On Some Types Of Ideals On Supra Topological Space

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Abstract - This paper introduces some types of ideals on supra topological Space called I^*,I^{**}, αI^*&αI^{**}, and it is studying the relations among ideal, $I^*$and I^{**}, αI^*&αI^{**}, on supra topological spaces .Also introduce a new class of sets and functions between topological spaces called supra αI^*-open sets and αI^{**} and supraαI^{**}-open sets . Finally investigate some properties between them

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Finally investigate some properties between them .

2. Preliminaries

In this section we introduced some definitions and results which are need of this paper.

Definition2.1.[3]

A nonempty collection I of subsets of X is called an ideal on X
if :
(i). $A \in I$and $B \subset A$ implies B $\in I$(heredity) ;
(ii). $A \in I$and $B \in I$implies $A \cup B \in I$ (finite additivity).

Definition2.2.[6]

A subfamily μ of the power set $P(X)$ of a nonempty set X iscalled a supra topology on X if μ satisfies the following conditions:
1. μcontains$\phi$and X,
2. $\mu$is closed under the arbitrary union.

The pair (X,μ) is called a supra topological space.In this respect, the member of μ is called supra open set in (X,μ). The complementof supra open set is called supra closed set.

Definition2.3. [8]

We called the triple of (X,μ,I) ideal supra topological space if(X,μ) is supra topological space and I is ideal on X, and we use(X,I) instead of (X,μ,I) for simply .

Definition2.4.[8]

Let (X,μ)be a supra topological space and $A \subset X$. Then supra interior and supra closure of A in (X,μ)defined as

1. Introduction

Vaidyanathswamy[11] and Kuratowski [3] introduced the concept of ideal topological space. Shyamapada Modakand Sukalyan Mistry[8] defined local function in ideal supra topological space .Further Shyamapada Modakand Sukalyan Mistry studied the properties of ideal supra topological space and they have introduced three operator called( )$*^{\mu}$, $\psi_\mu$and ψ_μ −C sets .

Modakand Bandyopadhyay[7]in 2007have defined generalized open sets using ψ − operator. More recently AI-Omri and Noiri[1] have defined the ideal m-space and introduced two operators as like similar to the local function andψ −operator.

Different types of generalized open sets like semi-open[4], pre open[5], semi per open[2], α open[10] , I– semi continuity points [9] already are there in literature and these generalized sets have a common property which is closed under arbitrary union.

Mashhour [6] put all of the sets in a pocket and defined a generalized space which is supra topological space .This paper introduces some types of ideals on supra topological space called I^*, I^{**}, αI^*&αI^{**}, and it is studying the relations among ideal,$I^*$and $I^{**}\alpha I^*$&αI^{**}, on supra topological spaces .

Also introduce a new class of sets and functions between topological spaces called supraαI^*-open sets and supra αI^{**}-open sets .

The following relation are independent .
ideal$\nrightarrow I^*$ & $I^* \nrightarrow$ ideal

Example 2.12.

Let $X = \{a, b, c, d\}$, $I = \{\{a\}, \{b, c\}, \{c\}, \{a, c\}\}$, $A = \{a, c\}$, $B = \{a\}$. Then (X, I) is ideal because satisfies two conditions of ideal , but is not (X, I^*) because $(A \cup B)^c \notin I$.

Example 2.13.

Let $X = \{a, b, c\}$, $I = \{\{a\}, \{b, c\}, X\}$, $A = \{b, c\}$, $B = \emptyset$. Then (X, I^*) satisfies two conditions of I^*, but (X, I) is not ideal because $B \notin I$.

Remark 2.14.

The following relation are independent .
ideal$\nrightarrow I^{**}$ & $I^{**} \nrightarrow$ ideal

Example 2.15.

Let $X = \{a, b, c\}$, $I = \{\{a\}, \{c\}, \{a, c\}\}$, $A = \{a, c\}$, $B = \{a\}$. Then (X, I) is ideal because satisfies two conditions of ideal , but is not (X, I^{**}) because $(A \cap B)^c \notin I^{**}$.

Example 2.16.

Recall Example 2.12 we see that (X, I^{**}) satisfies two conditions of I^{**}, but (X, I) is not ideal because $B \notin I$.

Remark 2.17.

The following relation are independent .
$I^* \rightarrow I^{**}$ & $I^{**} \nrightarrow I^*$

Proposition2.18.

Every (X, I^*) is(X, I^{**}) .

Proof :

suppose (X, I^*) and $A \in I, B \subset A$ implies $B^c \in I$.*Since* $(A \cup B)^c \in I$ then $A^c \cap B^c \in I$, since every $A \cap B \subseteq A \cup B$ then $A^c \cup B^c \supseteq A^c \cap B^c$. So that $(A \cap B)^c \in I$. Therefore (X, I^{**}) is satisfy.

Remark 2.19.

The converse of Proposition 2.17 is not true in general .The counter example show the converse is not true .

Example 2.20.

Let $X = \{a, b, c\}$, $I = \{\{a\}, \{b, c\}, \{a, c\}\}$, $A = \{a, c\}$, $B = \{A\}$. Then (X, I^{**}) satisfies two conditions of I^{**}, but (X, I^*) is not satisfy because$(A \cup B)^c \notin I$.

3- Basic properties of supra αI^*-open set

$\cup\{\ : U \subseteq A, U \in \mu\}$ and $\cap\{F : A \subseteq F, X - F \in \mu\}$ respectively.

The supra interior and supra closure of A in (X, μ) are denoted as $Int^\mu(A)$ and $Cl^\mu(A)$ [13] respectively .From definition, $Int^\mu(A)$ is a supra open set and $Cl^\mu(A)$ is a supra closed set.

Definition2.5.[8]

Let (X, μ)be a supra topological space and $M \subset X$. Then M is said to a supra neighborhood of a point x of X if for some supra open set $U \in \mu, x \in U \subset M$.

Theorem2.6. [8]

Let (X, μ) be a supra topological space and $A \subset X$. Then
(i). $Int^\mu(A) \subseteq A$.
(ii). $A \in \mu$ if and only if $Int^\mu(A) = A$.
(iii). $Cl^\mu(A) \supseteq A$.
(iv). A is a supra closed set if and only if $Cl^\mu(A) = A$.
(v). $x \in Cl^\mu(A)$ if and only if every supra open set $U x$containing x, $U_x \cap A \neq \phi$.

Theorem2.7.[8]

Let (X, μ)be a supra topological space and $A \subset X$. Then $Int^\mu(A) = X - Cl^\mu(X - A)$.

Definition2.8.

A nonempty collection I of subsets of X is called I^* on X if
(i). $A \in I$and $B \subset A$implies $B^c \in I$;
(ii). $A \in I$and$B \subset A$ implies $(A \cup B)^c \in I$.

Definition2.9.[6]

We called the triple of $(X, \mu, I^*), (X, \mu, I^{**})$ respectively,Ideal supra topological space if (X, μ) is supra topological space and I^* (*resp.* I^{**}) on$X$ is *ideal** (*resp. ideal***) on X.

Now, our work introduce two concepts of I^*&I^{**} on supra topological space.

Definition2.10.

A nonempty collection I of subsets of X is calledI^{**} on X if
(i). $A \in I$and $B \subset A$implies $B^c \in I$;
(ii). $A \in I$and $B \subset A$ implies $(A \cap B)^c \in I$.

Now we study the relations among them ideal , $I^*$and I^{**}.

Remark 2.11.

Let A^c and B^c be two supra αI^*-open sets. Then supra int(supra cl(supra int(A^c))) $\subseteq A^c$ and supra int(supra cl(supra int(B^c))) $\subseteq B^c$, this implies, supra int(supra cl(supra int($A^c \cup B^c$)))$\subseteq A^c \cup B^c$.Therefore $A^c \cup B^c$ is supra αI^*–open set.

Remark 3.9.

Finite intersection of ideal supra αI^*-open sets may fail to be ideal supraαI^*-open set .

Example 3.10.

Let (X, μ, I^*)be an ideal supra topological space .Where $X = \{a, b, c\}$ and $\mu = \{\phi, X, \{a\}, \{a, b\}\}$. Then $\{a\}, \{a, b\}$ are supra αI^*-open sets, But their intersection is a not supra αI^*-open set.

Definition 3.11.

Complement of ideal supra αI^*-open is a supra αI^*-closed set.

Proposition 3.12.

(i) Finite intersection of ideal supra αI^*-closed sets is always a supraαI^*-closed set.

Proof:

(i) This follows immediately from Proposition 3.8.

Remark 3.13.

Finite union of ideal supra αI^*-closed set may fail to be supra αI^*– closed set.

Example 3.14.

Let (X, μ, I^*)be an ideal supra topological space .Where $X = \{a, b, c, d\}$ and $\mu = \{\phi, X, \{a\}, \{a, b\}\}$. Then $\{a, d\}, \{c, d\}$ are supra αI^*- Closed sets, but their union is not a supra αI^*-closed set.

Definition 3.15.

The supra αI^*-closure of a set A^c is denote by supra αI^* cl(A^c) and defined as, supra αI^*cl(A^c) = $\cap\{B^c : B^c$ is ideal supra αI^*-closed set and $A^c \subseteq B^c\}$. The supra αI^*- interior of a set is denoted by supra αI^*int(A^c), and defined as, supraαI^*int(I^*) = $\cup\{B^c : B^c$ is ideal supra αI^*-closed set and $B^c \subseteq A^c\}$.

Remark 3.16.

It is clear that supra αI^*int(A^C) is a supra αI^*-open set and supra αI^*cl(A^c) is a supra αI^*-closed set.

Proposition 3.17.

In this section we introduce a new class of sets .

Definition 3-1.

Let (X, μ, I^*)be an ideal supra topological space . a set A^c is called supraαI^*-open set if supra int (supra cl(A^c))$\subseteq A^c$.

Proposition 3.2.

Every ideal supra open set is supra αI^*-open set

Proof:

Let A^c be an ideal supra open set in (X, μ, I^*) .Since $A^c \supseteq$ supra cl(A^c) , then supra int (A^c) $\supseteq$ supra int (supra cl(supra int(A^c))).
Hence $A^c \supseteq$ supra int(supra cl(supra int(A^c))).

Remark 3.3.

The converse of the above proposition need not be true. This is shown by the following example.

Example 3.4.

Let (X, μ, I^*)be an ideal supra topological space .Where $X = \{a, b, c\}$and$\mu = \{\phi, X, \{a\}\}$. Then $\{a, c\}$ is a supra αI^*-open set, but not Ideal supra open.

Proposition 3.5.

Every supra αI^*-open set is ideal supra semi-open set.

Proof:

Let A^c be a supra αI^*-open set in (X, μ, I^*) . Therefore,
supra int (supra cl(supra int(A^c))).$\subseteq A^c$ .It is obvious that,supra cl(supra int(A^c))$\subseteq$supra int(supra cl(supra int(A^c))) . Hence supra cl(supra int(A^c))$\subseteq A^c$.

Remark 3.6.

The converse of the above proposition need not be true. This is shown by the following example.

Example 3.7.

Let (X, μ, I^*)be an ideal supra topological space .Where $X = \{a, b, c\}$and$\mu = \{\phi, X, \{a\}\}$. Then $\{b, c\}$ is ideal supra semi open set, but not αI^*-open.

Proposition 3.8.

Finite union of ideal supra αI^*-open sets is always ideal supraαI^*- open set.

Proof:

The converse of the above proposition need not be true. This is shown by the following example.

Example 4.7.

Let (X,μ,I^{**})be an ideal supra topological space .Where $X = \{a,b,c\}$and$\mu= \{\phi,X,\{b\}\}$. Then $\{a,c\}$ is ideal supra semi open set, but not αI^{**}-open.

Proposition 4.8.

Finite intersection of ideal supra αI^{**}-open sets is always ideal supraαI^{**}-open set .

Proof.

Let A^c and B^c be two supra αI^{**}-open sets. Then supra int (supra cl(supra int(A^c))) $\subseteq A^c$ and supra int(supra cl(supra int(B^c))) $\subseteq B^c$, this implies, supra int(supra cl(supra int($A^c \cap B^c$)))$\subseteq A^c \cap B^c$ .Therefore $A^c \cap B^c$ is supra αI^{**}–open set.

Remark 4.9.

Finite union of ideal supra αI^{**}-open sets may fail to be ideal supraαI^{*}-open set .

Example 4.10.

Let (X,μ,I^{**})be an ideal supra topological space .Where $X = \{a,b,c\}$ and $\mu =\{ \phi , X,\{a\},\{a,c\},\{b\}\}$. Then$\{a\},\{a,b\}$are supra αI^{**}- open sets ,but their intersection is a not supra αI^{**}-open set.

Definition 4.11.

Complement of ideal supra αI^{**}-open is a supra αI^{**}-closed set.

Proposition 4.12.

(i) Finite union of ideal supra αI^{**}-closed sets is always a supraαI^{**}-closed set.

Proof:

(i) This follows immediately from Proposition 4.8.

Remark 4.13.

Finite intersection of ideal supra αI^{*}-closed set may fail to be supra αI^{*}–closed set .

Example 4.14.

Let (X,μ,I^{**})be an ideal supra topological space .Where $X = \{a,b,c\}$ and $\mu = \{\phi,X,\{a,b\}\}$. Then$\{a,c\},\{b,c\}$are supra αI^{*}-closed set but their union is not a supra αI^{**}-closed set.

Definition 4.15.

(i) X − supra αI^{*}int(A^c) = supra αI^{*}cl(X − A^c) .

(ii) X − supra αI^{*}cl(A^c) = supra αI^{*}int(X– A^c) .

Proof:

(i) and (ii) are clear from definition 3.15 and remark 3.16.

Proposition 3.18.

The following statements are true for every $A^c$and B^c.

(1)supraαI^{*}int(A^c) ∪ supra αI^{*}int(B^c) = supra αI^{*}int($A^c \cup B^c$)

(2)supraαI^{*}cl(A^c) ∩ supra αI^{*}cl(B^c) = supra αI^{*}cl($A^c \cap B^c$).

Proof:

Obvious.

4- Basic properties of supra αI^{**}-open sets

In this section we introduce a new class of sets .

Definition 4-1.

Let (X,μ,I^{**})be an ideal supra topological space .a set A^c is called supra αI^{**}-open set if supra int (supra cl(A^c))$\subseteq A^c$.

Proposition4.2.

Every ideal supra open set is supra αI^{**}-open set

Proof:

The prove is same as prove Proposition 3.2.

Remark 4.3.

The converse of the above proposition need not be true. This is shown by the following example.

Example 4.4.

Let (X,μ,I^{**})be an ideal supra topological space .Where $X = \{a,b,c\}$ and $\mu = \{\phi,X,\{b,c\}\}$. Then $\{a\}$ is a supra αI^{**}-open set, but not ideal supra open.

Proposition4.5.

Every supra αI^{**}-open set is ideal supra semi-open set.

Proof:

The prove is same as prove Proposition 3.5.

Remark 4.6.

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The supra αI^{**}-closure of a set A^c is denote by supra αI^* cl(A^c) and defined as, supra αI^{**}cl(A^c) $= \cap\{B^c : B^c$ is ideal supra αI^{**}-closed set and $A^c \subseteq B^c\}$. The supra αI^{**}-interior of a set is denoted by supra αI^{**}int(A^c), and defined as supraαI^{**}int(I^*) $= \cup\{B^c : B^c$ is ideal supra αI^{**}-closed set and $B^c \subseteq A^c\}$.

Remark 4.16.

It is clear that supra αI^{**}int(A^C) is a supra αI^{**}-open set and supra αI^{**}cl(A^C) is a supra αI^{**}-closed set.

Proposition4.17.

(i) X − supra αI^{**}int(A^c) = supra αI^{**}cl(X − z) .

(ii) X − supra αI^{**}cl(A^c) = supra αI^{**}int(X− A^c) .

Proof:

(i) and (ii) are clear from Definition 4.15 and Remark 4.16.

Remark4.18.

The following statements are true for every $A^c$and B^c.

(1)supra αI^{**} int(A^c) ∪ supra αI^{**} int(B^c) ≠ supra αI^{**}int($A^c \cup B^c$)

(2)supraαI^{**}cl(A^c) ∩ supra αI^{**}cl(B^c) ≠ supra αI^{**}cl($A^c \cap B^c$).

Example 4.19

Let (X, μ, I^{**})be an ideal supra topological space .Where $X = \{a, b, c\}$ and $\mu = \{\phi, X, \{a, b\}, \{b\}\}$. Then supra αI^{**} int(A^c)= $\{a, c\}$, supra αI^{**} int(B^c)= $\{a, b\}$ and supra αI^{**} int($A^c \cup B^c$) $= \{a, c\}$ .We see that supra αI^{**} int(A^c) ∪ supra αI^{**} int(B^c) = $\{a, b, c\} \neq \{a, c\}$ =supra αI^{**}int($A^c \cup B^c$) .Thus supra αI^{**}int(A^c) ∪ supra αI^{**}int(B^c) ≠ supra αI^{**}int($A^c \cup B^c$) .

Example 4.20.

Let (X, μ, I^{**}) be an ideal supra topological space .Where $X = \{a, b, c, d\}$ and $\mu = \{\phi, X, \{a, b\}, \{b, d\}, \{b\}\}$. Then supra αI^{**} int(A^c) $= \{a, b, d\}$, supra αI^{**} int(B^c)= $\{a, c\}$and supra αI^{**}int($A^c \cap B^c$)= $\{a, d\}$ .We see that supra αI^{**} int(A^c) ∩ supra αI^{**} int(B^c) $= \{a\} \neq \{a, c\} =$ supra αI^{**} int($A^c \cup B^c$) . Thus supra αI^{**} cl(A^c) ∩ supra αI^{**}cl(B^c) ≠ supra αI^{**}cl($A^c \cap B^c$).

حول بعض أنواع المثاليات في الفضاء التبولوجي الفوقي

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المستخلص :

قدمنا في هذا البحث بعض أنواع المثاليات في الفضاء التبولوجي الفوقي سميناه I^* و I^{**} و αI^* و αI^{**} , ودرسنا العلاقة بين المثالي I^* و I^{**} , αI^* و αI^{**} في الفضاء التبولوجي الفوقي . كذلك درسنا صف جديد للمجموعات والدوال بين الفضاء التبولوجي αI^* و αI^{**} والفضاء التبولوجي الفوقي αI^{**} وأخيرا استقصينا بعض الخواص بينهم .

الكلمات المفتاحية : المثالي , التوبولوجي الفوقي , المجموعة المفتوحة , الدوال , التبولوجي الفوقي المثالي .

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