# On some Types of coc-functions

Received:21/5/2015 Accepted :16/8/2015

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**Abstract :**

In this work , we investigate the properties of product of some coc- functions namely coc- continuous ,coc-irresolute , strongly coc-closed and introduce the definition of coc-perfect, and investigate the properties of composition, restrictions and product of this type. Also, We give the relation among them.

**Keywords:** coc-open, coc-irresolute, strongly coc-closed and coc-perfect.

**Math.Sub. Classifications: 54H,54 H 15, 22 F 05 .**

*<u>**The Research is apart of on MSC.Thesis in the case of the second researcher**</u>

**Introduction** :

One of the very important concepts in a topological spaces is the concept of functions . There are several types of fu nction related to types of open set in atopological spaces [3],[5], Al Ghour S.and Samarah. S in [1] introduce the definition of coc-open set . *Al-Hussaini* F.H.[2] introduce coc-continuity as a generalization of continuity , Also they give the concept of coc-closed function. In this work , we study the properties of product of some coc-function namely (coc--open, coc-irresolute, strongly coc-closed) and construet coc-perfect function ) . Several results and concepts related to them will be introduced. Throughout this paper , we use $\mathbb{R}$ , $\mathbb{Z}$ , $\mathbb{Q}$and N to denote the set of real numbers , the set of rational numbers the set of Integer numbers and the set of natural numbers .For a subset A of topological space (X,T) the closure of A and the interior of A will be denoted by $\bar{A}$ and $A^{\circ}$ respectively. Also, We write $T_A$to denote the relative topology on A.For a non empty set X $T_{dis}$, $T_{ind}$ will denote respectively the discrete and indiscret topology on X .We use$T_U$ to denote the usual topologY on R and$T_{fin}$ to denote the final segment topology on $\mathbb{N}$ , i.e $T_{fin}$ to contain N, $\varphi$ ,and every set {n,n+1,...} Where n any positive integer. C(X,T) denote the set of all compact sets in X.

*The research is a part of m.s.c. thesis in case of the second researcher**

## 1. COC-Open sets:

In this section , we recall the definition of coc-open set and investigate some properties .

**1.1.Definition : [1]**

A subset A of a space $(X,\tau)$ is called coc-compact open set (For brief : coc-open set ) if for every $x \in A$ there exists an open set $U \subseteq X$ and a compact subset K ∈ C$(X,\tau)$ such that $x \in U - K \subseteq A$. The complement of coc-open set is called coc-closed set.The family of all coc-open set of X is denoute by $T^k$ .

**1.2.Remark : [1] [2]**

i. Every open set is a coc-open set .
ii. Every closed set is a coc-closed set.
The converse of (i, ii) is not true in general as the following example shows:

Let $X = \mathbb{N}$ , $T = T_{fin}$.The set A= {1,5,6,7, ... } is coc-open set , but it's not an open set and B= {5,6} is an coc- closed set, but it's not an closed set.

**1.3 . Theorem :[1]**

Let $(X, T)$ be a topological space Then
i.The collection $T^k$ forms a topology on $X$.
ii.The collection $\beta^k(\tau)$ forms a base for $T^k$where $\beta^k(\tau) = \{U - K: U\epsilon\tau \text{ and } K \in C(X,\tau)\}$.
iii. $T \subseteq T^k$.

The converse of (iii) is not true as the following example shows: Let $X = \mathbb{N}$ , $T = T_{ind}$ then $T^k = T_{dis}$ and then $T^k \nsubseteq T$ .

**1.4.Corollary:**

**1.5. Theorem:[4]**

i.Let Y be asubspace of a topological space $X$ and A⊆ $Y$.Then A is compact relative to $X$ if and only A is compact relative to $Y$.

ii. Every closed subset of a compact space is compact.
ii. In a Hausdorff space , a point and a compact set not containg it can be separated by open sets .
iv. Every compact subset of a Hausdorff space is closed.

**1.6 . Definition:[1]**
A space $X$ is called $CC$ if every compact set in $X$ is closed.

**1.7 . Theorem :[1]**
Let $(X, \tau)$ be a space. Then the following statements are equivalent:
i. $(X, \tau)$ is $CC$.
ii. $T = T^k$.

**1.8.Corollary :[1]**
Let $(X,T)$ be a space then the intersection of an open set with a coc-open set is a coc-open set

Let $(X, \tau)$ be a $T_2$-space, then $T = T^k$.

**1.9 . Definition:[1]**
Let $X$ be a space and $A \subseteq X$ . The intersection of all coc-closed sets of $X$ containing A, is called coc- closure of A and is denoted by $\overline{A}^{coc}$ or coc-$Cl_\tau(A)$.
$\overline{A}^{coc} = \cap \{B : B \text{ is coc} - \text{closed in X and } A \subseteq B\}$ .

**1.10 . Definition: [2]**

Let $X$ be a space and $A \subseteq X$. The union of all coc-open sets of $X$ contained in $A$ is called coc- Interior of $A$ and denoted by $A^{\circ coc}$ or coc-$In_\tau(A)$.

coc-$In_\tau(A) = \cup \{B : B \text{ is coc} - \text{open in X and } B \subseteq A\}$ .

**1.11.Remark:**
It is clear that $A^\circ \subseteq A^{\circ coc}$. and $\overline{A}^{coc} \subseteq \overline{A}$ , but the converse is not true in general as the following example shows:
Let $X = \{1,2,3\}$ , and $T = T_{ind}$ and $A = \{2\}$. Then $A^\circ = \emptyset$ , $A^{\circ coc} = \{2\}$, $\overline{A}^{coc} = \{2\}$ and $\overline{A} = X$.

**1.12 . Definition:[2]**
Let $Y$ be a subspace of a space X. A subset $A$ of a space X is said to be an coc-open set in Y if for every $x \in A$ there exists an open set $U \subseteq Y$ and a compact subset $K \in C(Y, \tau_Y)$ such that $x \in U - K \subseteq A$.

**1.13 . Proposition:[1]**

Let X be a space and $Y$ be any nonempty closed set in X . If $B$ is a coc-open set in X then $B \cap Y$ is a coc-open set in $Y$.

**1.14. Proposition: [2]**
Let X be a space and $Y$ be any nonempty closed set in X.If B is an coc-closed set in X then $B \cap Y$ is an coc-closed set in $Y$.

Note that : If A is an coc-open set in a sub space Y then A is not necessary be an coc-open set in a space X , as the following example shows: Let R be the set of real numbers, $T_U$ be usual topology on R and let Y={1,2} then {1} is a coc-open set in Y , but {1} is not a coc- open in R .

**1.15 .Proposition :**
Let $X$ be a space and $Y$ be a coc-open set of $X$ , if $A$ is a coc-open set in $Y$ then $A$ is a coc-open set in $X$ .
**Proof:**
Let $x \in A$ , since A is a coc-open in Y then there exists an open set $W$ in $Y$ and a compact subset $K \in C(Y, T_Y)$ such that $x \in W - K \subseteq A$,since $W$ an open set in $Y$ then

$w = U \cap Y$(where $U$ an open set in $X$) then $U \cap Y$ is an coc-open set in $X$ (by Corollary1.4). Hence for each $x \in U \cap Y$ there exist an open set $V_x$ in $X$ and compact subset H∈ $C(X,T)$,such that $x \in V_x - H \subseteq U \cap Y = W$ . Therefore $A$ is an coc-open in $X$.

**1.16 .Proposition :**

Let $X$ be a space and $Y$ be an coc-closed set of $X$, if $A$ is a coc-closed set in $Y$ then $A$ is a coc-closed set in $X$ .

**Proof :**

To show that $X - A$ is an coc-open set in$X$.Let $x \in X - A$ then either $x \in X - Y$ or $x \in Y - A$ , if $x \in X - Y$ . Since $Y$ is an coc-closed in $X$ then $X - Y$ is an coc-open set in $X$, hence there is an open set $U$ in $X$ and a compact subset $K \in C(X,T)$ such that $x \in U - K \subseteq X - Y$ and since $A \subseteq Y$ then $X - Y \subseteq X - A$ .

Now if $x \in Y - A$. Since $Y - A$ is a coc-open set in $Y$ then there exist an open set $V$ in $Y$ and a compact subset $K \in C(Y,\tau_Y)$ such that $x \in V - K \subseteq Y - A$ hence $V = W \cap Y$ ($W$ an open set in $X$) Therefore $x \in W - K \subseteq Y - A$ .Thus $A$ be coc-closed set in $X$.

**1.17. Remark:**

Let X be a space and Y a sub space of X such that A ⊆ $Y$, if A a coc-open (coc-closed) subset in X then A is a coc-open (coc-closed) in Y.

**1.18. Remark:**

i.We use $T_{prod}$ to denote the product topology on$X \times Y$ of a topological spaces $(X,T)$ and$(Y,\sigma)$ and the family of all coc-open sets in product space X× $Y$ is denoted by $T^k_{prod}$.

ii. We use $T_{k-prod}$ to denoted the product topology on X× $Y$ of a topological space (X,$T^k$) and $(Y,\sigma^k)$.

Now , we study properties of product of coc -open sets in a given space .

**1.19. Proposition :**

Let X and Y be spaces and A,B are non empty subsets of X and Y (respectively) such that $A \times B$ be a coc-open set in $X \times Y$ then A and B are coc-open sets in X and Y (respectively).

**Proof :**

Let $x \in A$ then$(x,y) \in A \times B$ for some $y \in B$ and since $A \times B$ be an coc-open set in $X \times Y$ then there exist open set H containing$(x,y)$ in $X \times Y$ and a compact subset $W \in C(X \times Y, T_{prod})$ such that (x,y) $\in$ H $- W \subseteq A \times B$, then$P_{r1}(H)$ is open set in X contain $x$ , $P_{r1}(W)$ compact in X (x,y) ∈ H $- W \Rightarrow x \in P_{r1}(H) - P_{r1}(W) \subseteq P_{r1}$(H $- W$), (since$P_{r1}(W)$ is continuous and W open in $X \times Y$) thus $A$ is an coc-open set in $X$ .

In a similar way we can prove that B be an coc-open set in $Y$.

**1.20. Proposition :**

Let X and Y be spaces and A,B are non empty subsets of X and Y (respectively) such that $A \times B$ be a coc-closed set in $X \times Y$ then A and B are coc-closed sets in X and Y (respectively).

Proof:clear.

**1.21 . proposition :**

Let$(X,T)$ and $(Y,\sigma)$ be two spaces . then$T^k_{prod} \subseteq T_{k-prod}$.

**Proof**: Clear .

**1.22. Proposition:**

Let$(X,T)$ and $(Y,\sigma)$ be CC-spaces then :

i. If A is coc-open(coc-closed) set in $(X,T)$ then $A \times Y$ is coc-open(coc-closed) in ( $X \times Y, T_{prod}$).

ii. If B is coc-open(coc-closed) set in $(Y,\sigma)$ then $X \times B$ is coc-open(coc-closed) in $(X \times Y, T_{prod})$.

**Proof:**

i. Let A be coc-open set in $(X, T)$ .since $(X, T)$ is cc-space then $T^k = T$(proposition 1.8) so A is an open set in $(X, T)$ and then $A \times Y$is open in $(X \times Y, T_{prod})$ and consequently $A \times Y$is coc-open .

ii. In Similar way to proof in (i).

**1.23. Proposition:**

Let $X$and $Y$be spaces and let$A \subseteq X$ and $B \subseteq Y$ then :

i. If $A$, B are coc-closed subset of $X$and Y respectively ,then$\overline{(A \times B)}^{coc} = \bar{A}^{coc} \times \bar{B}^{coc}$.

ii. If $A$, B are coc-open subset of $X$and Y respectively , then$(A \times B)^{\circ coc} = A^{\circ coc} \times B^{\circ coc}$.

**2.COC- Irresoute functions:**

In this section , we recall the definitions of coc- continuous function and coc- irresolute function and investigate some properties of them.

**2.1 .Definition: [2]**

Let $f: X \to Y$ be a function of a space $X$ into a space $Y$ then $f$ is called a coc-continuous function if $f^{-1}(A)$ is a coc-open set in $X$ for every open set $A$ in $Y$.

Note that a function$f: (X, \tau) \to (Y, \sigma)$ is an coc- continuous if and only if $f: (X, \tau^K) \to (Y, \sigma)$ is a continuous.

The following definition introduced in [2].

**2.2 .Definition:[2]**

Let $f: X \to Y$ be a function of a space $X$ into a space $Y$ then $f$ is called a coc-irresolute function if $f^{-1}(A)$ is a coc-open set in $X$ for every coc-open set $A$ in $Y$.

Note that a function$f: (X, \tau) \to (Y, \sigma)$ is a $coc - irresolute$ funcion if and only if $f: (X, \tau^K) \to (Y, \sigma^K)$ is a continuous.

**2.3 .Example :[2]**

i. The constant function is a coc- irresolute continuous function.

ii. Let $X$and $Y$ be finite sets and $f: X \to Y$ be a function of a space $X$ into a space $Y$ then $f$ is $coc -$ irresolute function.

**2.4.Remark:[2]**

i. Every a continuous function is a coc-continuous function, but the converse not true in general as the following example shows:

$Let\ X = \{a, b\}$ and $Y = \{c, d\}, T = T_{ind}\ X$ and$\sigma = \{\phi, Y, \{c\}\}$ be a topology on $Y. Let\ f: X \to Y$ be a function defined by $f(a) = c, f(b) = d\ then f$ is an coc-continuous ,but is not continuous.

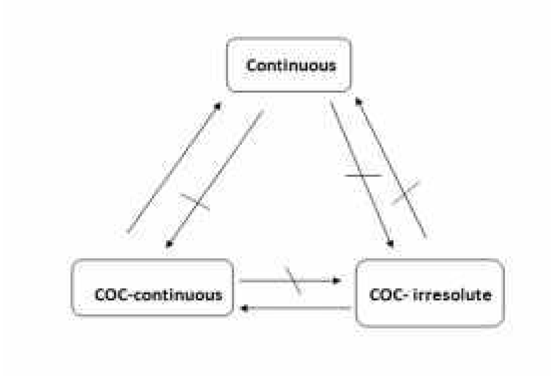
ii. Every a $coc -$ irresolute continuous function is a coc-continuous function, but the converse not true in general as the following example shows:

Let $T_u$ be usual topology on $\mathbb{R}$ and $T = T_{ind}$on $Y = \{1,2\}$.Let $f: \mathbb{R} \to Y$ be a function defined by

$$f(x) = \begin{cases} 1 & if x \in Q \\ 2 & if x \in Q^c \end{cases}$$

Then $f$is coc- continuous but is not $coc -$ irresolute continuous since$f^{-1}(\{1\}) = Q$ is not coc-open in $\mathbb{R}$.

The following diagram shows the relations among the differet types of continuous function.




**2.5. Proposition:[2]**

Let $f: X \to Y$ be a function then :

i. f is coc- continuous if and only if $f^{-1}(A)$is coc-closed set in $(X, T)$.for every closed set A in $(Y, \sigma)$.

ii. f is $coc - irresolutte$ continuous function if and only if $f^{-1}(A)$is coc – closed set in (X,T),every coc-closed in (Y,σ).

Now study restriction of coc-continuous( coc- $rresolutte$ continuous) functions ,[1],[2].

**2.6 .Proposition:**

Let $f: X \to Y$ be a function and $A$ be a nonempty closed set in $X$ .Then

i If f coc- continuous, then $f|_A: A \to Y$ is coc- continuous.

ii.If f coc- irresolute continuous, then $f|_A: A \to Y$ is coc- irresolute continuous.

Now, we study the composition of coc- continuous functions.

**2.7 .Remark:[2]**

A composition of two coc-continuous functions not necessary be a coc- continuous function as the following example shows:

Let $X = \mathbb{R}$ the set of real numbers, $Y = \{0,1,2\}$ ,$W = \{a, b\}$ ,$\tau = \{X\} \cup \{U \subseteq X: 1 \notin U\}$,the compact set are $\{K \subseteq X: 1 \in K\} \cup \{K \subseteq X: 1 \notin K \text{ is finite}\}$ hence $\tau^k = \tau \cup \{U \subseteq X: 1 \in U \text{ and } X - U \text{ is finite}\}$.
$\tau' = \{\phi, Y, \{0\}, \{0,1\}\}, \tau'' = \{\phi, W, \{a\}\}$ be topologies on $Y$ and $W$ respectively .If $f: X \to Y$ be function defined by $f(x) = \begin{cases} 2 & if x \in \{0,1\} \\ 1 & otherwise \end{cases}$ and $g: Y \to W$ is a function defined by $g(0) = g(2) = a$ and $g(1) = b$.Then $f, g$ are coc- continuous functions .But $g \circ f$ is not a coc-continuous since $(g \circ f)^{-1}(\{a\}) = \{0,1\}$ is not coc-open set in $X$.

**2.8.Proposition:[2]**

Let $X, Y$ and $Z$ be spaces and $f: X \to Y$ be coc-continuous if $g: Y \to Z$ is continuous then $g \circ f: X \to Z$ is coc-continuous.

**2.9 .Proposition: [2]**

Let $X, Y$ and $Z$ be spaces and $f: X \to Y$ , $g: Y \to Z$ be functions. Then if $f$ is an coc- irresolute continuous and $g$ is a coc-continuous then $g \circ f: X \to Z$ is coc-continuous .

**2.10 .Proposition: [2]**

Let $f: X \to Y$ and $g: Y \to Z$ be coc- irresolute continuous. Then $g \circ f: X \to Z$ is coc- irresolute continuous.

**2.11 .Theorem:[1]**

Let $f: X \to Y$ be a function and $(X, \tau)$ is $CC$ then the following statements are equivalent:

i. $f$ is continuous.

ii. $f$ is coc-continuous.

**2.12 .Theorem:[2]**

Let $f: X \to Y$ be a function and $(X, \tau)$ is $CC$ then the following statements are equivalent:

i. $f$ is continuous.

ii. $f$ is $coc -$ irresolute continuous.

**2.13 .Proposition:**

Let X,Y and Z be spaces and , f:X→ $Y$ and $g: Y \to Z$ are functions then If f and g are coc-continuous functions and Y is an cc-space then $g \circ f$ is an coc-continuous .

**Proof :**

Since Y is cc-space and g is a coc-continuous so g is continuous by theorem (2.8) then $g \circ f$ is an coc-continuous.

Now, we study the product of coc-continuous functions

**2.14 .Theorem :[1]**

Let $f: (X,T) \to (Y,\sigma)$ and $g: (X,T) \to (Z,\mu)$ be two function .Then the function $h: (X,T) \to (Y \times Z, \tau_{prod})$ defined by $h(x) = (f(x), g(x))$ is coc-continuous if and only if f and g are coc-continuous.

**2.15 .Remark :**

The product of two coc-continuous functions is not necessary be coc- continuous function as the following example shows:Let $X_i = Y_i = \mathbb{N}, T_i = T_{fin}, \sigma_i = T_{ind}$, i=1,2 and $f_i: (X_i, T_i) \to (Y_i, \sigma_i)$ be identity function then $f_i$ is coc-continuous .But $f_1 \times f_2: (X_1 \times X_2, T_{prod}) \to (Y_1 \times Y_2, \sigma_{prod})$ is not coc-continuous, since $\{1,2\} \times \mathbb{N}$ is a closed set in $Y_1 \times Y_2$ ,but $(f_1 \times f_2)^{-1}(\{1,2\} \times \mathbb{N})$ is not coc-closed set in $X_1 \times X_2$.

**2.16.Proposition:**

Let $f_i: (X_i, T_i) \to (Y_i, \sigma_i)$ $i = 1,2$ , be functions such that $f_1 \times f_2: (X_1 \times X_2, T_{prod}) \to (Y_1 \times Y_2, \sigma_{prod})$ be an coc- continuous function then $f_i$ are coc –continuous .

**Proof :**

To prove $f_1: X_1 \to Y_1$ be a coc-continuous . Let V be an open set in $Y_1$ , then $V \times Y_2$ is an open set in $Y_1 \times Y_2$ , since $f_1 \times f_2$ is a coc-continuous then $(f_1 \times f_2)^{-1}(V_1 \times Y_2) = f_1^{-1}(V_1) \times f_2^{-1}(Y_2)$ is an coc-open set in $X_1 \times X_2$ . Hence $f_1^{-1}(V_1)$ is a coc-open subset in $X_1$ , therefore $f_1: X_1 \to Y_1$ is an coc-continuous.In similar way we can prove that $f_2: X_2 \to Y_2$ is an coc-ontinuous .

**2.17 .Remark :**

The product of two coc-irresolute continuous functions is not necessary be coc-irresolute function as shown in example in Remark(2.15).

**2.18.Proposition:**

Let $f_i: (X_i, T_i) \to (Y_i, \sigma_i)$ $i = 1,2$ , be functions such that $f_1 \times f_2: (X_1 \times X_2, T_{prod}) \to (Y_1 \times Y_2, \sigma_{prod})$ be an coc- irresolute continuous function then $f_i$ are coc – irresolute continuous .

**Proof :**Similar to proof of proposition(2.16) .

**3. Strongly coc-closed functions:**

In this section , We recall the definition of coc-closed function and introduce a strongly coc –closed function and investigate the properties of them .

**3.1 .Definition: [2]**

Let $f: X \to Y$ be a function of a space $X$ into a space $Y$ then:

i. $f$ is called an coc-closed function if $f(A)$ is an coc-closed set in $Y$ for every closed set $A$ in $X$.

ii. $f$ is called an coc-open function if $f(A)$ is an coc-open set in $Y$ for every open set $A$ in $X$.

i. The constant function is an coc-closed function.
ii. Let $f: X \to Y$ be a function of a space $X$ into a space $Y$ such that $Y$ is a finite set then $f$ is an coc-open function.

**3.3 . Remark: [2]**
i. A function $f:(X,\tau) \to (Y,\sigma)$ is an coc-open if and only if $f:(X,\tau^k) \to (Y,\sigma^k)$ is a open function.
ii. Every closed (open) function is an coc-closed (an coc-open) function, but the converse not true in general as the following example shows:

**3.4.Example:[2]**
Let $X = \{1,2,3\}$ ,$Y = \{4,5\}$, $\tau = \{\phi, X, \{3\}\}$ be a topology on X and $\tau' = T_{ind}$ be topology on $Y$. Let $f: X \to Y$ be a function defined by $f(1) = f(2) = 4$ , $f(3) = 5$ then $f$ is an coc-closed (an coc-open) function but is not a closed (an open) function.

**3.5. Remark:**
The composition of two coc-closed function is not necessary be coc-closed function , so we put conditions either on function or on atopology spaces to obtain the result as show in the following proposition :

**3.6.Proposition:**
Let $f:(X,T) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\mu)$ be functions then:
i. if $f$ is a closed and $g$ is an coc-closed then $g \circ f$ is a coc-closed function.
ii. If $f$ and $g$ be coc-closed and $Y$ cc-space then $g \circ f$ is a coc-closed function.

**Proof:**
i.see in [2]
ii. let A be closed set in $(X,T)$then f(A) is coc-closed in $(Y,\sigma)$, since $(Y,\sigma)$,is cc-space then f(A) is closed in $(Y,\sigma)$,(proposition1.7 )A nd then $g(f(A)) = (gof)(A)$is coc-closed set in $(Z,\mu)$.

**3.2 . Example:[2]**

**3.7.Proposition:[2]**
Let $f: X \to Y$ be a coc-closed function then the restriction of $f$ to a closed subset $F$ of $X$ is an coc-closed of $F$ into $Y$.

**3.8 .Proposition:[2]**
A bijective function $f: X \to Y$ is an coc-closed function if and only if $f$ is an coc-open function.

**3.9 .Proposition: [2]**
Let $f: X \to Y$ be bijective function from a space $X$ into a space $Y$. Then:
i. $f$ is an coc-open function if and only if $f^{-1}$ is an coc-continuous.
ii. $f$ is an coc-closed function if and only if $f^{-1}$ is an coc-continuous.

**3.10 .Proposition : [2]**
Let $f:(X,T) \to (Y,\sigma)$ be coc-closed function and A be closed subset of $(X,T)$then $f|_A : (A, T|_A) \to (Y,\sigma)$ is coc-closed.

**3.11 .Proposition :**
Let $f_i: (X_i,\tau_i) \to (Y_i,\sigma_i)$ ,i=1,2 be a functions such that $f_1 \times f_2: (X_1 \times X_2, T_{prod}) \to (Y_1 \times Y_2, \sigma_{prod})$ be coc-closed function then $f_i$ is coc-closed function , i=1,2.

**Proof :**

To prove $f_1:(X_1,\tau_1) \to (Y_1,\sigma_1)$ is coc-closed
Let A be closed set in $(X_1,\tau_1)$ then $A \times X_2$ is closed set in $(X_1 \times X_2, T_{prod})$ ,so $(f_1 \times f_2)(A \times X_2) = f_1(A) \times f_2(X_2)$ is coc-closed set in $(Y_1 \times Y_2, \sigma_{prod})$ so $f_1(A)$ is coc-closed set in $(Y_1,\sigma_1)$By proposition (1.20)So $f_1$ is coc-closed function .
In similar way $f_2$ is coc-closed function .

Now we recall the definition of strongly coc-closed function and introduce some propositions about it.

**3.12 .Definition:[2]**

Let $f: X \to Y$ be a function of a space $X$ into a space $Y$ then:

i. $f$ is called an strongly coc −closed function if $f(A)$ is an coc-closed set in $Y$ for every coc-closed set $A$ in $X$.

Note that a function $f:(X,T) \to (Y,\sigma)$ is strongly coc-closed if and only if $f:(X,T^k) \to (Y,\sigma^k)$ is closed .

**3.13 .Example:[2]**

The constant function is an strongly coc-closed function.

**3.14 . Remark :[2]**

It is clear that every strongly coc-closed function is coc-closed but the converse is not true in general.

**3.15 .Proposition:[2]**

Let $X, Y$ and $Z$ be spaces and $f: X \to Y$ , $g: Y \to Z$ be functions. Then:

i. If $f$ and $g$ are strongly coc-closed function, then $g \circ f$ is strongly coc-closed function.

ii. If $g \circ f$ is strongly coc-closed function, $f$ is strongly coc-closed -continuous and onto ,then $g$ is strongly coc − closed

iii.If $g \circ f$ is strongly coc-closed function, $g$ is strongly coc-closed -continuous and onto, then $f$ is strongly coc − closed .

**3.16 .Remark :**

if $f: (X,T) \to (Y,\sigma)$ be strongly coc-closed function and A⊆ $X$ then it is not necessary $f|_A : (A, T|_A) \to (Y,\sigma)$ be strongly coc-closed as the following example shows: $X = Y = Z, T = \{\emptyset, \mathbb{Z}, \mathbb{Z} - \{1,2\}\}, \sigma = T_{ind}$ , $A = \mathbb{Z}_0$ and $f: (X,T) \to (Y,\sigma)$ be identity function then f is coc-closed function but $f|_A : (A, T|_A) \to (Y,\sigma)$ is not coc-closed.

**3.17 .Proposition :**

Let $f: (X,T) \to (Y,\sigma)$ be strongly coc-closed function and A be coc-closed set in $(X,T)$ then $f|_A : (A, T|_A) \to (Y,\sigma)$ be strongly coc-closed

**Proof:**

Since A is coc-closed set in $(X,T)$ then inclusion function $i_A: (A, T_A) \to (X,T)$ is strongly coc-closed so $f|_A = f \circ i_A: (A, T|_A) \to (Y,\sigma)$ is strongly coc-closed .

**3.18 .Remark :**

The product of two strongly coc-closed function is not necessary strongly coc-closed function as the following example shows : $X_i = Y_i = \mathbb{N}, T_i = T_{ind}, \sigma_i = T_{fin}, i = 1,2$ and $f_i: (X_i,T_i) \to (Y_i,\sigma_i), i = 1,2$ are identity functions then $f_i$ is coc-closed function . But $f_1 \times f_2$is not coc-closed function .

**3.19 .Proposition :**

let $f_i: (X_i,T_i) \to (Y_i,\sigma_i), i = 1,2$ be afunction if $f_1 \times f_2: (X_1 \times X_2, T_{prod}) \to (Y_1 \times Y_2, \sigma_{prod})$ is strongly coc-closed function and $(X_i, T_i)$is $cc - space$ then $f_i$ is coc-closed function , i=1,2 .

**Proof** :

To prove $f_1: (X_1,T_1) \to (Y_1,\sigma_2)$ be strongly coc-closed function .

Let A be coc-closed set in $(X_1,T_1)$ then $A \times X_2$ is coc-closed in $(X_1 \times X_2, T_{prod})$,propostion(1.22) So $(f_1 \times f_2)(A \times X_1) = f_1(A) \times f_2 (X_1)$ is coc-closed set in $(Y_1 \times Y_2, \sigma_{prod})$ ,so $f_1(A)$ is coc-closed set in $(Y_1,\sigma_1)$ propostion (1.20) ,So $f_1$ *is* strongly coc-closed function .

In similar way $f_2$ is strongly coc-closed function .

**4 .COC-Perfect functions**

In this section , We introduce the definition of coc-perfect function and investigate the properties of it. Also we give the relation between coc-perfect and perfect .

**4.1 .Definition : [5]**

A function $f:(X,T)\rightarrow(Y,\sigma)$ is called perfect if f is continuous closed surjection and each fiber $f^{-1}(y)$ is compact , $\forall y\in Y$ .

Now we introduce the following definition .

**4.2 .Definition :**

A function $f:(X,T)\rightarrow(Y,\sigma)$ is called coc-perfect if :

(i) f is coc- continuous function ,
(ii) f is coc-closed function ,
(iii) The fibers of f are coc-compact (i.e.$f^{-1}(y)$ is coc-compact $\forall y\in Y$ ).

**4.3.Example:**

Let X={1,2,3} , Y={2,4,6} and $T=T_{ind}$ , $T=\{\emptyset,Y,\{4\}\}$be topologies on X ,Y(resp). A function $f:X\rightarrow Y$defined as f(x) =2x , $\forall x\in X$is coc-perfect .

**4.4. Example:**

Let $X=Y=\{a,b,c\}$ and $T=\{\emptyset,X,\{b\},\{a,b\}\}$, $\sigma=\{\emptyset,Y,\{b\},\{a,b\},\{b,c\}\}$ be topologies on X and $Y$ respectively . Then a function$f:(X,T)\rightarrow(Y,\sigma)$ Which defined by $f(a)=c$; $f(b)=b$; $f(c)=$a . Then f is not coc-perfect function , since it is not coc-continuous function .

**4.5. Remark:**

(i)Every perfect function from $CC$ −space into any topological space is coc-perfect.
(ii)Every homeomorphism from $CC$ −spaceinto any topological space coc-perfect .

From the definition (4.2) , every coc-perfect function is coc-closed , but the converse is not true in general as the following example shows:

**4.6. Example:**

Let $R$be the real numbers, $N$ be a subset of $R$.
$T=\{U\subseteq R: U=R\ or\ U\cap N=\emptyset\}$be a topology on$R$ , a function $f:R\rightarrow R$ ,which defined as $f(x)=0$ for all $x\in R$ is coc-closed , but not coc-perfect function. Since$f^{-1}(\{0\})=R$ is not coc-compact.

**4.7.Rremark :**

If $f:(X,T)\rightarrow(Y,\sigma)$ be coc-perfect and A$\subseteq X$ then it is not necessary $f|_A:(A,T|_A)\rightarrow(Y,\sigma)$ is coc-perfect since the restriction of coc-closed function not necessary coc-closed .

**4.8.Proposition :**

Let $f:(X,T)\rightarrow(Y,\sigma)$ be coc-perfect and A be closed set in (X,T) then $f|_A:(A,T|_A)\rightarrow(Y,\sigma)$ is coc-perfect.

**Proof:**

$f|_A$ is coc- continuous (Proposition2.6 i).And ccc –closed (proposition 3.10) . Now, let $y\in Y$ then $f^{-1}(y)$is coc-compact in $(X,T)$ so $f^{-1}(y)\cap A$ is coc-compact in$(X,T)$since A is closed in $(X,T)$there fore $(f|_A)^{-1}(y)=f^{-1}(y)\cap A$is coc-compact in $(A,T|_A)$ hence f(A)is coc-perfect .

**4.9 . Remark :**

A composition of two coc-perfect functions is not necessary coc-perfect function .

Now , we put the condition either on a function or on topological spaces to satisfy the composition of two coc-perfect function is coc-perfect.

**4.10 .Proposition:**

Let $f:X\rightarrow Y$and$h:Y\rightarrow Z$be functions, then:

**(i)**If $f$ and $h$ are coc-perfect functions, $Y$ is a $cc$-space,then $hof$ is coc-perfect.

**(ii)**If $f$ is perfect and $h$ is coc-perfect functions, where $X$, $Y$ are $cc$ −spaces, then $hof$ is coc-perfect.
**(iii)**If $f$ is coc-perfect and $h$ is perfect function $Y$ is $cc$ −space, then $hof$ is coc-perfect

**Proof:**

**(i)**By theorem (2.13), $hof$ is coc-continuous and by theorem , (3.6.ii) $hof$ is coc-closed .Now, to prove that $(hof)^{-1}\{z\}$ is coc-compact set in $X$ for every $z \in Z$ . Since $h$ is coc-perfect, then $h^{-1}\{z\}$ is coc-compact set in $Y$ for every $z \in Z$ .
But $f$ is coc-perfect, then $f^{-1}(h^{-1}\{z\}) = (hof)^{-1}\{z\}$ is coc-compact set in $X$ for every $z \in Z$ .
Hence $hof$ is coc-perfect function.

**(ii)** By theorem (2.8), $hof$ is coc-continuous and by theorem (2.6.i), $hof$ is coc-closed . Now, to prove that $(hof)^{-1}\{z\}$ is coc-compact set in $X$ for every $z \in Z$ Since $h$ is coc-perfect, then $h^{-1}\{z\}$ is coc-compact set in $Y$. But $Y$ is cc-space pace, then $h^{-1}\{z\}$ is compact in Y , since $f$ is a perfect, then $f^{-1}(h^{-1}\{z\}) = (hof)^{-1}\{z\}$ is compact set in $X$, but $X$ is cc-space then $(hof)^{-1}\{z\}$ is coc-compact . $hof$ is coc-perfect.

**(iii)**Clear.

.

**4.11 .Remark** :

The product of two coc-perfect function is not necessaty be coc- perfect function .

**4.12.Proposition :**

Let $f_i: (X_i, T_i) \to (Y_i, \sigma_i) i = 1,2$ be functions such that $f_1 \times f_2: (X_1 \times X_2, T_{prod}) \to (Y_1 \times Y_2, \sigma_{prod})$ is coc-perfect function then $f_i$ is coc-perfect function .

**Proof :**

To prove that $f_1: (X_1, T_1) \to (Y_1, \sigma_1)$ is coc-perfect .

(a) Since $f_1 \times f_2$ is continuous then $f_1$ is coc-continuous (by proposition 2.16)

(b) Since $f_1 \times f_2$ is coc-closed then $f_1$ is coc-closed (by proposition 3.12)

(c) Let $y_1 \in Y$ then $(y_1, y_2) \in Y_1 \times Y_2$ for each $y_2 \in Y$ and $(f_1 \times f_2)^{-1}(y_1, y_2) = f_1^{-1}(y_1) \times f_2^{-1}(y_2)$ is coc-compact in $(X_1 \times X_2, T_{prod})$ So $f_1^{-1}(y_1)$ is coc-compact in $(X_1, T_1)$ .

$f_2$ is we can prove coc-perfect. From (a), (b) and (c) $f_1$ coc-perfect and in similar way

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# حول بعض الانماط من الدوال –COC

تاريخ الاستلام : 2015/5/21 تاريخ القبول :2015/8/16

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المستخلص :

في هذا العمل درسنا خصائص لا لبعض الأنماط من الدوال–coc وبالتحديد الدالة المنحلة من النمط –coc والمغلقة من النمط –coc,وقدمنا تعريف الدالة التامة من النمط –cocوأعطينا خصائص التركيب والقصر والضرب لهذه الدوال وكذلك العلاقة بينهم .

كلمات افتتاحية :

المجموعة المفتوحة من النمط –co , المنحلة من النمط –coc والمغلقة من النمط –coc, التامة من النمط –coc,

*البحث مستل من رسالة ماجستير للباحث الثاني .