_1 (ut)^{-1} \in A$ and $b_2 (ut) \in A$.

Proof of Theorem 1.5:For a nonzero $f \in R$, let $\alpha(f)$ denote the first nonzero coefficient of f. In order to show that (e) implies (d), let $a,b \in A$ and $s \in S$, such that $s \mid_A ab$. By the second condition of (e) applied for $b/s \in B$, there exists $t \in S$ such that $at^{-1} \in A$, $bt/s \in A$. Let $b = c/a \in B$, where $a,c \in A$ are nonzero. So $ba = c \in A$, hence , by (e), there exists $t \in S$ such that $bt \in A$, thus $b \in A_S$. The inclusion $A_S \subseteq B$ is obvious. Conversely, let $a \in A$, $b \in B$ be nonzero elements such that $c = ab \in A$. Then $b = c/a \in K \cap B = A_S$. So there exist $s \in S$, $d \in A$ such that $b = d/s$. Since S is good, there exists $t \in S$ such that $at^{-1} \in A$ and $bt = (d/s)t \in A$.

(b) $\Rightarrow$ (e). In order to see that the second part of (e) holds, let $a \in A$, $b \in B$ nonzero elements such that $ab = c \in A$. Then $X^n \mid_R a(bX^n)$, , so $X^n = fg$, $a = ff'$, $bX^n = gg'$ for some $f, f', g' \in R$ of order 0 and $g \in R$ of order n. Then $t = \alpha(f) \in S$, because $\alpha(f)\alpha(g) = 1$. Also $a = \alpha(f)\alpha(f')$, so $t \mid_A a$, and $b = \alpha(g)\alpha(g')$, hence $bt = \alpha(g') \in A$. To show the first part of (e), let $0 \neq b \in B$. Then $X^n \mid_R (bX^n)(bX)$,so $X^n = fg$, $bX^n = ff'$, $bX = gg'$, for some $f, f', g, g' \in R$. Then $1 = \alpha(f)\alpha(g)$ and $b = \alpha(f)\alpha(f') = \alpha(g)\alpha(g')$ with $\alpha(f') \in A$ or $\alpha(g') \in A$. Hence $\alpha(f),\alpha(g) \in U(B)$ and of the elements $b\alpha(f)$, $b\alpha(g)$, belongs to A. (a) $\Rightarrow$ (e) can be proved similarly. (c) $\Rightarrow$ (b) and (c) $\Rightarrow$ (a) are trivial . (e) $\Rightarrow$ (c). Assume that $f, g \in R$ are nonzero, $X^n \mid_R fg$ and $\mathrm{ord}(g) = j \geq \mathrm{ord}(f) = i$. Obviously, if $j \geq n + 1$, then $X^n \mid_R g$. We consider the following three cases.

If $i=0$ and $j=n$, then $\alpha(f) \in A$, $\alpha(g) \in B$ and $\alpha(f)\alpha(g) \in A$. By (e), there exists $t \in S$ such that $t^{-1}\alpha(f)$, $t\alpha(g) \in A$. Hence $X^n = t(t^{-1}X^n)$ with $t \mid_R f$ and $t^{-1}X^n \mid_R g$, because t divides each nonconstant monomial in f. If $i+j=n$ and $i \geq 1$, then $\alpha(f), \alpha(g) \in B$ and $\alpha(f), \alpha(g) \in A$. By Remark 1.6 there exists $w \in U(B)$ such that $w\alpha(f) \in A$ and $w^{-1}\alpha(g) \in A$. Hence $X^n = (w^{-1}X^i)(wX^j)$ with $w^{-1}X^i \mid_R f$ and $wX^j \mid_R g$. If $i+j \geq n+1$ and $i \geq 1$, then $J \geq 2$ and $\alpha(f), \alpha(g) \in B$. By (e), there exists $c \in U(B)$ that such $\alpha(f)c \in A$. Hence $X^n = (c^{-1}X^i)(cX^{j-1})$ with $c^{-1}X^i \mid_R f$ and $cX^{j-1} \mid_R g$.

Example 1.7: (i).Let A be a domain and $S \subseteq A$ a multiplicative system. In $A + X A_S[X]$, X is primal if and only if X^2 is primal .

(ii).Let K be a field and A a subring of K. Then X^2 is primal in $A + X K[X]$, but X is primal in $A + X K[X]$ if and only if $K = Q(A)$. For instance, in the ring $Z + X R[X]$, X^2 is primal, but X is not primal.

(iii). X^2 is not primal in $Z + X Z[\sqrt{2}][X]$.

(iv). If $A' \subseteq B$ is an extension of domains such that $bU(B) \cap A' \neq \emptyset$ for each $b \in B$, then X^2 is primal in $A + X B[X]$, where $A = Q(A') \cap B$. Indeed, $Q(A) \cap B = A$, whence $U(B) \cap A = U(A)$.

We recall that an element x of integral domain in R is said to be an extractor if $Rx \cap Ry$ is principal for each $y \in R \setminus \{0\}$. Obviously each extractor is primal. By (D. D. Anderson et al 1995, Theorem 4.1)[4] if x is an extractor, then so is every divisor of x. By the proof of [T. Dumitrescu, Proposition 2.11][11], a completely primal element x is an extractor if and only if for each y, the elements x.y have a maximal common divisor (abbrev.MCD), that is, a common divisor z such that x/z and y/z are relatively prime.

Proposition 1.8: Let A be a domain and $S \subseteq A$ a saturated multiplicative system of A. Then X is an extractor in $A + X A_S[X]$ or $A + X A_S[[X]]$ if and only if S is splitting and consists of extractors.

Proof:Let R denote $A + X A_S[X]$ or $A + X A_S[[X]]$. Assume that X is an extractor in R. By the above remark, the elements of S are extractors in R, hence extractors in A. By Remark 1.3 (b), S is good. Let $a \in A$ be nonzero and $f = GCD_R(X,a)$. Then $f(0) = t \in S$ and if $w \mid_A a$ with $w \in S$, then $w \mid_R a$, X, so $w \mid_R t$, hence $w \mid_A t$. By Remark1.3(e), S is splitting.

Conversely, assume that S is splitting and consists of extractors. By Corollary 1.4,, X is completely primal in R. So, by the remarks made before Proposition 1.8, it suffices to see that X, f have an MCD, for each nonzero $f \in R$. If $a = f(0) \neq 0$, a can be written $a = bs$ with $s \in S$ and b LCM-prime to S. Thus s is an MCD of X and f in R. If ord(f) = 1, say $f = (a/t)X + c_2X^2 + c_3X^3 + \dots$ with $a \in A$, $t \in S$, we write again $a = bs$ with $s \in S$ and b LCM-prime to S. Since s,t are extractors, there exists $w \in S$ such that $(s,t)_v = Aw$. Then $(w/t)X$ is an MCD of X and f in R.

Corollary 1.9: Let A be a domain and $S \subseteq A$ a saturated multiplicative system of A. Then $A + X A_S[X]$ is GCD-domain if and only if X is an extractor in $A + X A_S[X]$ and A_S is a GCD-domain.

Proof: Apply proposition 1.8[8,12]

2. SEMIRIGID GCD-DOMAINS

Let R be an integral domain. We recall that a element $x \in R$ is said to be rigid, if whenever $r,s \in R$ and $r,s \mid x$, we have $r \mid s$ or $s \mid r$. so an irreducible element is obviously rigid (we recall that a nonzero nonunit $x \in R$ is called irreducible if whenever $y \in R$ and $y \mid x$, we have $y \mid 1$ or $x \mid y$. Then R is called semirigid if every nonzero element of R can be expressed as a product of a finite number of rigid elements. So, each atomic domain, that is a domain in which every nonzero nonunit element is a product of irreducible elements, is semirigid.

Lemma 2.1: Let A be a domain, $S \subseteq A$ a saturated multiplicative system and $R = A + x A_S[X]$.

(a). X is rigid in R if and only if for every $r,s \in S$, $r \mid_A s$ or $s \mid_A r$. If X is rigid in R and $s \in S$, then X/s is rigid in R.

(b). A nonzero element $a \in A$ is rigid in A if and only if a is rigid in R.

Proof:Is a consequence of the following remarks. The set of all divisors of X in R is $S \cup \{ X/s \; ; \; s \in S \}$ and any nonzero $a \in A$ has the same divisors in A and R. Also if $b,c \in A$, then $b \mid_A c$ if and only if $b \mid_R c$. When $s \in S$, R has an A-algebra automorphism sending X into X/s .

Let A be a domain and $S \subseteq A$ a saturated multiplicative system. In [D. Costa et al. , Theorem 1.1][12], is show that $A + X A_S[X]$ is a GCD-domain if and only if A is a GCD-domain and S is splitting. Our next result characterizes the semirigid GCD-domain of type $A + X A_S [X]$.

Theorem 2.2: Let A be a domain and $S \subseteq A$ a splitting (saturated) multiplicative system. Then $A + X A_S[X]$ is a semirigid GCD-domain if and only if A is a semirigid GCD-domain and for every $s,t \in S$, $s \mid_A t$ or $t \mid_A s$.

Proof: Set $R = A + X A_S[X]$. Using Lemma 2.1, and the comments above, it suffices to show that R is semirigid provided A is a semirigid GCD-domain and X/s is rigid in R for each $s \in S$. We use freely Lemma 2.1. Let $f \in R$ be a nonzero polynomial of minimal degree among all nonzero polynomials of R which cannot be written as product of rigid elements of R. Then any divisor of f in R is constant or have the same degree with f. Since a rigid element of A is still rigid in R, it follows that $f \notin A$. If $f(0) = 0$, then $f = (X/s)a$ for some $s \in S$ and $a \in A$. This contradicts the choice of f because X/s is rigid and A semirigid. If $f(0) \neq 0$, then factoring out from f an appropriates element of A, we may assume that

$$f = a_0 + (a_1/s)X + \dots + (a_n/s)X^n$$

with $a_i \in A$, $s \in S$ and $GCD(a_0, \cdots a_n) = 1$. Moreover, since S is splitting, we may assume that a_0 is LCM-prime to S. Then f has no nonunit factor in A, thus f is irreducible, a contradiction.

There is as certain resemblance between the proofs of Theorem 2.2 and [T. Dumitrescu, Corollary 1.8][11].

Corollary 2.3: If A is a semirigid GCD-domain, so is the polynomial ring A [X].

Corollary 2.4:[M. Zafrullah, Example 4] [6]. If V is a valuation domain with quotient field K, then V + X K[X] is a semirigid GCD-domain.

Corollary 2.5: If A is a semirigid GCD-domain and $p \in A$ a prime element such that $\cap_{n\geq 1} p^n A = 0$, (e.g. if A is a factorial domain and $p \in A$ a prime), then A + X A[1/p][X] is a semirigid GCD-domain.

Proof: The saturated multiplicative system generated by p is splitting cf[3].

Example 2.6: Iterating the preceding corollary, we obtain successively that the following rings are semirigid GCD-domains

$$A_1 = Z + X\, Z[1/2][X_1],\ \ A_2 = A_1 + X_2\, A[1/3][X_2], \cdots,$$

$$A_n = A_{n-1} + X_n\, A_{n-1}[1/p_{n-1}][X_n],$$

Where p_n is the nth prime number.

Indeed, if q is a prime distinct from $p_1, p_2, \cdots, p_n$, then

$$A_n/qA_n \simeq (A_{n-1}/qA_{n-1})[X_n] \simeq \cdots \simeq (Z/qZ)[X_1, X_2, \cdots, X_n] \text{ and}$$

$$\cap_{k\geq 1} q^k A_n \subseteq \cap_{k\geq 1} q^k Z[1/p_1, 1/p_2, \cdots, 1/p_n][X_1, X_2, \cdots, X_n] = 0 .$$

REFERENCES:

(1) P. M. Cohn, Bezout rings and their subring. Proc. Cambridge Philios. Soc. 64 (1968), 251-264.

(2) M. Roitman, Polynomial extensions of atomic domains. J. Pure and Appl. Algebra, 87 (1993), 187-199.

(3) D. D. Anderson, D. F. Anderson and M. Zafrullah, Factorization in integral domains,II. J. Algebra, 152 (1992), 78-93.

(4) D. D. Anderson and M. Zafrullah, P. M. Cohn s completely primal elements, Zero dimensional Commutative Rings, pp. 115-123. Lecture Notes in Pure and Appl. Math. 171, Marcel Dekker, Inc., New York, 1995.

(5) M. Zafrullah, Semirigid GCD-domain elements. Manuscripta Math. 17 (1975), 55-66.

(6) M. Zafrullah, On a property of preSchreier domains. Comm. Algebra, 15 (1987), 1895-1920.

(7) M. Zafrullah, The $D+XD_S[X]$ construction from GCD domains. J, Pure Appl. Algebra, 50(1988), 93-107.

(8) R. Gilmer, Multiplicative Ideal Theory. Marcel Dekker, New York, 1972.

(9) R. Gilmer and T. Parker, Divisibility properties in semigroup rings. Michigan J. Math 21 (1974) , 65-86.

(10) V. Barucci, L. Izelgue and S, Kabbaj, Some factorization properties of A+XB[X] domains. In: Commutative Ring Theory. 69-78. Lecture Notes in Pure and Appl. Math., 185, Marcel Dekker, New York, 1997.

(11) T. Dumitrescu, On some examples of atomic domains and of G-rings. To appear n Comm. Algebra.

(12) 6-D. Costa, J. Mott and M. Zafrullah, The construction $D+XD_S[X]$. J. Algebra, 53 (1978), 423-439.

العنصر الاساسي في الساحة العددية

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جامعة تكريت - كلية التربية للبنات – قسم الرياضيات

الملخص

يدعى العنصر x في الساحة العددية R عنصرا اساسيا عندما x يقسم حاصل ضرب $a_1 a_2$ حيث a_1, a_2 في R ويمكن كتابة x كالشكل $x=x_1 x_2$ بحيث ان x_i يقسم a_i لكل $i=1,2$. يتناول البحث x^2 عندما يكون عنصرا اساسيا في $A+xB[x]$ او في $A+xB[[x]]$, حيث ان $A \subseteq B$ هو توسعة للساحات وكذلك تبين الدراسة انه اذا كان A ساحة عددية و $S \subseteq A$ هو نظام ضربي منفلق، فان $A+A_S[x]$ ساحة- GCD شبه صارم اذا وفقط اذا كان A ساحة- GCD شبه صارم ولكل عنصرين في S احدهما يقسم الاخر.