Identification Critical Paths On Fuzzy Network Project

Alauldin N. Ahmed and SaadS. Mohsen

Alnahrain University/ Science College/ Math. &Comptr. Aplcs.

Abstract

In this paper, a new procedure for identifying fuzzy critical paths is presented, based on the parameters set of the fuzzy activities, the probability density function PDF of each fuzzy path, and its penalty cost index are calculated in order to identify and schedule all critical paths. A numerical tested problem is presented to demonstrate our approach.

Key Words: Fuzzy Network, Stochastic Network, Scheduling.

1. Introduction

Recently, projects planning and optimal timing, under uncertainty are extremely critical for many organizations. As a result, the uncertainty associated with such risky projects should be reduced. The problem of identifying critical activities (CA) in deterministic problems is well understood. Since a project could be delayed if these activities were not completed in the scheduled time, Standard Critical Path Method (CPM) analyses can be used to identify the longest path(s), known as the critical path(s), in an activity network. However, there are many cases where the activity times may not be presented in a precise manner. To deal quantitativelywith imprecise data, the (PERT) based on the probability theory can be employed, and the probability distributions of each activities is needed, it is difficult to use in some situations when the priori data of the activity probability distributions are absent. Identifying critical activities in a fuzzy project (FP) is difficult problems. Several methods had been proposed contain series draw backs which lead to identifying critical fuzzy activities incorrectly, leaving project mangers without means to identify the most probable sources of project delays. A new direction for identifying fuzzy critical path activitiesin stochastic project (SP) is based on a different philosophy, than in a deterministic project (DP), where each critical activity must correspond to zero time slack activity, while such condition need not to be necessary in (SP).

An alternative way to deal with imprecise data is to employ the concept of***fuzziness;*** the main advantages of methodologies based on fuzzy theory are thatthey do not require prior predictable regularities or posterior frequencydistributions . The problems of computing the intervals of possible values ofthe latest starting times and floats of activities with imprecise durationsrepresented by fuzzy or intervalnumbers, andmany solution methods have been proposed, in which most of them arestraightforward extension of deterministic CPM.

In(ghoseirl k. at el.), a nice literature survey about this subject mentioned the works in [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]is presented.

Numerous papers have been written in identifying (CA) with Stochastic activity durations. In [17], a recursive algorithm is developed indetermine the CDF of the project distribution. In [18, 19] bounds are obtained for the PDF, and developed the distribution for project network with exponentially distributed activity times. In [9] a Branch –and-Bound algorithm is presented for solving a discrete version of continuous density functions, when activity times must be either normal or crashed, see also [20]. Several methods had been proposed contain series draw backs which lead to identifying critical activities incorrectly, leaving project mangers without means to identify and rank the most probable sources of project delays, and with activities represent the best opportunities for successfully addressing schedule risk. The activities times (durations) are random variables. Activity starting and ending times, as well as activity slack times, are therefore random variables. A new direction for identifying critical path activities (CPA) in stochastic project (SP) is based on different philosophy, than in deterministic project (DP), where each critical activity must correspond to zero time slack activity, while such condition need not to be necessary in (SP). We immediately encounter difficulties developing concepts analogous to total slack and "critical" activities for stochastic project. Such concept is a criticality index, defined as the probability that an activity will lie on a critical path. However, an activity may lie on a critical path without introducing risk of project delay. Based on such concept, a critical degrees is constructed to schedule fuzzy critical paths,[21].

Most of the systems work with fuzzy values, which have to be mapped to non-fuzzy (crisp) values after conversion processing called **defuzzification**. Various defuzzification methods have been proposed in [22, 23, 24, 25]. The most popular methods are the center of gravity method and the mean of maxima method, which are computationally inexpensive and easy to implement within fuzzy hardware chips although a full scientificreasoning has not been established. Many researchers attempted to understand the logic of the defuzzification process. Yager and Filev[26, 27, 28]contributed to the process of defuzzification from the perspective of invariant transformation between different uncertainty paradigms, including basic defuzzification distribution, semi-linear defuzzification and generalized level set defuzzification. They all can be seen extention of the center of gravity method. Similarly, Roychowdhury and Wang [22]attempt to understand the defuzzification problem from the scope of optimal selection of an element from a fuzzy set. They used the concepts of interaction, variability, and voting techniques to compute an optimal solution. The authors, Genther, *et al.*[29]proposed a fuzzy clustering based defuzzification method. Ezzati and Saneifard[30], proposed a defuzzification method with most typical values. Saneifard[31], proposed a method for defuzzification with weighted distance. Smith [32], proposed a dynamic switching defuzzification method for fuzzy control.Ming *et al.*[33], proposed a defuzzification method with the "nearest" symmetric triangular fuzzy number of a fuzzy set. Mabuchi [34], proposed a procedure to defuzzify fuzzy subsets and interval values by employing the concept of sensitivity analysis with a kind min-max principle. Leekwijck*et al*[24], proposed a continuous maxima defuzzification method. Saneifard[31], proposed weighted distance method for defuzzification and used it for solving fuzzy linear equations system. There are also researchers who tried to build an axiomatic foundation for the defuzzification theory [35, 36]. They are also researchers on the fast computation of the center of gravity defuzzification method [37, 38]. It should be noted that with the developments of intelligent technologies, some adaptive and parameterized defuzzification methods that caninclude human knowledge have been proposed. Halgamogeet, *et* *al*.[39], used neural networks for defuzzification. Song, *et al* [40], proposed an adaptive learning defuzzification technique. Yager[41], proposed a knowledge based defuzzification process become more intelligent. Similar to the methods of Yager and Filev[26, 42, 28, 43]also proposed a parameterized defuzzification method with Gaussian based distributionand polynomial transformation, but it is mainly based on intuition and there is no explicit decision making meaning for these parameters. For more comparison details on most of these methods. Although so many defuzzification methods have been proposed so far, no one method gives a right effectivedefuzzified output. The computational results of these methods often conflict, and they don't have an uniform framework in theoretical view. We often face difficulty in selecting appropriate defuzzification methods for some specific application problems. Most of the existing defuzzification methods tried to make the estimation of a fuzzy set in an objective way. However, an important aspect of the fuzzy set application is that it can represent the subjective knowledge of the decision maker, different decision makers may have different perception for the defuzzification results.

In this paper, we are presented a new identifying algorithm based ondefuzzificationparameters set of fuzzytypesactivities, by using the concept probability density of each fuzzy path. Some expressions based on the project lateness costs to obtain the corresponding penalty cost index are constructed.A numerical example ispresented,and explain the proposed procedure.

Atrapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$with membership function (MPF)$\mu_{\tilde{A}}$, is considered, and if we are assumed that all the activities $\tilde{A}_i$ $(i \in N)$ are lexicographically ordered. By defuzzification the PF$\mu_{A_i}(t)$, whereA_i=($a_{1i}, a_{2i}, a_{3i}, a_{4i}$) is an arbitrary fuzzy number, intothe Probability Density Function "*pdf* " associated with activity A_i, defined as following

$$p_i(t) = c_i \mu_{A_i}(t), \quad (1)$$

where $c_i = \frac{2}{a_{4i}+a_{3i}-a_{2i}-a_{1i}}$, obtained by the property $\int_{-\infty}^{\infty} p_i(t)dt = 1$, see [44].

Therefore,the probability density function of the path (***we mean about path is a set of connected activities starting from the first activity to ending activity***) can be calculated using the following formula:

$$p_{1\ldots n}(t) = p_1(t)p_{2|1}(t) \ldots p_{n|1\ldots n-1}(t) \quad (2)$$

2. Proposed Fuzzy Critical Path Method[15]

Critical path method schedules are important technique, that has been used since 1950. A fuzzy project network is an acyclic digraph, where the vertices represent events, and the directed edges represent the activities, to be performed in a project. Activity is then represented by one, and only one, arrow with a tail event and a head event. For each activity, a fuzzy number t_{ij} is defined, where $t_{ij}$is the fuzzy time required for the completion of activity (i,j). A critical path is a longest path, and an activity on a critical path is called a critical activity. Let $FE_i$and FL_ibe the earliest fuzzy event time, and the latest fuzzy event time for event i, respectively. Let FE_j and FL_jbe the earliest event time, and the latest event time for event j, respectively. Let Dj be a set of events obtained from event j and i < j. We then obtain $E\ j$ using the following equations

$$FE_j = \max_{i \in D_j} [FE_i \oplus t_{ij}] \text{ and } FE_1 = FL_1 = 0. \quad (3)$$

Similarly, let H_i be a set of events obtained from event i and i < j . We obtain FL_i using the following equations

$$FL_i = \min_{j \in H_i} [FL_j \ominus t_{ij}] \text{ and } FL_n = FE_n \quad (4)$$

The interval [FE_i, FL_j] is the time during which the activity (i,j) must be completed. When the earliest fuzzy event time and latest fuzzy event time have been obtained, we can calculate the total float of each activity. For activity (i,j) in a fuzzy project network, the total float FT_{ij} of the activity (i,j) can be computed as follows :

$$FT_{ij} = FL_j \ominus FE_i \ominus t_{ij} \quad (5)$$

Hence we can obtain the earliest fuzzy event time, latest fuzzy event time, and the total float of every activity by using the above last three equations. Where, the arithmetic operators $\oplus$ and $\ominus$ are defined as following:

Let $A_1 = (a_1, b_1, c_1, d_1)$ and

$A_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers, the arithmetic operations between them are as follow :

$$(a_1, b_1, c_1, d_1) \oplus (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$$

$$(a_1, b_1, c_1, d_1) \ominus (a_2, b_2, c_2, d_2) = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$$

Then, from (3), (4) and (5), the total finishing time F_p is calculated from fuzzy duration times, and the total late L_p distribution for path p, is calculated, using an expression for determining the total expected lateness, given that the total latest finishing time is greater than or equal to the known due date D, constructed as following :

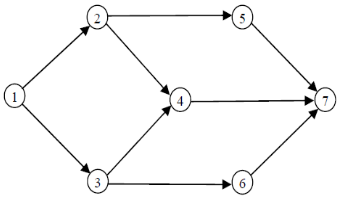
$$E(L | F_p \geq D, TL_p) = \int_D^{\infty} (t - D) p_p (t | L_p) dt \quad (6)$$

where $\boldsymbol{L = F_p - D}$ represents project lateness and a new approach is developed to identify and schedule all possible fuzzy critical paths problem, by considering c_{p}as the cost of path p per time unit.The index cost penalty of the of each path, which effect to the project result, is constructed as follow:

$$r_{\mathrm{p}} = \left(\frac{c_{\mathrm{p}}}{E\left[FT_{P_k}\right]}\right)\left(E\left(L\middle|F_{L,\mathrm{p}} \geq D, TL_{\mathrm{p}}\right)\right),\ \ forallpaths \quad (7)$$

3. Tested Problem

A fuzzy project network problem, is presented in [45] has been consideredto demonstrate and verify the computational process of our proposed method.



Activity	Duration t_{ij}
(1,2)	(25,28,32,35)
(1,3)	(40,55,65,70)
(2,4)	(32,37,43,48)
(3,4)	(20,25,35,40)
(2,5)	(35,38,42,45)
(3,6)	(42,45,55,60)
(4,7)	(60,65,75,85)
(5,7)	(65,75,85,90)
(6,7)	(15,18,22,26)

From the above table, we obtain all possible fuzzy paths are: 1-2-5-7, 1-2-4-7, 1-3-4-7, 1-3-6-7.Then using (1), we obtain:

Activity	PDF
(1,2)	(t+27)/84
(1,3)	(75-t)/600
(2,4)	(t+23)/330
(3,4)	(t-5)/150
(2,5)	(37-t)/105
(3,6)	(7t-255)/420
(4,7)	(t-45)/175
(5,7)	2/25
(6,7)	(t-30)/180

and using (2), we obtain:

Path	PDF
1-2-5-7	0.009t+0.335
1-2-4-7	0.001t+0.024
1-3-4-7	-0.001t+0.011
1-3-6-7	-0.003t+0.463

By using (3) & (4), we obtain:

Event FE_i FL_i

1	(0,0,0,0)	(0,0,0,0)
2	(25,28,32,35)	(25,32,48,60)
3	(45,55,65,70)	(45,55,65,70)
4	(60,80,100,110)	(65,80,100,110)
5	(60,66,74,80)	(60,70,90,105)
6	(82,100,120,130)	(110,127,153,169)
7	(125,145,175,195)	(125,145,175,195)

and using (5), we obtain:

Activity	FT_{ij}
1,2	***(0,4,16,25)***
1,3	***(0,0,0,0)***
2,4	***(8,15,25,27)***
3,4	***(0,0,0,0)***
2,5	***(0,4,16,25)***
3,6	***(27,27,33,39)***
4,7	***(0,0,0,0)***
5,7	***(0,4,16,25)***
6,7	***(27,27,33,39)***

By using formula (6)& (7):

Path	***Penalty***
1-2-4-7	***0.51***
1-3-4-7	***0.87***
1-2-5-7	***0.30***
1-3-6-7	***0.70***

Finally, according to the cost penalties indices, the scheduling critical paths are in the following order:

1-3-4-7, 1-3-6-7, 1-2-4-7, 1-2-5-7.

4. <u>*References & Bibliographies*</u>

(1) Mon D. L., Cheng C. H., Lu H. C. (1995);"Application of Fuzzy Distributions on project management". Fuzzy Set Syst. 73, pp. 227-234.

(2) Chanas P. and Zielinski (2001): "Critical path analysis in the network with fuzzy activity times". *Fuzzy Sets Syst*. 122 pp. 195–204.

(3) Liberatore M.J. and J.F. Connelly (2001);"Applying fuzzy logic to critical path analysis". Porceedings of International Conference on Management of Energy and Technology, Portland Vol. 1, Portland. OR, USA, pp. 419-419.

(4) Santiago I. and Vakili , P. (2005); "On the value of flexibility in R & D projects". O.R. 24(1), 177-182.

(5) Chanas S., Dubois D and Zielinski P. (2002): "On the sure criticality of tasks in activity networks with

imprecise durations". IEEE Trans. on Systems. Man and Cybernetics-Part B. Cybernetics 32 pp. 393-407.

(6) Chanas S. and Zielinski P. (2002): "The computational complexity of the criticality problems in a network with interval activity times". Eur. Jr. Opr. Res. 136, pp. 541-550.

(7) Dubois D., Fargier H. and Galvagnon V. (2003): On latest starting times and floats in activity networks with ill-known durations. Eur. Jr. Opr. Res., 147, pp. 266-280.

(8) Zielinski P. (2005);"On computing the latest starting times and floats of activities in a network with imprecise durations". Fuzzy Set.Sys.150,pp.53-76.

(9) Chen S.P. (2006): "Analysis of critical paths in a project network with fuzzy activity times". Eur. Jr. Opr. Res. 183: pp. 442-459.

(10) Chen C.T. and S.F. Huang (2007): "Applying fuzzy method for measuring criticality in project network". Inform. Sci. 177.

(11) Shanker N.R. and BP. Saradhi (2011);"Fuzzy Critical Path Method in Interval-Valued Activity Networks". Int. J. Pure Appl. Sci. Technol. 3(2), pp. 72-79.

(12) Shankar N. R., Sireesha V. and Rao P. P.B.. (2010);"Critical Path Analysis in the Fuzzy Project Network". Advances in Fuzzy Math. 5(3) pp. 285-294.

(13) Shankar N.R. ,Sireesha V., Siresha S. and Madhuri K. U. (2011);"Measuring Risk Element Criticality in a Fuzzy Project Network Using Trapezoidal Fuzzy Number Method". Applied Math. Sci. 5(11) pp. 529-539.

(14) Sireesha V., N. Ravi Shankar , RaoK.S.andRao P. P. B. (2012);"On the Latest Times and Float Times of Activities in a Fuzzy Project Network

with LR Fuzzy Numbers". 2,(12), pp. 91-101.

(15) Shakeela S. and Ganesan K. (2011);"A simple approach to fuzzy critical path analysis in project networks". Intr. Jr. of Sci. Res. 4(12).

(16) Stefan C. and Zielinskib P. (2001);" Critical path analysis in the network with fuzzy activity times". Fuzzy sets and systems, vol. 122, no.2, pp.195-04.

(17) Hagstrom, J. (1990); "Commuting the probability distribution of project duration in a PERT network". Networks, 20:231-244.

(18) Cho, J. G. & B. J. Yum, (1997); "An uncertainty importance measure activities in PERT networks". Int. Jr. Production. Res., 35, 2737-2757.

(19) Shahsavari P. N. Zeynali S., Kheradmand M. (2012);"Calculating the Fuzzy Project Network Critical Path". Intr. Jr. of Eng. and Tech. 1(2), pp.55-66.

(20) Mitchell, G. and Klastorin T. (2007); "An effective methodology for the stochastic project compression problem". IIE Trans., 39:957-969.

(21) Alauldin N. A. and Saad M. S. (2013);"Scheduling Algorithm of fuzzy critical paths in project network". Jr. of Eng. & Technology. To be appeared.

(22) Roychowdhury, S., B.H. Wang, 1996. Cooperative neighbors in defuzzification, Fuzzy Sets and Systems, 78: 37-49. *J. Appl. Sci. Res., 7(2): 102-110, 2011* 110.

(23) Roychowdhury, S., Pedrycz, W., 2001. A survey of defuzzification strategies, International Journal of Intelligent.

(24) Leekwijck, W.V., E.E. Kerre, 1999. Defuzzification criteria an classification, Fuzzy Sets and Systems, 108: P. 159-178.

(25) Kosko, B., 1992. Neural Networks and

Fuzzy Systems, Prentice Hall, NJ.
(26) Filev, D.P., R.R. Yager, 1991. A generalized defuzzification method via BADD distribution, International Journal of Intelligence Systems, 6: 687-697.
(27) Yager, R.R., D.P. Filev, 1993. SLIDE: A simple adaptive defuzzification method, IEEE Transaction on Fuzzy systems, 1: 69-78.
(28) Filev, D.P., R.R. Yager, 1993. An adaptive approach to defuzzification based on level sets, Fuzzy Sets and Systems, 53: 355-360.
(29) Genther, H., T. Runkler, M. Glenser, 1994. Defuzzification based on fuzzy clustering, Third IEEE Conference on fuzzy Systems, 1646-1648.
(30) Ezzati, R., R. Saneifard, 2010. A new approach for ranking of fuzzy numbers with continuous weighted quasiarithmetic means, Mathematical Sciences, 4: 143-158.
(31) Saneifard, R., 2009. A method for defuzzification by weighted distance , International Journal of Industrial Mathematics., 3: 209-217.
(32) Smith, M., 1994. Tuning membership functions, tuning AND and OR operations, tuning defuzzification: which is best?, Proceeding of the 1st International Joint Conference of the North American, 347-351.
(33) Ming, M., A. Kandel, M. Friedman, 2000. A new approach for defuzzification, Fuzzy Sets and Systems, 111: 351-356.
(34) Mabuchi, S., 1993. A proposal for a defuzzification strategy by the concept of the sensitivity analysis, Fuzzy Sets and Systems, 55: 1-14.
(35) Runkler, T.A., M. Glesner, 1993. A set of

axioms for defuzzification strategies-towards a theory of rational defuzzification operators, Proceeding of the 2nd IEEE Internat. Conf. on fuzzy Systems, San Francisco, 1161-1166.

(36) Thiele, H., 1998. Towards axiomatic foundations for defuzzification theory, 2nd International Conference on Knowledge-Based Intelligent Electronic Systems, 2: 243-250.

(37) Wang, W., Luoh, L., 2000. Simple computation for the defuzzification of center of sum and center of gravity, Journal of Inteligentand fuzzy Systems, 9: 53-59.

(38) Broekhoven, E.V., B.D. Baets, 2006. Fast and accurate center of gravity defuzzification of fuzzy system outputs defined on trapezoidal fuzzy partitions, Fuzzy Sets and Systems, 157: 904-918.

(39) Halgamuge, S., Runkler T. , M. Glesner, 1996. On the neural defuzzification methods, Proceeding of the 5th IEEE International Conference on Fuzzy Systems, 463-469.

(40) Song, Q., Leland R.P., 1996. Adaptive learning defuzzification techniques and applications, Fuzzy Sets and Systems, 81: 321-329.

(41) Yager, R.R., 1996. Knowledge-based defuzzification, Fuzzy Sets and Systems, 80: 177-185.

(42) Yager, R.R., D.P. Filev, 1994. Parameterized and-like and or-like OWA operators, International Journal of General Systems, 22: 297-316.

(43) Jiang, T., Y. Li, 1996. Generalized defuzzification strategies and their parameter learning procedure, IEEE Transactions on Fuzzy Systems, 4: 64-71.

(44) Rahim S. and Rasoul S. (2011); " A Modified Method For Defuzzification By Probability Density Function". Journal of Applied Sciences Research, 7(2): 102-110.

(45) Ahmed S. and Rasoul H. (2007): "A Project Scheduling Method on Fuzzy Theory". Jr. Indust. & Sys. Eng. 1(1) pp.70-80.