Soft Semi-Norm Spaces &Soft Pseudo Metric Spaces

Received: 8/10/2015 accepted: 28/12/2015

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Abstract

The paper introduces definitions for some properties of the soft set as balanced, absorbing and convex. The study discusses a number of theorems and observations about these properties. A definition for soft semi-norm space is introduced. The paper also proves one of the soft pseudo metric space axioms which are introduced by [7]. It discusses some of the propositions and theories related to the two spaces. A theory that explains the relation between the two spaces is introduced.

Keywords: Soft set, semi-norm, soft semi-norm, pseudo metric, soft pseudo metric, soft balanced set, soft absorbing and soft convex set.

Math. Sub. classifications:QA150-272.5

1. Introduction

In the year 1999, Molodtsov [2] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties which cannot be dealt with by classical methods. He has shown several applications for his theory in solving many practical problems in economics, engineering, social science, and medicine. Research works in soft sets theory and its applications in various fields have been progressing rapidly since Maji et al. [5] [6] introduced several operations on soft sets and applied it to decision making problems. In the line of reduction and addition of parameters of soft sets some works have been done by Chen [1], Pei and Miao [3], Kong et al. [9], Zou and Xiao [8].

This paper deals with some of the properties of soft set such as the balanced set, absorbing set, convex set. The study discusses a number of theorems and observations about these properties.

2. Preliminaries

Definition 1 [2]: A pair (F, E) is called a soft set over X, where F is a mapping given by $F: E \to P(X)$.

Definition 2 [6]: A soft set (F, E) over X is said to be a null soft set denoted by φ ,if for all $e \epsilon E$, $F(e) = \varphi$ (null set).

Definition 3 [6]: A soft set (F, E) over X is said to be an absolute soft set denoted by $\tilde{X}$, if for all $\varepsilon \epsilon E, F(\varepsilon) = X$.

Definition 4 [4]: Let R be the set of real numbers and $B(R)$ be the collection of all non-empty bounded subset of R and E

taken as a set of parameters. Then a mapping $F: E \to B(R)$ is called a soft real set. If a soft real set is a singleton soft set, it will be called a soft real number and denoted by $\tilde{r}, \tilde{s}, \tilde{t}$ etc. $\tilde{0}, \tilde{1}$ are the soft real numbers where $\tilde{0}(e) = 0$, $\tilde{1}(e) = 1$ for all $e \epsilon E$, respectively.

Definition 5 [7]: A soft set (F, E) over X is said to be a soft point if there is exactly one $e \epsilon E$, such that $F(e) = \{x\}$ for some $x \epsilon X$ and $F(\acute{e}) = \varphi$, $\forall \acute{e} \epsilon E/\{e\}$.It will be denoted by $\tilde{x}_e$.

Definition 6 [7]: Two soft point $\tilde{x}_e, \tilde{y}_e$ are said to be equal if $e = \acute{e}$ and $x = y$. Thus $\tilde{x}_e \neq \tilde{y}_{\acute{e}} \Leftrightarrow x \neq y$ or $e \neq \acute{e}$.

Definition 7 [4]: Let (F, E) be a soft set over X .The soft set (F, E) is said to be a soft vector and denoted by $\tilde{x}_e$ if there is exactly one $e \epsilon E$, such that $F(e) = \{x\}$ for some $x \epsilon X$ and $F(\acute{e}) = \varphi$, $\forall \acute{e} \epsilon E/\{e\}$.

The set of all soft vectors over $\tilde{X}$ will be denoted by $SV(\tilde{X})$.

Definition 8 [4]: The set $SV(\tilde{X})$ is called a soft vector space.

Proposition 1 [4]: The set $SV(\tilde{X})$ is a vector space according to the following operations;

1. $\tilde{x}_e + \tilde{y}_{\acute{e}} = \widetilde{(x+y)}_{(e+\acute{e})}$ for every $\tilde{x}_e, \tilde{y}_{\acute{e}} \tilde{\epsilon} SV(\tilde{X})$;
2. $\tilde{r}\tilde{x}_e = \widetilde{(rx)}_{(re)}$ for every $\tilde{x}_e \tilde{\epsilon}\ SV(\tilde{X})$ and for every soft real number $\tilde{r}$.

3. Main Result

A. Soft Balanced, Soft Absorbing And Soft Convex set

Definition [9]: A soft subset $\tilde{A}$ of a soft vector space $\tilde{X}$ over a field R is said the balanced if $\tilde{r}\tilde{A} \tilde{\subset} \tilde{A}$ for every soft real number $\tilde{r}$ with $|\tilde{r}| \lesssim \tilde{1}$.

Definition [10]: Let $\tilde{A}$ and $\tilde{B}$ be two soft subsets in a soft vecter space $\tilde{X}$ over F. we say that $\tilde{A}$ is a soft absorbs $\tilde{B}$ if there exist $\tilde{r}_o \tilde{\epsilon} \tilde{A}$ such that $\tilde{B} \tilde{\subset} \tilde{r}_o \tilde{A}$ for all $|\tilde{r}| \gtrsim |\tilde{r}_o|$, and we say that $\tilde{A}$ is a soft absorbing if for every $\tilde{x}_e \tilde{\epsilon} \tilde{X}$, there exists $\tilde{r} \gtrsim \tilde{0}$ such that $\tilde{x}_e \tilde{\epsilon} \tilde{r}\tilde{X}$.

Proposition 2: The set $\tilde{A} = \{\tilde{x}_e \tilde{\epsilon} \tilde{X}: \tilde{P}(\tilde{x}_e) \lesssim \tilde{1}\}$ is a soft balanced, soft absorbing set.

Proof: Let $\tilde{r}$ is a soft real number with $|\tilde{r}| \lesssim \tilde{1}$,let $\tilde{x}_e \tilde{\epsilon} \tilde{r}\tilde{A}$ then $\tilde{x}_e = \tilde{r}\tilde{y}_{\acute{e}}$ where $\tilde{y}_{\acute{e}} \tilde{\epsilon} \tilde{A}$, $\tilde{P}(\tilde{y}_{\acute{e}}) \lesssim \tilde{1} \Rightarrow |\tilde{r}|\tilde{P}(\tilde{y}_{\acute{e}}) \lesssim \tilde{1} \Rightarrow \tilde{P}(\tilde{r}\tilde{y}_{\acute{e}}) \lesssim \tilde{1} \Rightarrow \tilde{P}(\tilde{x}_e) \lesssim \tilde{1}$

$\tilde{x}_e \tilde{\epsilon} \tilde{A} \Rightarrow \tilde{r}\tilde{A} \tilde{\subset} \tilde{A} \Rightarrow \tilde{A}$ is a soft balanced set.

Let $\tilde{x}_e \tilde{\epsilon} \tilde{A}$ and let $\tilde{P}(\tilde{x}_e) \lesssim \tilde{r}$, $\tilde{r} \gtrsim \tilde{0} \Rightarrow \tilde{P}\left(\frac{1}{\tilde{r}}\tilde{x}_e\right) \lesssim \tilde{1}$

$\frac{1}{\tilde{r}}\tilde{x}_e \tilde{\epsilon} \tilde{A} \Rightarrow \tilde{x}_e \tilde{\epsilon} \tilde{r}\tilde{A}$ therefore $\tilde{A}$ is a soft absorbing set.

Proposition 3: Let $(F, \tilde{A})$ and $(G, \tilde{B})$ be two soft balanced sets in $\tilde{X}$ then $\tilde{A} \tilde{\cap} \tilde{B}$ is also soft balanced.

Proof: Let $\tilde{x}_e \tilde{\epsilon}\ \tilde{r}(\tilde{A} \tilde{\cap} \tilde{B})$, $|\tilde{r}| \lesssim \tilde{1} \Rightarrow$ $\tilde{x}_e = \tilde{r}\tilde{y}_{\acute{e}}$ suth that $\tilde{y}_{\acute{e}} \tilde{\epsilon}\ (\tilde{A} \tilde{\cap} \tilde{B}) \Rightarrow \tilde{y}_{\acute{e}} \tilde{\epsilon} \tilde{A}$ and $\tilde{y}_{\acute{e}} \tilde{\epsilon} \tilde{B} \Rightarrow \tilde{x}_e \tilde{\epsilon}\ (\tilde{A} \tilde{\cap} \tilde{B})$ therefore $\tilde{r}(\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\subset} (\tilde{A} \tilde{\cap} \tilde{B})$, $\tilde{A} \tilde{\cap} \tilde{B}$ is a soft balanced.

Proposition 4: The union of two soft balanced sets $(F, \tilde{A})$ and $(G, \tilde{B})$ over $\tilde{X}$ is the soft balanced set, where $\tilde{C} = \tilde{A} \tilde{\cup} \tilde{B}$.

Proof: Let $\tilde{y}_{\acute{e}} \tilde{\epsilon} \tilde{r}(\tilde{A} \tilde{\cup} \tilde{B})$, $|\tilde{r}| \lesssim \tilde{1} \Rightarrow$ $\tilde{y}_{\acute{e}} = \tilde{r}\tilde{x}_e$ such that $\tilde{x}_e \tilde{\epsilon} \tilde{r}(\tilde{A} \tilde{\cup} \tilde{B}) \Rightarrow$

$$H(\tilde{x}_e) = \begin{cases} F(\tilde{x}_e) & if \quad \tilde{x}_e \tilde{\epsilon} \tilde{A} - \tilde{B} \\ G(\tilde{x}_e) & if \quad \tilde{x}_e \tilde{\epsilon} \tilde{B} - \tilde{A} \\ F(\tilde{x}_e) \tilde{\cup} G(\tilde{x}_e) & if \quad \tilde{x}_e \tilde{\epsilon}\ (\tilde{A} \tilde{\cap} \tilde{B}) \end{cases}$$

$\tilde{x}_e \tilde{\in} \tilde{A}$ or $\tilde{x}_e \tilde{\in} \tilde{B}$ or $\tilde{x}_e \tilde{\in} (\tilde{A} \tilde{\cap} \tilde{B}) \Rightarrow \tilde{y}_{\acute{e}} = \tilde{r}\tilde{x}_e \tilde{\in} \tilde{r}\tilde{A} \tilde{\subset} \tilde{A}$ or $\tilde{y}_{\acute{e}} = \tilde{r}\tilde{x}_e \tilde{\in} \tilde{r}\tilde{B} \tilde{\subset} \tilde{B}$ or $\tilde{y}_{\acute{e}} = \tilde{r}\tilde{x}_e \tilde{\in} \tilde{r}(\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\subset} (\tilde{A} \tilde{\cap} \tilde{B})$
$\Rightarrow \tilde{y}_{\acute{e}} \tilde{\in} (\tilde{A} \tilde{\cup} \tilde{B})$

$\Rightarrow \tilde{r}(\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\subset} \tilde{A} \tilde{\cup} \tilde{B}$, therefore $\tilde{A} \tilde{\cup} \tilde{B}$ is a soft balanced set.

Definition 11: A soft subset $\tilde{A}$ of a soft vector space $\tilde{X}$ over a field R is said the soft convex set if $\tilde{r}\tilde{A} + (\tilde{1} - \tilde{r})\tilde{A} \tilde{\subseteq} \tilde{A}$, $\tilde{0} \tilde{\leq} \tilde{r} \tilde{\leq} \tilde{1}$. Or equivalently if $\tilde{r}\tilde{x}_e + (\tilde{1} - \tilde{r})\tilde{y}_{\acute{e}} \tilde{\in} \tilde{A}$, whenever $\tilde{x}_e, \tilde{y}_{\acute{e}} \tilde{\in} \tilde{A}$, and $\tilde{0} \tilde{\leq} \tilde{r} \tilde{\leq} \tilde{1}$.

Proposition 5: The set $\tilde{A} = \{\tilde{x}_e \tilde{\in} \tilde{X}: \tilde{P}(\tilde{x}_e) \tilde{<} \tilde{1}\}$ is a soft convex set in $\tilde{X}$.

Proof: Let $\tilde{x}_e, \tilde{y}_{\acute{e}} \tilde{\in} \tilde{A}$ and $\tilde{0} \tilde{\leq} \tilde{r} \tilde{\leq} \tilde{1}$ then $\tilde{P}(\tilde{x}_e) \tilde{<} \tilde{1}, \tilde{P}(\tilde{y}_{\acute{e}}) \tilde{<} \tilde{1}$

$\tilde{P}(\tilde{r}\tilde{x}_e + (\tilde{1} - \tilde{r})\tilde{y}_{\acute{e}}) \tilde{\leq} \tilde{P}(\tilde{r}\tilde{x}_e) + \tilde{P}((\tilde{1} - \tilde{r})\tilde{y}_{\acute{e}}) = |\tilde{r}| \ \tilde{P}(\tilde{x}_e) + |\tilde{1} - \tilde{r}| \ \tilde{P}(\tilde{y}_{\acute{e}}) = \tilde{r}\,\tilde{P}(\tilde{x}_e) + (\tilde{1} - \tilde{r})\,\tilde{P}(\tilde{y}_{\acute{e}})$

Since $\tilde{P}(\tilde{x}_e) \tilde{<} \tilde{1}$, $\tilde{P}(\tilde{y}_{\acute{e}}) \tilde{<} \tilde{1} \Rightarrow \tilde{r}\tilde{P}(\tilde{x}_e) \tilde{<} \tilde{r}, (\tilde{1} - \tilde{r})\ \tilde{P}(\tilde{y}_{\acute{e}}) \tilde{<} (\tilde{1} - \tilde{r})$

$\Rightarrow \tilde{r}\,\tilde{P}(\tilde{x}_e) + (\tilde{1} - \tilde{r})\ \tilde{P}(\tilde{y}_{\acute{e}}) \tilde{<} \tilde{r} + (\tilde{1} - \tilde{r}) = \tilde{1}$

$\Rightarrow \tilde{P}(\tilde{r}\tilde{x}_e + (\tilde{1} - \tilde{r})\tilde{y}_{\acute{e}}) \tilde{<} \tilde{1}$, therefore $\tilde{r}\tilde{x}_e + (\tilde{1} - \tilde{r})\tilde{y}_{\acute{e}} \tilde{\in} \tilde{A}$

$\tilde{A}$ is soft convex set.

Remark 1: If $\tilde{A}$ is a soft subset of a soft vector space $\tilde{X}$, then $(\tilde{r} + \tilde{s})\tilde{A} \tilde{\subset} \tilde{r}\tilde{A} + \tilde{s}\tilde{A}$.

Theorem (1): If $\tilde{A}$ is a soft subset of a soft vector space $\tilde{X}$, then is soft convex set if and only if $(\tilde{r} + \tilde{s})\tilde{A} = \tilde{r}\tilde{A} + \tilde{s}\tilde{A}$ such that $\tilde{r}, \tilde{s}$ are soft real numbers.

Proof: Suppose that $\tilde{A}$ is soft convex set, since $(\tilde{r} + \tilde{s})\tilde{A} \tilde{\subseteq} \tilde{r}\tilde{A} + \tilde{s}\tilde{A}$ we most prove $[\ \tilde{r}\tilde{A} + \tilde{s}\tilde{A} \tilde{\subseteq} (\tilde{r} + \tilde{s})\tilde{A}]$

Let $\tilde{x}_e \tilde{\in}\ \tilde{r}\tilde{A} + \tilde{s}\tilde{A} \Rightarrow \tilde{x}_e = \tilde{r}\tilde{y}_{e_1} + \tilde{s}\tilde{z}_{e_2}$ such that $\tilde{y}_{e_1}, \tilde{z}_{e_2} \tilde{\in} \tilde{A}$

$\tilde{x}_e = (\tilde{r} + \tilde{s})\ \ (\frac{\tilde{r}}{\tilde{r}+\tilde{s}}\ \tilde{y}_{e_1} + \frac{\tilde{s}}{\tilde{r}+\tilde{s}}\tilde{z}_{e_2})$, put $\tilde{t} = \frac{\tilde{r}}{\tilde{r}+\tilde{s}}, \tilde{1} - \tilde{t} = \frac{\tilde{s}}{\tilde{r}+\tilde{s}}, \tilde{0} \tilde{\leq} \tilde{t} \tilde{\leq} \tilde{1}$

Since $\tilde{A}$ is convex soft set then $\tilde{t}y_{e_1} + (\tilde{1} - \tilde{t})\tilde{z}_{e_2} \tilde{\in} \tilde{A} \Rightarrow$

$\frac{\tilde{r}}{\tilde{r}+\tilde{s}}\tilde{y}_{e_1} + \frac{\tilde{s}}{\tilde{r}+\tilde{s}}\tilde{z}_{e_2} \tilde{\in} \tilde{A} \Rightarrow \tilde{x}_e \tilde{\in} (\tilde{r} + \tilde{s})\tilde{A}$, hence $\tilde{r}\tilde{A} + \tilde{s}\tilde{A} \tilde{\subseteq} (\tilde{r} + \tilde{s})\tilde{A} \Rightarrow (\tilde{r} + \tilde{s})\tilde{A} = \tilde{r}\tilde{A} + \tilde{s}\tilde{A}$.

The converse: let $(\tilde{r} + \tilde{s})\tilde{A} = \tilde{r}\tilde{A} + \tilde{s}\tilde{A}$ such that $\tilde{r}, \tilde{s}$ are soft real numbers

Let $\tilde{0} \tilde{\leq} \tilde{t} \tilde{\leq} \tilde{1} \Rightarrow \tilde{1} - \tilde{t} \tilde{\geq} \tilde{0}$ then $\tilde{t}\tilde{A} + (\tilde{1} - \tilde{t})\tilde{A} = \left(\tilde{t} + (\tilde{1} - \tilde{t})\right)\tilde{A} = \tilde{A}$

$\Rightarrow \tilde{t}\tilde{A} + (\tilde{1} - \tilde{t})\tilde{A} \tilde{\subseteq} \tilde{A}$ then $\tilde{A}$ is soft convex set.

Theorem (2): If $\tilde{A}$ and $\tilde{B}$ are soft convex sets in a vector space $\tilde{X}$ and $\tilde{r}$ is soft real number then :

1. $\tilde{A} \tilde{\cap} \tilde{B}$ 2. $\tilde{r}\tilde{A}$ 3. $\tilde{A} + \tilde{B}$ are also soft convex sets in $\tilde{X}$.

Proof:

1. Let $\tilde{x}_e, \tilde{y}_{\acute{e}} \tilde{\in} \tilde{A} \tilde{\cap} \tilde{B}$ and $\tilde{0} \tilde{\leq} \tilde{r} \tilde{\leq} \tilde{1} \Rightarrow \tilde{x}_e, \tilde{y}_{\acute{e}} \tilde{\in} \tilde{A}$ and $\tilde{x}_e, \tilde{y}_{\acute{e}} \tilde{\in} \tilde{B}$

Since $\tilde{A}$ and $\tilde{B}$ are soft convex sets then $\tilde{r}\tilde{x}_e + (\tilde{1} - \tilde{r})\tilde{y}_{\acute{e}} \tilde{\in} \tilde{A}$ and $\tilde{r}\tilde{x}_e + (\tilde{1} - \tilde{r})\tilde{y}_{\acute{e}} \tilde{\in} \tilde{B} \Rightarrow \tilde{r}\tilde{x}_e + (\tilde{1} - \tilde{r})\tilde{y}_{\acute{e}} \tilde{\in} \tilde{A} \tilde{\cap} \tilde{B}$

$\Rightarrow \tilde{A} \tilde{\cap} \tilde{B}$ is soft convex set.

2. Let $\tilde{x}_e, \tilde{y}_{\acute{e}} \tilde{\in} \tilde{r}\tilde{A}$ and $\tilde{0} \tilde{\leq} \tilde{t} \tilde{\leq} \tilde{1}$, $\tilde{x}_e = \tilde{r}\tilde{z}_{e_1}, \tilde{y}_{\acute{e}} = \tilde{r}\widetilde{w}_{e_2}$ such that $\tilde{z}_{e_1}$

, $\widetilde{w}_{e_2} \tilde{\in} \tilde{A}$, since $\tilde{A}$ is soft convex set, then $\tilde{t}\tilde{z}_{e_1} + (\tilde{1} - \tilde{t})\widetilde{w}_{e_2} \tilde{\in} \tilde{A} \Rightarrow$

$\tilde{r}(\tilde{t}\widetilde{z_{e_1}} + (\tilde{1} - \tilde{t})\widetilde{w_{e_2}})\tilde{\in}\tilde{r}\tilde{A}$, since $\tilde{r}(\tilde{t}\widetilde{z_{e_1}} + (\tilde{1} - \tilde{t})\widetilde{w_{e_2}}) =$

$\tilde{t}(\tilde{r}\widetilde{z_{e_1}}) + (\tilde{1} - \tilde{t})\tilde{r}\,\widetilde{w_{e_2}})\tilde{\in}\tilde{r}\tilde{A} \Rightarrow$ $\tilde{r}\tilde{A}$ is soft convex set.

3. Let $\tilde{x}_e, \tilde{y}_{\acute{e}} \tilde{\in} \tilde{A} + \tilde{B}$ and $\tilde{0} \lesssim \tilde{t} \lesssim \tilde{1}$, $\tilde{x}_e = \widetilde{z_{e_1}} + \widetilde{w_{e_2}}, \tilde{y}_{\acute{e}} = \widetilde{E_{e_3}} + \widetilde{F_{e_4}}$ such that $\widetilde{z_{e_1}}, \widetilde{E_{e_3}} \tilde{\in} \tilde{A}$ and $\widetilde{w_{e_2}}, \widetilde{F_{e_4}} \tilde{\in} \tilde{B}$ and $\tilde{A}$ and $\tilde{B}$ are soft convex sets, then $\tilde{t}\widetilde{z_{e_1}} + (\tilde{1} - \tilde{t})\widetilde{E_{e_3}} \tilde{\in} \tilde{A}$ and $\tilde{t}\widetilde{w_{e_2}} + (\tilde{1} - \tilde{t})\widetilde{F_{e_4}} \tilde{\in} \tilde{B}$, since

$\tilde{t}\tilde{x}_e + (\tilde{1} - \tilde{t})\tilde{y}_{\acute{e}} = \tilde{t}(\widetilde{z_{e_1}} + \widetilde{w_{e_2}}) + (\tilde{1} - \tilde{t})(\widetilde{E_{e_3}} + \widetilde{F_{e_4}}) =$

$[\tilde{t}\widetilde{z_{e_1}} + (\tilde{1} - \tilde{t})\widetilde{E_{e_3}}] + [\tilde{t}\widetilde{w_{e_2}} + (\tilde{1} - \tilde{t})\widetilde{F_{e_4}}] \Rightarrow \tilde{t}\tilde{x}_e + (\tilde{1} - \tilde{t})\tilde{y}_{\acute{e}} \tilde{\in} \tilde{A} + \tilde{B}$

$\tilde{A} + \tilde{B}$ is soft convex set.

B. Soft Semi-Norm and Soft Pseudo Metric

Definition 12: Let X be a vector space over F, $p: X \to R^+$ is semi-norm function if and only if it satisfies the following conditions for all $x, y, z \epsilon X$, $\lambda \epsilon F$:

1. $p(x) \geq 0$;
2. $p(x) = 0$ if $x = 0$;
3. $p(\lambda x) = |\lambda| p(x)$
4. $p(x + y) \leq p(x) + p(y)$

The vector F over X together with p is called a semi-normed space and is denoted by (X, p).

Remark 2: we can prove the conditions 2 of the above definition to make the definition consisted of three conditions as following:

From condition 3 and by take $\lambda = 0$ we can get condition 2.

Definition 13: Let $SV(\tilde{X})$ be a soft vector space. Then a mapping $\tilde{P}$: $SV(\tilde{X}) \to R^+(E)$ is said to be a soft semi-norm on $SV(\tilde{X})$, if $\tilde{P}$ satisfies the following conditions:

N1: $\tilde{P}(\tilde{r}\,\tilde{x}_e) = |\tilde{r}|\tilde{P}(\tilde{x}_e)$ for all $\tilde{x}_e \tilde{\in} SV(\tilde{X})$ and for every soft scalar $\tilde{r}$;

N2: $\tilde{P}(\tilde{x}_e + \tilde{y}_{\acute{e}}) \lesssim \tilde{P}(\tilde{x}_e) + \tilde{P}(\tilde{y}_{\acute{e}})$ for all $\tilde{x}_e, \tilde{y}_{\acute{e}} \tilde{\in} SV(\tilde{X})$.

The soft vector space $SV(\tilde{X})$ with a soft semi-norm $\tilde{P}$ on $\tilde{X}$ is said to be a soft semi-norm linear space and is denoted by$(\tilde{X}, \tilde{P})$.

Theorem (3): Let $\tilde{P}: SV(\tilde{X}) \to R^+(E)$ is a soft semi-norm, then for all $\tilde{x}_e, \tilde{y}_{\acute{e}} \tilde{\in} SV(\tilde{X})$:

1. $\tilde{P}(\tilde{0}) = \tilde{0}$;
2. $\tilde{P}(-\tilde{x}_e) = \tilde{P}(\tilde{x}_e)$;
3. $\tilde{P}(\tilde{x}_e - \tilde{y}_{\acute{e}}) = \tilde{P}(\tilde{y}_{\acute{e}} - \tilde{x}_e)$;
4. $|\tilde{P}(\tilde{x}_e) - \tilde{P}(\tilde{y}_{\acute{e}})| \lesssim \tilde{P}(\tilde{x}_e - \tilde{y}_{\acute{e}})$;
5. $\tilde{P}(\tilde{x}_e) \gtrsim \tilde{0}$;
6. The set $N(\tilde{P}) = \{\tilde{x}_e \tilde{\in} \tilde{X}: \tilde{P}(\tilde{x}_e) = \tilde{0}\}$ is a soft subspace of $SV(\tilde{X})$.

Proof: For all $\tilde{x}_e, \tilde{y}_{\acute{e}} \tilde{\in} SV(\tilde{X})$:

1. $\tilde{P}(\tilde{0}) = \tilde{P}(\tilde{0}.\tilde{x}_e) = |\tilde{0}|\tilde{P}(\tilde{x}_e) = \tilde{0}$.
2. $\tilde{P}(-\tilde{x}_e) = |-\tilde{1}|\tilde{P}(\tilde{x}_e) = \tilde{P}(\tilde{x}_e)$.
3. $\tilde{P}(\tilde{x}_e - \tilde{y}_{\acute{e}}) = \tilde{P}(-(\tilde{y}_{\acute{e}} - \tilde{x}_e)) = |-\tilde{1}|\tilde{P}(\tilde{y}_{\acute{e}} - \tilde{x}_e) = \tilde{P}(\tilde{y}_{\acute{e}} - \tilde{x}_e)$.
4. $\tilde{x}_e = (\tilde{x}_e - \tilde{y}_{\acute{e}}) + \tilde{y}_{\acute{e}} \Rightarrow \tilde{P}(\tilde{x}_e) = \tilde{P}[(\tilde{x}_e - \tilde{y}_{\acute{e}}) + \tilde{y}_{\acute{e}}] \lesseqqgtr \tilde{P}(\tilde{x}_e - \tilde{y}_{\acute{e}}) + \tilde{P}(\tilde{y}_{\acute{e}})$

$\tilde{P}(\tilde{x}_e) - \tilde{P}(\tilde{y}_{\acute{e}}) \lesssim \tilde{P}(\tilde{x}_e - \tilde{y}_{\acute{e}})$.....*

$\tilde{y}_{\acute{e}} = (\tilde{y}_{\acute{e}} - \tilde{x}_e) + \tilde{x}_e \Rightarrow \tilde{P}(\tilde{y}_{\acute{e}}) = \tilde{P}[(\tilde{y}_{\acute{e}} - \tilde{x}_e) + \tilde{x}_e] \lesseqqgtr \tilde{P}(\tilde{y}_{\acute{e}} - \tilde{x}_e) + \tilde{P}(\tilde{x}_e)$
$-\tilde{P}(\tilde{x}_e - \tilde{y}_{\acute{e}}) \lesssim \tilde{P}(\tilde{x}_e) - \tilde{P}(\tilde{y}_{\acute{e}})$....**

From *&** $|\tilde{P}(\tilde{x}_e) - \tilde{P}(\tilde{y}_{\acute{e}})| \lesssim \tilde{P}(\tilde{x}_e - \tilde{y}_{\acute{e}})$.

5. Since $|\tilde{P}(\tilde{x}_e) - \tilde{P}(\tilde{y}_{\acute{e}})| \lesssim \tilde{P}(\tilde{x}_e - \tilde{y}_{\acute{e}})$,take $\tilde{y}_{\acute{e}} = \tilde{0} \Rightarrow |\tilde{P}(\tilde{x}_e)| \lesssim \tilde{P}(\tilde{x}_e)$

Since $|\tilde{P}(\tilde{x}_e)| \gtrsim \tilde{0} \Rightarrow \tilde{P}(\tilde{x}_e) \gtrsim \tilde{0}$.

6. Since $\tilde{P}(\tilde{0}) = \tilde{0} \Rightarrow \tilde{0} \tilde{\in} N(\tilde{P}) \Rightarrow N(\tilde{P}) \neq \varphi$

Let $\tilde{x}_e, \tilde{y}_{\acute{e}} \tilde{\in} N(\tilde{P})$ and $\tilde{r}, \tilde{s}$ are soft real numbers,

$\tilde{P}(\tilde{x}_e) = \tilde{0}$, $\tilde{P}(\tilde{y}_{\acute{e}}) = \tilde{0}$,

$\tilde{P}(\tilde{r}\tilde{x}_e + \tilde{s}\,\tilde{y}_{\acute{e}}) \lesssim \tilde{P}(\tilde{r}\tilde{x}_e) + \tilde{P}(\tilde{s}\,\tilde{y}_{\acute{e}})$

$= |\tilde{r}|\tilde{P}(\tilde{x}_e) + |\tilde{s}|\tilde{P}(\tilde{y}_{\acute{e}}) = \tilde{0}$ ....*

Since $\tilde{x}_e, \tilde{y}_{\acute{e}} \tilde{\in} N(\tilde{P}) \Rightarrow \tilde{x}_e, \tilde{y}_{\acute{e}} \tilde{\in} SV(\tilde{X})$ and $SV(\tilde{X})$ is a soft vector space, then $\tilde{r}\tilde{x}_e + \tilde{s}\,\tilde{y}_{\acute{e}} \tilde{\in} SV(\tilde{X}) \Rightarrow \tilde{P}(\tilde{r}\tilde{x}_e + \tilde{s}\,\tilde{y}_{\acute{e}}) \gtrsim \tilde{0}$....**

By (*&**)

$\tilde{P}(\tilde{r}\tilde{x}_e + \tilde{s}\,\tilde{y}_{\acute{e}}) = \tilde{0} \Rightarrow \tilde{r}\tilde{x}_e + \tilde{s}\,\tilde{y}_{\acute{e}} \tilde{\in} N(\tilde{P})$, therefore $N(\tilde{P})$ is a soft subspace.

Example: Let $\tilde{X}$ be a soft semi-norm space, for every $\tilde{x}_e \tilde{\in} SV(\tilde{X})$

$\tilde{P}(\tilde{x}_e) = |e| + p(x)$ is a soft semi-norm such that p is a semi-norm on X.

Proof: (N1) $\tilde{P}(\tilde{r}\tilde{x}_e) = \tilde{P}[(\widetilde{rx})_{(re)}]$ $=|re| + p(rx)= |r||e| + |r|p(x)$

$=|r|\{|e| + p(x)\}=|\tilde{r}|\tilde{P}(\tilde{x}_e)$

(N2) $\tilde{P}(\tilde{x}_e + \tilde{y}_{\acute{e}}) = \tilde{P}((\widetilde{x+y})_{(e+\acute{e})})$ $= |e + \acute{e}| + p(x + y) \lesssim |e| + |\acute{e}| + p(x) + p(y) =(|e| + p(x)) +(|\acute{e}| + p(y))$ $=\tilde{P}(\tilde{x}_e) +\tilde{P}(\tilde{y}_{\acute{e}})$. Therefore $\tilde{P}(\tilde{x}_e)$ is a soft semi-norm.

Definition 14: Let $SV(\tilde{X})$ be a soft vector space, then $\tilde{d}: SV(\tilde{X}) \times SV(\tilde{X}) \to R^{+}(E)$ is said to be a soft pseudo metric function on $\tilde{X}$ if it satisfies the following conditions for all $\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{z}_{e_3} \tilde{\in} \tilde{X}$:

1. $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) \geq \tilde{0}$
2. $\tilde{d}(\tilde{x}_{e_1}, \tilde{x}_{e_1}) = \tilde{0}$;
3. $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) = \tilde{d}(\tilde{y}_{e_2}, \tilde{x}_{e_1})$
4. $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) \lesssim \tilde{d}(\tilde{x}_{e_1}, \tilde{z}_{e_3}) + \tilde{d}(\tilde{z}_{e_3}, \tilde{y}_{e_2})$

The soft set $\tilde{X}$ with a soft pseudo metric $\tilde{d}$ is called a soft pseudo metric space and denoted by $(\tilde{X}, \tilde{d}, E)$.

Remark 3: We can prove the conditions 1 of the above definition to make the definition consisted of three conditions as following:

Proof: $\tilde{d}(\tilde{x}_{e_1}, \tilde{x}_{e_1}) \lesssim \tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) + \tilde{d}(\tilde{y}_{e_2}, \tilde{x}_{e_1}) = \tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) + \tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2})$

$\Longrightarrow 2\,\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) \gtrsim \tilde{d}(\tilde{x}_{e_1}, \tilde{x}_{e_1}) = \tilde{0} \Longrightarrow \tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) \gtrsim \tilde{0}$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2} \tilde{\in} \tilde{X}$.

Definition 15: Let $SP(\tilde{X})$ be a soft point space, then $\tilde{d}: SP(\tilde{X}) \times SP(\tilde{X}) \to R^{+}(E)$ is said to be a soft pseudo metric function on $\tilde{X}$ if it satisfies the following conditions for all $\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{z}_{e_3} \tilde{\in} \tilde{X}$:

1. $\tilde{d}(\tilde{x}_{e_1}, \tilde{x}_{e_1}) = \tilde{0}$;
2. $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) = \tilde{d}(\tilde{y}_{e_2}, \tilde{x}_{e_1})$
3. $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) \lesssim \tilde{d}(\tilde{x}_{e_1}, \tilde{z}_{e_3}) + \tilde{d}(\tilde{z}_{e_3}, \tilde{y}_{e_2})$

The soft set $\tilde{X}$ with a soft pseudo metric $\tilde{d}$ is called a soft pseudo metric space and denoted by $(\tilde{X}, \tilde{d}, E)$.

Theorem (4): Every soft semi-normed space is a soft pseudo metric space.

Proof: Let $(\tilde{X}, \tilde{P})$ be a soft semi-normed space. If we define the soft pseudo metric by $\tilde{d}(\tilde{x}_{e_1},\tilde{y}_{e_2}) = \tilde{P}(\tilde{x}_{e_1} - \tilde{y}_{e_2})$ for every $\tilde{x}_{e_1},\tilde{y}_{e_2},\tilde{z}_{e_3}\tilde{\in}\tilde{X}$,

1. $\tilde{d}(\tilde{x}_{e_1},\tilde{x}_{e_1}) = \tilde{P}(\tilde{x}_{e_1} - \tilde{x}_{e_1}) = \tilde{P}(\tilde{0}) = \tilde{0}$;
2. $\tilde{d}(\tilde{x}_{e_1},\tilde{y}_{e_2}) = \tilde{P}(\tilde{x}_{e_1} - \tilde{y}_{e_2}) = \tilde{P}(\tilde{y}_{e_2} - \tilde{x}_{e_1}) = \tilde{d}(\tilde{y}_{e_2},\tilde{x}_{e_1})$;
3. $\tilde{d}(\tilde{x}_{e_1},\tilde{y}_{e_2}) = \tilde{P}(\tilde{x}_{e_1} - \tilde{y}_{e_2}) = \tilde{P}(\tilde{x}_{e_1} - \tilde{z}_{e_3} + \tilde{z}_{e_3} - \tilde{y}_{e_2}) \tilde{\leq} \tilde{P}(\tilde{x}_{e_1} - \tilde{z}_{e_3}) + \tilde{P}(\tilde{z}_{e_3} - \tilde{y}_{e_2}) = \tilde{d}(\tilde{x}_{e_1},\tilde{z}_{e_3}) + \tilde{d}(\tilde{z}_{e_3},\tilde{y}_{e_2})$.

Then $(\tilde{X},\tilde{d})$ is a soft pseudo metric space.

Theorem (5): Let $\tilde{d} : SP(\tilde{X}) \times SP(\tilde{X}) \to R^+(E)$ be a soft pseudo metric function. $SV(\tilde{X})$ is a soft semi-normed space if and only if the following conditions for all $\tilde{x}_{e_1},\tilde{y}_{e_2},\tilde{z}_{e_3}\tilde{\in}\tilde{X}$:

1. $\tilde{d}(\tilde{x}_{e_1} + \tilde{z}_{e_3},\tilde{y}_{e_2} + \tilde{z}_{e_3}) = \tilde{d}(\tilde{x}_{e_1},\tilde{y}_{e_2})$
2. $\tilde{d}(\tilde{r}\tilde{x}_{e_1},\tilde{r}\,\tilde{y}_{e_2}) = |\tilde{r}|\tilde{d}(\tilde{x}_{e_1},\tilde{y}_{e_2})$ satisfied.

Proof: If $\tilde{d}(\tilde{x}_{e_1},\tilde{y}_{e_2}) = \tilde{P}(\tilde{x}_{e_1} - \tilde{y}_{e_2})$ then $\tilde{d}(\tilde{x}_{e_1} + \tilde{z}_{e_3},\tilde{y}_{e_2} + \tilde{z}_{e_3}) = \tilde{P}(\tilde{x}_{e_1} + \tilde{z}_{e_3} - \tilde{y}_{e_2} - \tilde{z}_{e_3}) = \tilde{P}(\tilde{x}_{e_1} - \tilde{y}_{e_2}) = \tilde{d}(\tilde{x}_{e_1},\tilde{y}_{e_2})$ and

$\tilde{d}(\tilde{r}\tilde{x}_{e_1},\tilde{r}\,\tilde{y}_{e_2}) = \tilde{P}(\tilde{r}\tilde{x}_{e_1} - \tilde{r}\tilde{y}_{e_2}) = \tilde{P}(\tilde{r}(\tilde{x}_{e_1} - \tilde{y}_{e_2}))$

$= |\tilde{r}|\,\tilde{P}(\tilde{x}_{e_1} - \tilde{y}_{e_2}) = |\tilde{r}|\tilde{d}(\tilde{x}_{e_1},\tilde{y}_{e_2})$

Suppose that the conditions of the proposition are satisfied. Taking $\tilde{P}(\tilde{x}_{e_1}) = \tilde{d}(\tilde{x}_{e_1},\tilde{0})$ for every $\tilde{x}_{e_1}\tilde{\in}\tilde{X}$ we have:

$N1.\ \tilde{P}(\tilde{r}\tilde{x}_{e_1}) = \tilde{d}(\tilde{r}\tilde{x}_{e_1},\tilde{0}) = \tilde{d}(\tilde{r}\tilde{x}_{e_1},\tilde{r}\,\tilde{0}) = |\tilde{r}|\tilde{d}(\tilde{x}_{e_1},\tilde{0}) = |\tilde{r}|\,\tilde{P}(\tilde{x}_{e_1})$

$N2.\ \tilde{P}(\tilde{x}_{e_1} + \tilde{y}_{e_2}) = \tilde{d}(\tilde{x}_{e_1} + \tilde{y}_{e_2},\tilde{0}) = \tilde{d}(\tilde{x}_{e_1},-\tilde{y}_{e_2}) \tilde{\leq} \tilde{d}(\tilde{x}_{e_1},\tilde{0}) + \tilde{d}(\tilde{0},-\tilde{y}_{e_2}) = \tilde{P}(\tilde{x}_{e_1}) + |-\tilde{1}|\,\tilde{P}(\tilde{y}_{e_2}) = \tilde{P}(\tilde{x}_{e_1}) + \tilde{P}(\tilde{y}_{e_2})$.

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الفضاءات شبه المعيارية اللينة والفضاءات شبه المترية اللينة

تاريخ الاستلام : 8\10\2015 تاريخ القبول: 28\12\2015

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الخلاصة: يعرض هذا البحث تعريفات لبعض من خصائص المجموعه اللينه,ومنها مجموعه متوازنه, وماصة, ومحدبة. تتناول الدراسة مجموعة من المبرهنات والملاحظات المتعلقة بتلك الخصائص, ايضا تم تقديم تعريف شبه المعياري اللين.تبرهن الدراسة ايضا واحدة من بديهيات الفضاء شبه المتري اللين التي ورت في[7]. كما تناقش بعض الطروحات والمبرهنات المتعلقة بالفضائين وقدمت ايضا مبرهنة توضح العلاقة بين الفضائين.

الكلمات المفتاحية: المجموعه اللينه ,شبه معياري,شبه معياري لين,شبه متري,شبه متري لين,المجموعات المتوازنه اللينه والماصه اللينه والمحدبه اللينه.

Math. Sub. classifications:QA150-272.5