Nonlinearity and carriers transport effects on the modulation response and relative intensity noisespectra in quantum dot lasers

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ABSTRACT

Noise and modulation response behavior in quantum dot semiconductor lasers under the influence the nonlinearity studied theoretically in this paper. The present rate equations model consists of five equations for the carriers density and one equation for photons density. The nonlinearity effect was added in two rate equations namely the photon density rate equation and the ground state rate equation. Two equationswere derived to calculate the noise and modulation response. Calculations in this paper focused on the effect of each of the nonlinear effect and carrier transport inside and outside quantum dots on the laserbehavior. The results indicate the weak effect of the nonlinearity on the behavior of the laser noise because of inability of the present formula to represent the nonlinear gain parameter. Also, results indicated the strong effect of the carriers relaxation from the wetting layer to the continuous state in comparison with other relaxation lifetimes in low energy states.

Key words: quantum dot lasers, lasers noise and rate equation model
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تاثيرات اللاخطية وانتقال الحاملات على استجابة التضمين والضوضاء في ليزرات النقاط الكمية

الخلاصة

سلوك الضوضاء واستجابة التضمين في ليزرات النقاط الكمية درست في هذا البحث نظريا تحت تاثير عاملي اللاخطية وانتقال الحاملات فيها. نموذج معادلات المعدل في هذا البحث مكون من خمس معادلات لوصف ديناميكية الحاملات ومعادلة واحدة لوصف تركيز الفوتونات. اضيف حد اللاخطية في معادلتين هما معادلة الفوتونات ومعادلة الحاملات في الحالة الارضية. تم اشتقاق معادلتين لحساب كل من الضوضاء واستجابة التضمين. الحسابات تركزت حول تاثير اللاخطية وانتقال الحاملات داخل وخارج النقاط الكمية. النتائج دلت على تاثير ضعيف للاخطية على سلوك الضوضاء بسبب قصور الصيغة الحالية لتمثيل هذا العامل. ايضا،النتائج دلت على اعتمادها بشكل واضح على زمن الانتقال من المنطقة الرخوة الى المنطقة المستمرة بالمقارنة مع ازمان الانتقال الخرى

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INTRODUCTION

In comparison, with other nanostructure lasers such as quantum well laser and quantum wire laser, quantum dot lasers (QD-LASERS) are expected to have the advantages of large gain, low chirp, high temperature stability and low threshold current. Therefore, QD-LASERS in particular GaAs-based 1.3μm wavelength range, are favorite as next-generation light sources in the metro/access optical fiber communication networks. The generation/absorption processes in the receiver and transmitter lead to a weak optical signal. So, the optical signal plays an important role in optical communication networks. Furthermore, quantum nature of photon and fluctuate in photons number are important in case of weak optical beam.Expermentaly, performance of optical communication networks is limited by the quantum nature of photon generation/absorption processes. The effect of such fluctuations can be described by shot noise when we consider the system performance. Phase fluctuations and amplitude fluctuations of semiconductor lasers affect the performance of optical communication networks. Phase fluctuations affect the line-width which is a very important factor for coherent communication networks. Amplitude fluctuations appear in both the total output and the output levels of individual longitudinal modes. Historically, there are three theories for quantum noise for finding the statistical properties of the laser cavity internal field, namely, the density matrix master equation, the Fokker- Planck equation and the Langevin equation [1-3]. Study the performance of these lasers is very important when the noise sources take into consideration. This is necessary to evaluate the actual performance of these devices in communication networks.study the performance of these lasers is very important when the noise sources take into consideration. This is necessary to evaluate the actual performance of these devices in communication networks.Langevin noises have common features observed not only in semiconductor lasers, but also in all other lasers and they are formulated as the same equations [4]. Analysis of laser noise by using the rate equations modelincluding Langevin noise sources can be done in three ways. Namely, small signal analysis, direct numerical integration to avoid the limitations of the small-signal analysis and Fourier transforms [5-18]. In real lasers, it is sometimes difficult apply the linear stability analysis for direct injection current modulation if the modulation Is not a small or the effect of spontaneous emissions is not negligible.So, the rate equations model must be solved numerically. In this paper, we will use the small signal analysis to study the laser noise in QD-LASER under the effect of both carriers transport and nonlinear properties.Several theoretical and experimental works had been published about the intensity noise behavior in QD-LASER [19-23]. This paper present a small signal analysis in six-levels rate equation model. The main result of this analysis is an analytical equation to calculate laser noise in QD-LASER. Also,from this equation the modulation response behavior can be calculated.Present model consist of several regions, some of these regions have been neglected in previous studies [21]. Rate equations model describes the carrier dynamics depending on carriers escape/relaxation lifetime between these regions. These lifetimes can be controlled by some factors such as doping, temperature and energy level values. One of the advantages of small signal analysis of the rate equations model is the possibility to predict the behavior of noise because of its dependence on the modulation

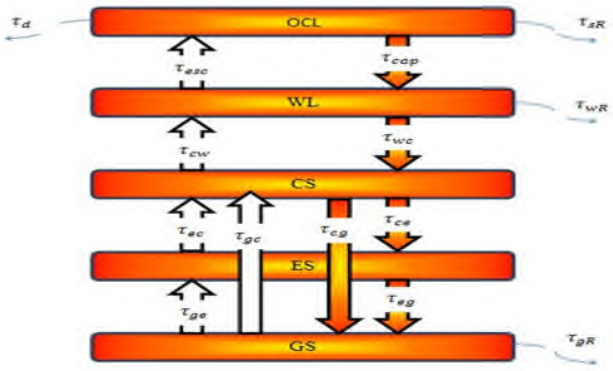
response.This advantage not available in direct numerical integration and Fourier transforms.

This paper is organized as follows. In Section 2, we present the six level rate equations model, including Langevin noise sources, five equations for carriers and one equation for photons, underlying our analysis and the main assumptions for QD laser. In Section 3, Small signal analysis of the rate equations model is obtained. Section 4 illustrates the results of simulations for QD laser. Finally, section 5 gives the main conclusions.

Six levels rate equations mode (6lrem)

As in [24], beginning with a description of the parameters in five levels rate equations model. These regions are, optical confinement layer (OCL), wetting layer (WL), continuous state (CS), excited state (ES) and ground state (GS). Because of the difference in energy between (WL) on the one hand and all of (ES) and (GS) on the other hand, the transfers of all kinds are forbidden. Therefore, escape\ relaxation lifetimes, not appear on the model. This description is repeated with the region (OCL) on the one hand and each of (GS), (ES) and (CS) on the other hand.The movement of carriers in the model can explain as follows, firstly, the carriers are directly injected into OCL region, and then they are captured in the first carrier reservoir (WL) within related time constant (τ_{cap}). Secondly, the carriers are captured into the CS within time(τ_{wc}). We can consider the CS as a second carrier reservoir to the ES and GS. Then, within time τ_{ce}andτ_{cg}respectively, the carriers in the CS are captured into the ES and GS.τ_{sR}andτ_{d} are the carriers spontaneous recombination lifetime and diffusion lifetime in OCL respectively.

The carrier relaxation lifetime and carrier escape lifetime, respectively, between ES and GS are given by τ_{eg}andτ_{ge} . The carrier can also be thermally remitted from WL to OCL, CS to WL, ES to CS and GS to CS with the scape lifetimes τ_{esc},τ_{cw},τ_{ec} and τ_{gc} respectively. The carrier spontaneous recombination lifetime in the (GS) and (WL) is τ_{gR} and τ_{wR} respectively. Since the hole has a larger effective mass in comparison with the electron effective mass and the small energy difference between hole energy levels, we are neglecting the hole dynamics effect. Fig. 1 shows the schematic of the carrier dynamics in the present model.



Figure(1) The present carrier dynamics model with all relaxation/escape lifetimes [20].

Figure(1) The present carrier dynamics model with all relaxation/escape lifetimes [20].

Based on [24], present model for carrier dynamics in the conduction band, including Langevin noise sources is consisted of the following set of six coupled rate equations for carriers in the OCL, WL, CS, ES, GS,and photons

$$\dot{N}_0 = \frac{J}{eb} + \frac{f_w\widetilde{NQ}}{\tau_{esc}} - \frac{N_O(1-f_w)}{\tau_{cap}} - \frac{N_O}{\tau_d} - \frac{N_O}{\tau_{sR}} + F_n(t) \quad \ldots(1)$$

$$\dot{f_w} = \frac{N_O(1-f_w)}{2D_w\widetilde{NQ}\tau_{cap}} + \frac{D_c f_c(1-f_w)}{D_w\tau_{cw}} - \frac{f_w(1-f_c)}{\tau_{wc}} - \frac{f_w\widetilde{NQ}}{2D_w\widetilde{NQ}\tau_{esc}} - \frac{f_w}{\tau_{wR}} + F_w(t) \quad \ldots(2)$$

$$\dot{f_c} = \frac{D_w f_w(1-f_c)}{D_c\tau_{wc}} - \frac{f_c(1-f_w)}{\tau_{cw}} - \frac{f_c(1-f_g)}{\tau_{cg}} + \frac{D_e f_e(1-f_c)}{D_c\tau_{ec}} - \frac{f_c(1-f_e)}{\tau_{ce}} + \frac{D_g f_g(1-f_c)}{D_c\tau_{gc}} + F_c(t) \quad \ldots(3)$$

$$\dot{f_e} = \frac{D_c f_c(1-f_e)}{D_e\tau_{ce}} - \frac{f_e(1-f_c)}{\tau_{ec}} + \frac{D_g f_g(1-f_e)}{D_e\tau_{ge}} - \frac{f_e(1-f_g)}{\tau_{eg}} + F_e(t) \quad \ldots(4)$$

$$\dot{f_g} = \frac{D_c f_c(1-f_g)}{D_g\tau_{cg}} - \frac{f_g(1-f_c)}{\tau_{gc}} + \frac{D_e f_e(1-f_g)}{D_g\tau_{eg}} - \frac{f_g(1-f_e)}{\tau_{ge}} - \frac{f_g}{\tau_{gR}} - \frac{v_g g_m(2f_g-1)S(1-\varepsilon S)}{2D_g\widetilde{NQ}} + F_g(t) \quad \ldots(5)$$

$$\dot{S} = v_g g_m(2f_g-1)S(1-\varepsilon S) - \frac{S}{\tau_p} + \frac{2D_g\widetilde{NQ}\beta f_g}{\tau_{gR}} + F_s(t) \quad \ldots(6)$$

Where J is the injection current, fw, f_c,f_e, and f_g are the electron occupation probabilities corresponding respectively to WL, CS, ES, and GS. e is the electron charge and b is the OCL thickness. $N_O$is 1the carrier density in OCL,$S$is the photon density and , b is the thickness of the WL.$\widetilde{NQ}$ is the effective volume density of the QDs which usually be defined as$\widetilde{NQ} = NQ/l_w$.Nonlinearities arise from inhomogeneous spectral hole-burning, stimulated emission carrier heating and free carrier absorption. These effects contribute to the phenomenological gain compression factor leading to the gain multiplication term(1 - εS) in eqs.(5-6).The factor (1 - εS)is important when the photon density is high. ε is called the gain suppression coefficient. $g_m$is the maximum modal gain, which depends on the confinement factor of each QD, the surface density of QDs, and the number of QD layers.The final term of each equation, $F_n(t)$,$F_w(t)$, $F_c(t)$,$F_e(t)$, $F_g(t)$and$F_s(t)$,is the effect of Langevin noise sources in the rate equations model. The photon lifetime is given by

$$\frac{1}{\tau_p} = v_g(\alpha_m + \alpha_i) \quad \ldots(7)$$

Where $v_g$is the group velocity of the mode of interest, including both material, α_i is the internal cavity losses and waveguide dispersion, $\alpha_m$is mirror loss which usually be defined as

$$\alpha_m = \frac{1}{L} ln\left(\frac{1}{r_1 r_2}\right) \quad \ldots (8)$$

Where L is cavity length, $r_1$is the front mirror reflectivity, and r_2, the back mirror reflectivity. The itradot relaxation/ escape lifetimes between CS, ES and GS, can be expressed as [24]

$$\tau_{ji} = \frac{D_j}{D_i} exp\left(\frac{\Delta E_{ij}}{KT}\right)\tau_{ij} \qquad i = c,e \;\; ; j = e,g; i \neq j \quad \ldots (9)$$

Where τ_{ij} is the phonon-dominated relaxation time, and $\Delta E_{ij}$is the energy separation between the ith state and the jth state in the conduction band of QDs. D, is the degeneracy of the corresponding electron state. K is the Boltzmann's constant and T is the room temperature. The electron relaxation/ escape lifetimes between WL and CS can be expressed as [24]

$$\tau_{cw} = \tau_{wc}\, exp\left(\frac{\Delta E_{wc}^{c}}{KT}\right) \quad \ldots (10)$$

Whereτ_{wc} is the electron capture lifetime, τ_{cw} is the electron escape lifetime, ΔE_{wc}^{c} is the energy separation between the WL bandedge and the CS in the conduction band of QDs.

Small-signal analysis and evaluate the lanegvin noise

In present paper, the Langevin noise sources are calculated based on the same procedure in [25] as a method to simplify the rigorous quantum description of noise in semiconductor lasers. Semiconductor laser noise is coming from shot noise associated with the discrete random flow of particles (carriers/photons) into and out of the reservoirs. To evaluate the Langevin noise density $\langle F_i F_i \rangle$, we simply sum over all rates of particle flow into and out of reservoir (i). Also, To determine cross-correlation strength$\langle F_i F_j \rangle$ between two reservoirs (i) and (j) we sum only over particle flow which affect both reservoirs simultaneously.

$$\langle F_n F_n \rangle = 2\left[\frac{f_w \widetilde{NQ}}{\tau_{esc}}\right] = 2D_{nn} \quad \ldots (11a)$$

$$\langle F_w F_w \rangle = 2\left[\frac{N_O(1-f_w)}{2D_w \widetilde{NQ}\tau_{cap}} + \frac{D_c f_c (1-f_w)}{D_w \tau_{cw}}\right] = 2D_{ww} \quad \ldots (11b)$$

$$\langle F_c F_c \rangle = 2\left[\frac{D_w f_w (1-f_c)}{D_c \tau_{wc}} + \frac{D_e f_e\,(1-f_c)}{D_c \tau_{ec}} + \frac{D_g f_g\,(1-f_c)}{D_c \tau_{gc}}\right] = 2D_{cc} \quad \ldots (11c)$$

$$\langle F_e F_e \rangle = 2\left[\frac{D_c f_c (1-f_e)}{D_e \tau_{ce}} + \frac{D_g f_g\,(1-f_e)}{D_e \tau_{ge}}\right] = 2D_{ee} \quad \ldots (11d)$$

$$\langle F_g F_g \rangle = 2\left[\frac{D_c f_c \left(1-f_g\right)}{D_g \tau_{cg}} + \frac{D_e f_e \left(1-f_g\right)}{D_g \tau_{eg}}\right] = 2D_{gg} \quad \ldots (11e)$$

$$\langle F_s F_s \rangle = 2\left[v_g g_m (2f_g - 1) N_{ph}(1+\varepsilon N_{ph}) + \frac{2D_g \widetilde{NQ}\beta f_g}{\tau_{gR}}\right] = 2D_{ss} \quad \ldots (11f)$$

$$\langle F_s F_n \rangle = \langle F_s F_w \rangle = \langle F_s F_c \rangle = \langle F_s F_e \rangle = \langle F_s F_g \rangle = \langle F_g F_e \rangle = \langle F_g F_c \rangle = \langle F_c F_e \rangle$$
$$= \langle F_g F_n \rangle = \langle F_g F_w \rangle = \langle F_c F_w \rangle = \langle F_e F_n \rangle = \langle F_w F_e \rangle = \langle F_c F_n \rangle = \langle F_n F_w \rangle = 0 \quad \ldots (11g)$$

To begin the small signal analysis, we must transform to the frequency domain in Eqs.(1-6) by the following relation

$$\delta P(\omega) = \int_{-\infty}^{+\infty} \delta P(t)\, e^{j\omega t} dt \qquad \dots (12)$$

Now, in order to study the (QD-Laser) noise by using present model, we need to set the following equation as a solution for Eqs. (1-6) as follows,

$$N_O = N_{O,0} + \delta N_O e^{iwt} \qquad \dots (13-a)$$

$$f_{w,c,e,g} = f_{w,c,e,g,0} + \delta f_{w,c,e,g} e^{iwt} \qquad \dots (13-b)$$

$$S = S_0 + \delta S e^{iwt} \qquad \dots (13-c)$$

where $N_{O,0}$, $N_{w,0}$, $N_{c,0}$, $N_{e,0}$, $N_{g,0}$, and S_0 are the solutions of the rate equations at the steady-state. Using Eqs. (13a-c) in Eqs. (1-6) we obtain:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \delta n_O \\ \delta f_w \\ \delta f_c \\ \delta f_e \\ \delta f_g \\ \delta S \end{bmatrix} = \begin{bmatrix} F_n(\omega) \\ F_w(\omega) \\ F_c(\omega) \\ F_e(\omega) \\ F_g(\omega) \\ F_s(\omega) \end{bmatrix} \qquad \dots (14)$$

Where

$$C_{11} = j\omega + \frac{(1-f_{w0})}{\tau_{cap}} + \frac{1}{\tau_d} + \frac{1}{\tau_{sR}} \qquad \dots (15a)$$

$$C_{12} = -\left[\frac{\widetilde{NQ}}{\tau_{esc}} + \frac{N_{O0}}{\tau_{cap}}\right] \qquad \dots (15b)$$

$$C_{21} = -\left[\frac{(1-f_{w0})}{2D_w \widetilde{NQ} \tau_{cap}}\right] \qquad \dots (15c)$$

$$C_{22} = j\omega + \frac{N_{O0}}{2D_w \widetilde{NQ} \tau_{cap}} + \frac{D_c f_{c0}}{D_w \tau_{cw}} + \frac{1-f_{c0}}{\tau_{wc}} + \frac{1}{2D_w \tau_{esc}} + \frac{1}{\tau_{wR}} \qquad \dots (15d)$$

$$C_{23} = -\left[\frac{D_c(1-f_{w0})}{D_w \tau_{cw}} + \frac{f_{w0}}{\tau_{wc}}\right] \qquad \dots (15e)$$

$$C_{32} = -\left[\frac{D_w(1-f_{c0})}{D_c \tau_{wc}} + \frac{f_{c0}}{\tau_{cw}}\right] \qquad \dots (15f)$$

$$C_{33} = j\omega + \frac{1-f_{w0}}{\tau_{cw}} + \frac{1-f_{e0}}{\tau_{ce}} + \frac{1-f_{g0}}{\tau_{cg}} + \frac{D_w f_{w0}}{D_c \tau_{wc}} + \frac{D_e f_{e0}}{D_c \tau_{ec}} + \frac{D_g f_{g0}}{D_c \tau_{gc}} \qquad \dots (15g)$$

$$C_{34} = -\left[\frac{D_e(1-f_{c0})}{D_c \tau_{ec}} + \frac{f_{c0}}{\tau_{ce}}\right] \qquad \dots (15h)$$

$$C_{35} = -\left[\frac{D_g(1-f_{c0})}{D_c \tau_{gc}} + \frac{f_{c0}}{\tau_{cg}}\right] \qquad \dots (15i)$$

$$C_{43} = -\left[\frac{D_c(1-f_{e0})}{D_e \tau_{ce}} + \frac{f_{e0}}{\tau_{ec}}\right] \qquad \dots (15j)$$

$$C_{44} = j\omega + \frac{1-f_{c0}}{\tau_{ec}} + \frac{1-f_{g0}}{\tau_{eg}} + \frac{D_c f_{c0}}{D_e \tau_{ce}} + \frac{D_g f_{g0}}{D_e \tau_{ge}} \quad \ldots(15k)$$

$$C_{45} = -\left[\frac{D_g(1-f_{e0})}{D_e \tau_{ge}} + \frac{f_{e0}}{\tau_{eg}}\right] \quad \ldots(15l)$$

$$C_{53} = -\left[\frac{D_c(1-f_{g0})}{D_g \tau_{cg}} + \frac{f_{g0}}{\tau_{gc}}\right] \quad \ldots(15m)$$

$$C_{54} = -\left[\frac{D_e(1-f_{g0})}{D_g \tau_{eg}} + \frac{f_{g0}}{\tau_{ge}}\right] \quad \ldots(15n)$$

$$C_{55} = j\omega - \frac{D_c f_{c0}}{D_g \tau_{cg}} + \frac{1-f_{c0}}{\tau_{gc}} + \frac{D_e f_{e0}}{D_g \tau_{eg}} + \frac{1-f_{e0}}{\tau_{ge}} + \frac{1}{\tau_{gR}} + \frac{v_g g_m S_0(1+\varepsilon S_0)}{D_g \widehat{NQ}} \quad \ldots(15o)$$

$$C_{56} = \frac{v_g g_m (2f_g - 1)(1+2\varepsilon S_0)}{2 D_g \widehat{NQ}} \quad \ldots(15p)$$

$$C_{65} = -2 v_g g_m S_0 (1-\varepsilon S_0) - \frac{2\beta D_g \widehat{NQ}}{\tau_{gR}} \quad \ldots(15q)$$

$$C_{66} = j\omega - v_g g_m (2f_g - 1 - 4\varepsilon f_g S_0 + 2\varepsilon S_0) + \frac{1}{\tau_p} \quad \ldots(15r)$$

$$C_{13} = C_{14} = C_{15} = C_{16} = C_{24} = C_{25} = C_{26} = C_{31} = C_{36} = C_{41} = C_{42} = C_{46} = C_{51} = C_{52} = 0 = C_{61} = C_{62} = C_{63} = C_{64} = 0 \quad \ldots(15s)$$

As in [24] the modulation response in quantum dot laser$H(\omega)$ can be calculated from Eq. (14) as follows

$$H(\omega) = \frac{\omega_R^2}{\Delta} \quad \ldots(16)$$

Where

$$\omega_R^2 = \begin{vmatrix} C_{11}-j\omega & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22}-j\omega & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33}-j\omega & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44}-j\omega & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55}-j\omega & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66}-j\omega \end{vmatrix} \quad \ldots(17)$$

it is clear that, ω_R^2 is the real part of the determinant

$$\Delta = \begin{vmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{vmatrix} \quad \ldots(18)$$

Now, to calculate the relative intensity noise (RIN) we must begin with calculating the photon number fluctuations(δs) as follows

$$\delta s = \frac{\begin{vmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & F_n(\omega) \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & F_w(\omega) \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & F_c(\omega) \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & F_e(\omega) \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & F_g(\omega) \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & F_s(\omega) \end{vmatrix}}{\Delta} \quad \ldots(19)$$

With aid of Eq. (16), we can rewrite Eq. (19) as follows

$$\delta s = \frac{H(\omega)}{\omega_R^2}\left[H_n(\omega)F_n + H_w(\omega)F_w + H_c(\omega)F_c + H_e(\omega)F_e + H_g(\omega)F_g + H_s(\omega)F_s\right] \ldots(20)$$

Where

$$H_n(\omega) = C_{21}C_{32}C_{44}C_{53}C_{65} - C_{21}C_{32}C_{43}C_{54}C_{65} \quad \ldots(21a)$$

$$H_w(\omega) = C_{11}C_{32}C_{43}C_{54}C_{65} - C_{11}C_{32}C_{44}C_{53}C_{65} \quad \ldots(21b)$$

$$H_c(\omega) = C_{11}C_{22}C_{44}C_{53}C_{65} - C_{11}C_{22}C_{43}C_{54}C_{65} + C_{12}C_{21}C_{43}C_{54}C_{65} - C_{12}C_{21}C_{44}C_{53}C_{65} \quad \ldots(21c)$$

$$H_e(\omega) = C_{11}C_{22}C_{33}C_{54}C_{65} - C_{11}C_{22}C_{43}C_{53}C_{65} - C_{11}C_{23}C_{32}C_{54}C_{65} - C_{12}C_{21}C_{33}C_{54}C_{65} + C_{12}C_{21}C_{43}C_{53}C_{65} \quad \ldots(21d)$$

$$H_g(\omega) = C_{11}C_{22}C_{43}C_{34}C_{65} - C_{11}C_{22}C_{33}C_{44}C_{65} + C_{11}C_{23}C_{32}C_{44}C_{65} - C_{12}C_{21}C_{34}C_{43}C_{65} + C_{12}C_{21}C_{33}C_{44}C_{65} \quad \ldots(21e)$$

$$\begin{aligned} H_s(\omega) = {} & C_{11}C_{22}C_{33}C_{44}C_{55} - C_{11}C_{22}C_{33}C_{45}C_{54} - C_{11}C_{22}C_{34}C_{43}C_{55} \\ & + C_{11}C_{22}C_{34}C_{45}C_{53} + C_{11}C_{22}C_{35}C_{43}C_{54} - C_{11}C_{22}C_{35}C_{44}C_{53} \\ & - C_{11}C_{23}C_{32}C_{44}C_{55} + C_{11}C_{23}C_{32}C_{45}C_{54} - C_{12}C_{21}C_{33}C_{44}C_{55} \\ & + C_{12}C_{21}C_{33}C_{45}C_{54} + C_{21}C_{12}C_{43}C_{34}C_{55} - C_{12}C_{21}C_{34}C_{45}C_{53} \\ & + C_{12}C_{21}C_{35}C_{43}C_{54} - C_{12}C_{21}C_{35}C_{44}C_{53} \quad \ldots(21f) \end{aligned}$$

In term of spectral density of the noise accompanying the signal, the (RIN) per unit bandwidthis define as, the ratio between the photon number fluctuations and the mean photon number$P$ as follows [21]

$$\frac{(\mathrm{RIN})}{\Delta f} = \frac{S_P(\omega)}{P^2} = \frac{1}{P^2}\lim_{T\to\infty}\frac{1}{T}|\delta s(\omega)|^2 \qquad (single-sided) \quad \ldots(22)$$

Now, the RIN can be expressed by using Eqs.(20-22) as follows

$$\frac{(\mathrm{RIN})}{\Delta f} = \frac{|H(\omega)|^2}{(\omega_R^2)^2}\left[2D_{nn}|H_n(\omega)|^2 + 2D_{ww}|H_w(\omega)|^2 + 2D_{cc}|H_c(\omega)|^2 + 2D_{ee}|H_e(\omega)|^2 + 2D_{gg}\left|H_g(\omega)\right|^2 + 2D_{ss}|H_s(\omega)|^2\right]/P^2 \quad \ldots(23)$$

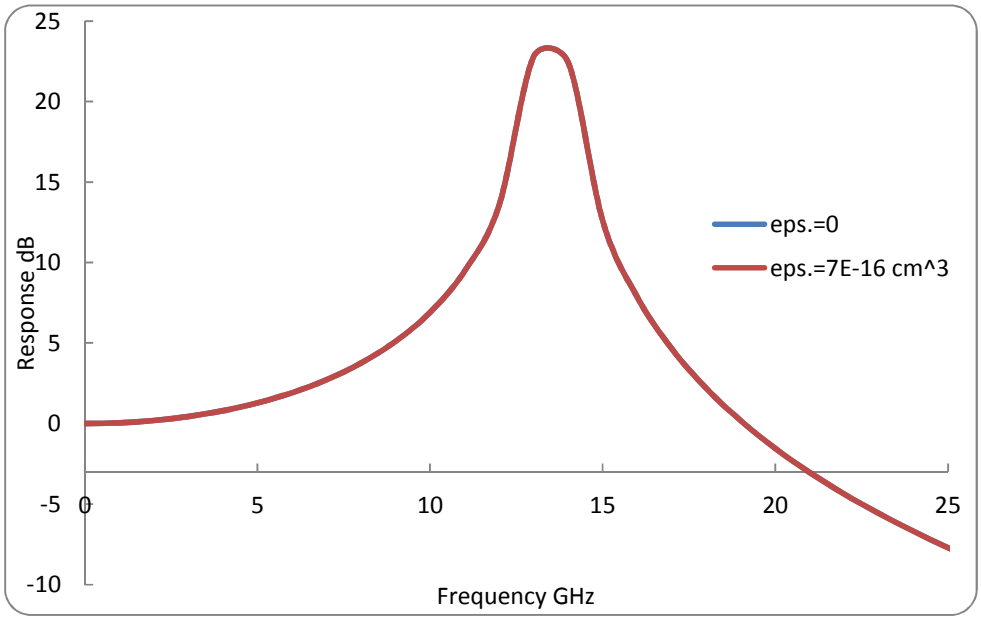
As is well known, each of photon lifetime, spectral hole burning, carrier heating, and carrier transport has an effect on the direct modulation of semiconductor laser$H(\omega)$. From Eq. (23), we can see that the RIN also affected from these physical processes. The present study is focused only on the carrier transport in/out quantum dot.

Results and discussion

Figs.2,3, 4, 5 and 6,shows the effect of nonlinearity and carriers transport on the small signalmodulation response.

Table (1) Symbols values used in present simulation [24]

NQ	5×10^{10}cm^{-2}	D_c	10
g_m	19.5 cm^{-1}	D_e	3
τ_p	1 ps	D_g	1
b	2×10^{-5}cm	D_W	250
e	1.6×10^{-19} C	ΔE_{wc}^{C}	134.565 meV
β	10^{-5}	ΔE_{ce}^{C}	111.077 meV
τ_{wr}	0.4 ns	ΔE_{eg}^{C}	51.836 meV
τ_{gr}	0.4 ns	v_g	9.1×10^{9} cm/s



Figure(2) Effect of nonlinearity on the small signal modulation response.

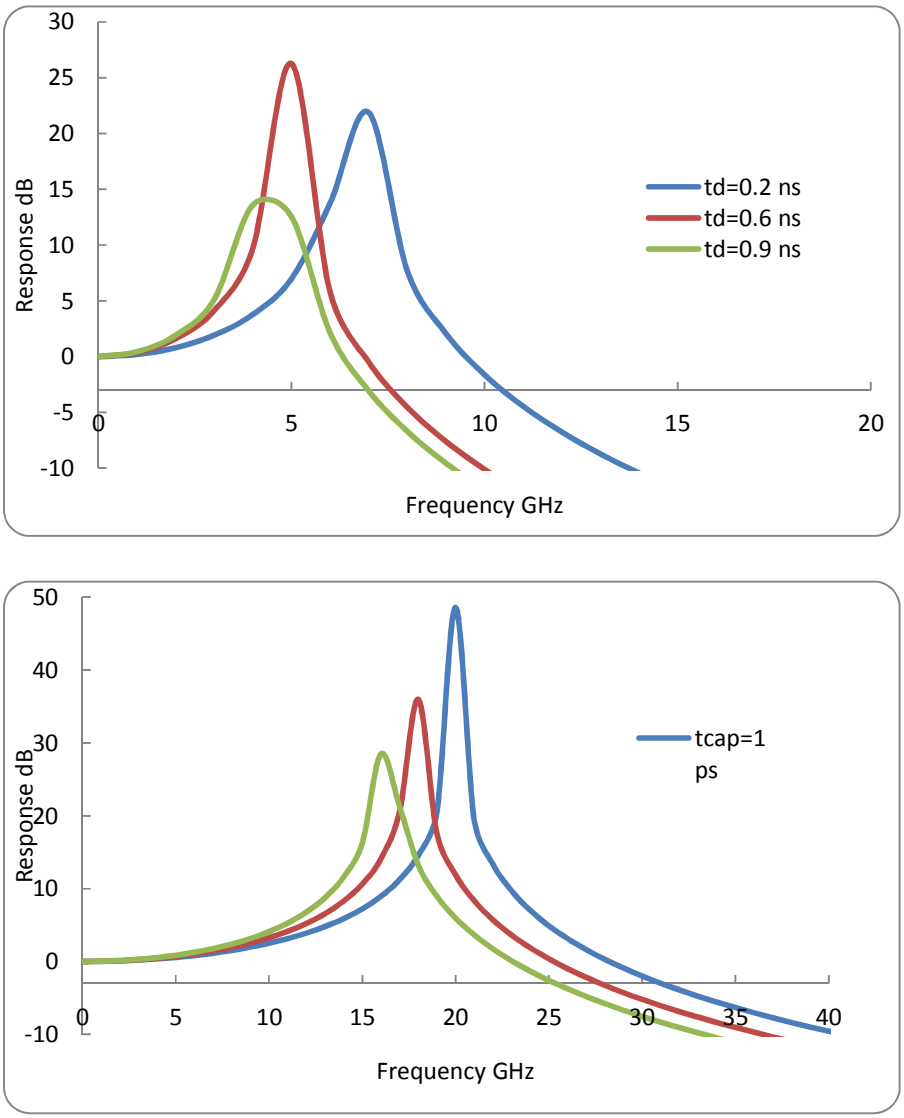
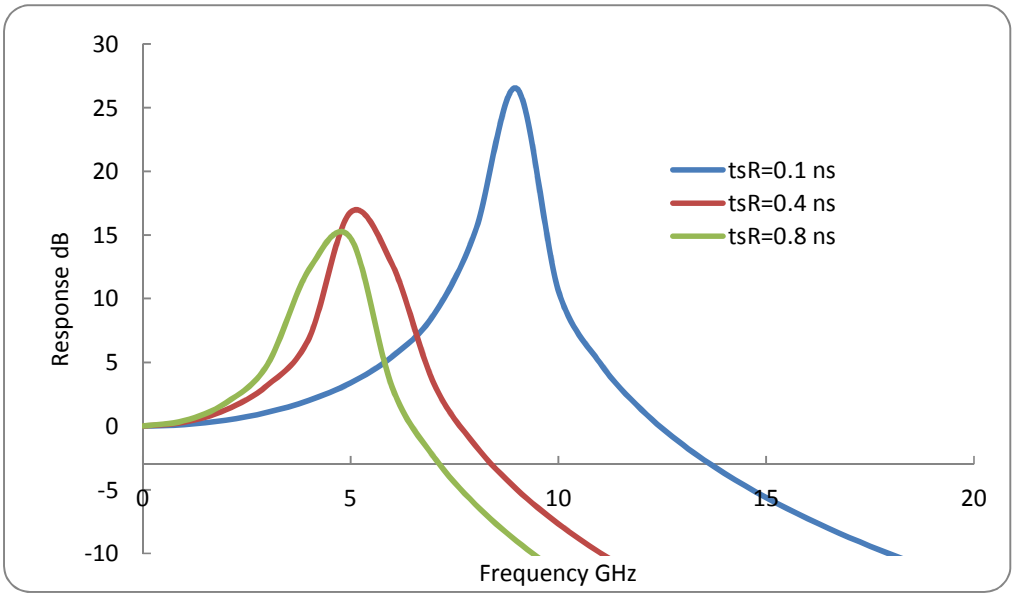


Figure (3) Effect of τ_d and τ_{cap} on the small signal modulation response.



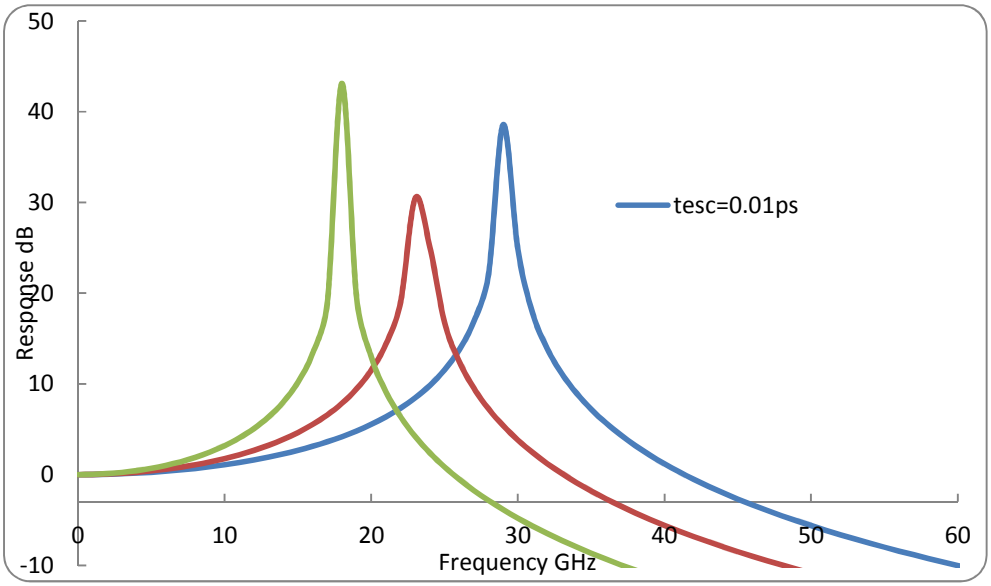
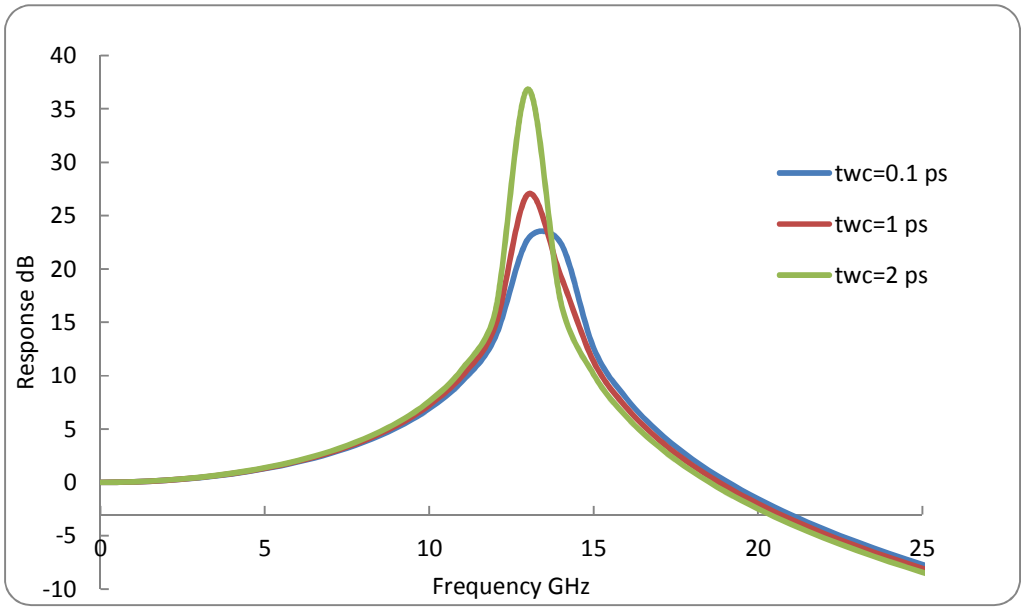


Figure (4) Effect of τ_{sR} and τ_{esc} on the small signal modulation response.



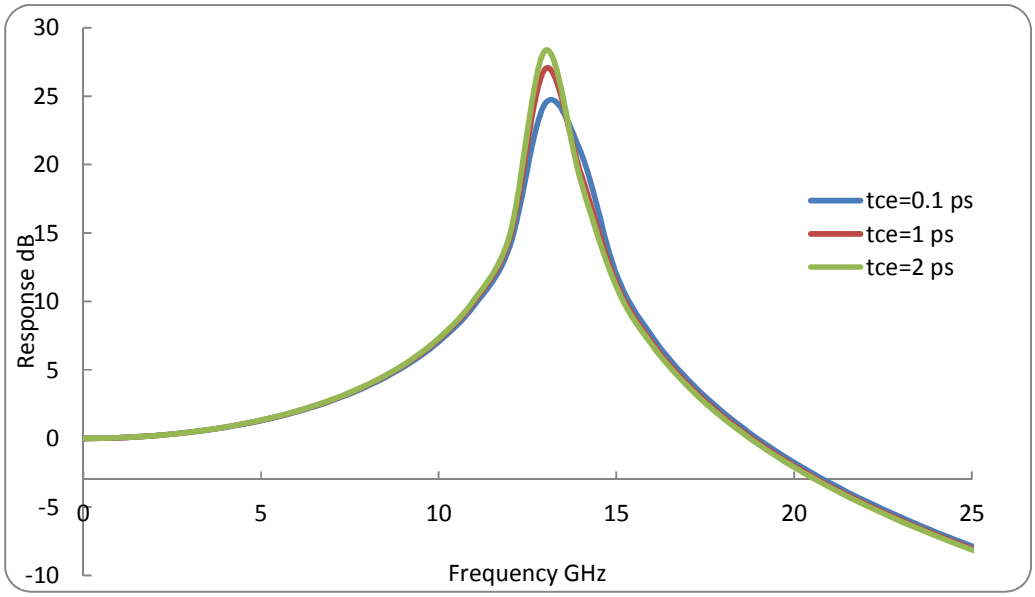
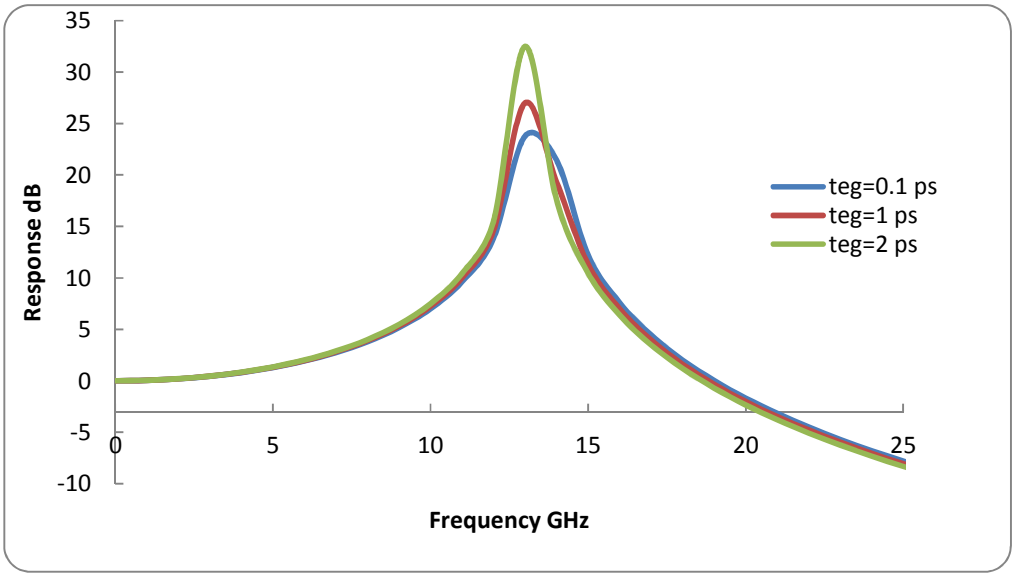


Figure (5) Effect of τ_{wc} and τ_{cg} on the small signal modulation response.



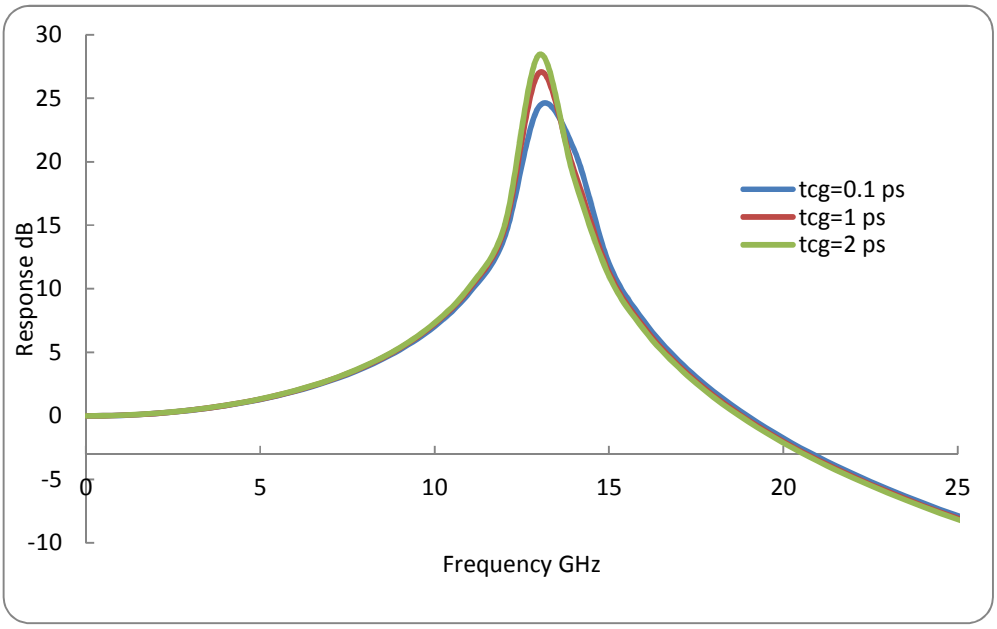
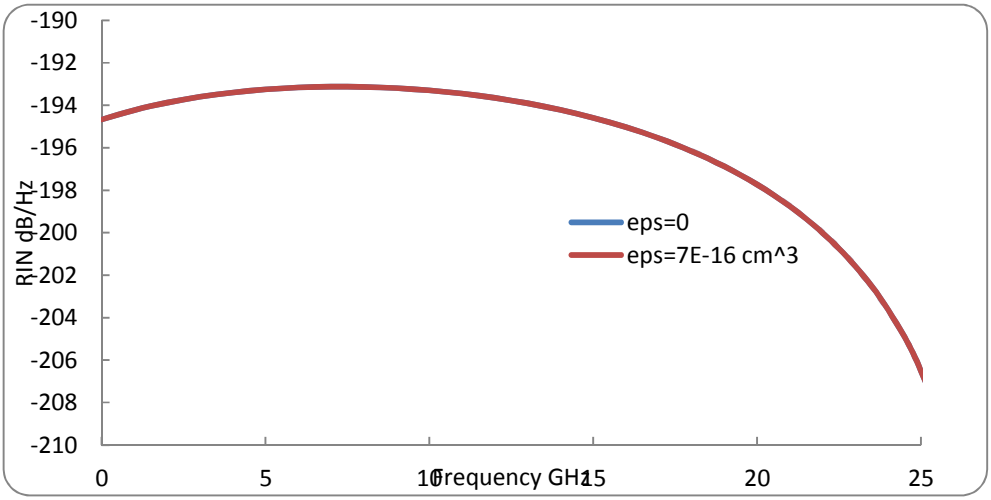


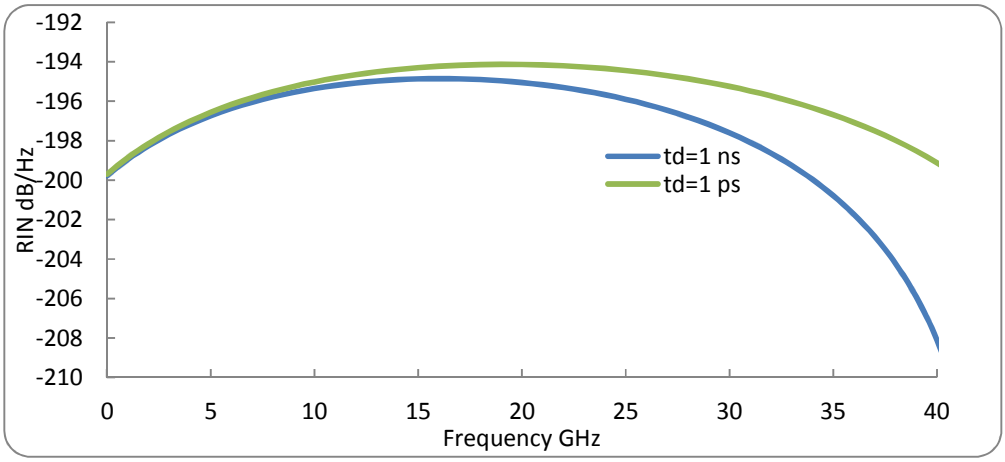
Figure (6) Effect of τ_{ce} and τ_{eg} on the small signal modulation response.

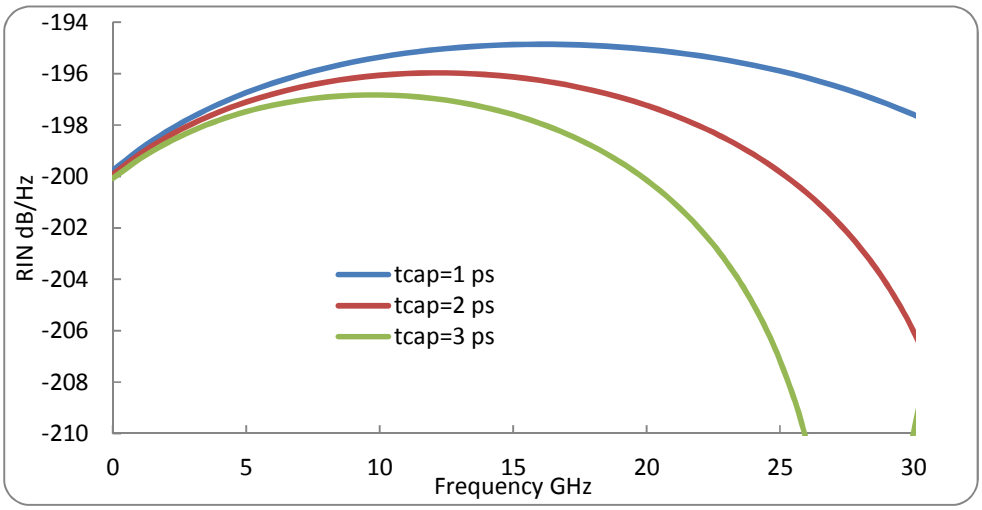
From figure 1,we cannot see any effect for nonlinear gain parameter ε on the present response behavior, we will see the same results in noise figures. This comes from the capture time in quantum dots be slower in comparison with QW (1 ps), since the change in modulation response is inversely proportional to this time. Also, the traditional formula for representing the effect of nonlinear gain parameter ε in present model is needed to improve by modifying ε to include injection heating, non stimulated recombination heating, stimulated recombination heating, and free-carrier absorption heating separately. In this case, fig. 2, we can improve the modulation bandwidth by decreasing the photon lifetime or by increasing the modal differential gain.In figures 3,4,5 and 6, the results show the modulation response fordifferent (capture, diffusion, escape,spontaneous recombination and relaxation in quantum dot states) times. The escape time and capture them in the present simulation calculated at capture cross section equal to $10^{-11}cm^2$. In all these figures, an increase in the time leads to increase the damping rate, also cause a significant low-frequency roll-off, note that the impact of the increase in time, weaker inside the dot. Our results give a good agreement with the experimental results [-]. Two solutions have been proposed and implemented to increase the modulation bandwidth in QD lasers: tunneling injection (TI) and acceptor (p) doping of the dots [6–13].In order to get more accurate results, both electron and hole transport must be considered together since the capture of holes is faster than that of electrons.From figures, we can see the effect of relaxation times τ_{wc} and τ_{eg} in comparison with other times on the band width of modulation response. Therefore, we can decrease these times to increase the bandwidth.The main increase in nonlinear gain parameter is coming from the increase in transport times outside the active region (QD), Therefore, the transport times (capture, diffusion, escape, spontaneous recombination) must be improve to increase the modulation bandwidth.Figs.7, 8, 9, 10 and 11, shows the effect of nonlinearity and carriers transport on the relative intensity noise.



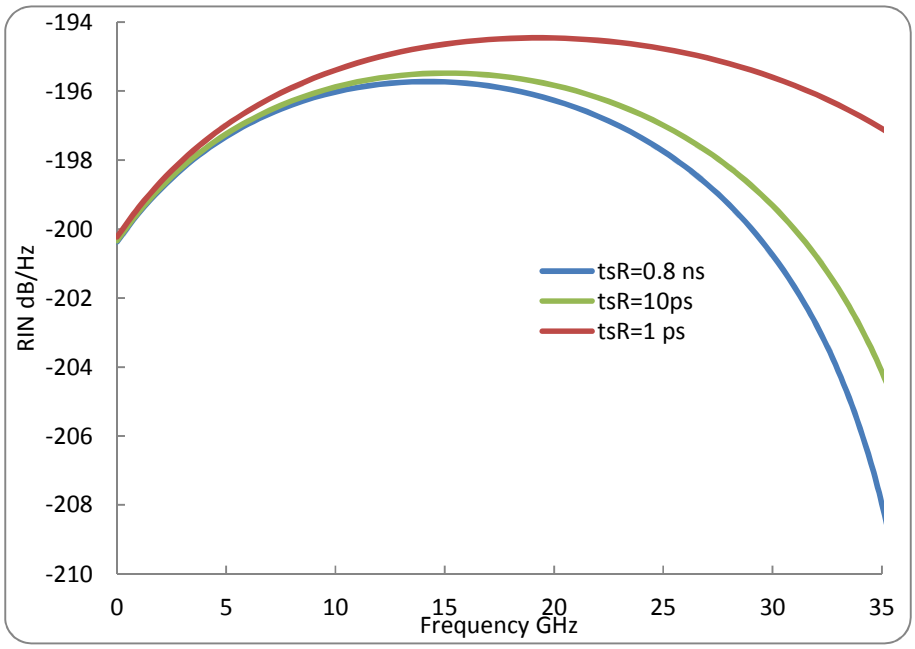
Figure(7) Effect of nonlinearity on the relative intensity noise.

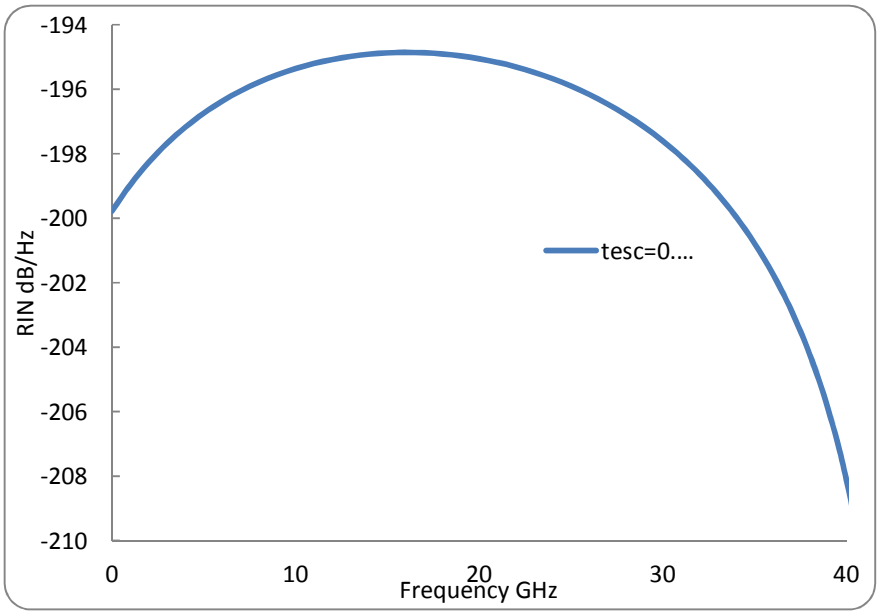
This is the same result in a figure (2). The effect of nonlinearity on long wavelength quantum dot lasers such as InAs/GaAs is very weak in comparing other short wavelength quantum dot laser such as III-Nitrides lasers because of the high photon intensity in the latter. Therefore, the correct formula for noise and modulation must be derived based on the density matrix formalism which includes relaxation times for both holes and electrons in the valence band and the conduction band. Figure 8 Figure 1 shows the effect of $\tau_d$and τ_{cap}, on the relative intensity noise. It is clear that the effect of transport time has strong effectsat high frequencies. In general, in all figure we have the following formula for the noise spectral power density $S(\omega)\sim\frac{1}{\omega}$faster transport time leads to decrease noise spectral power density because of the all carriers will be injected into the lower states and finally in radiative recombination process.





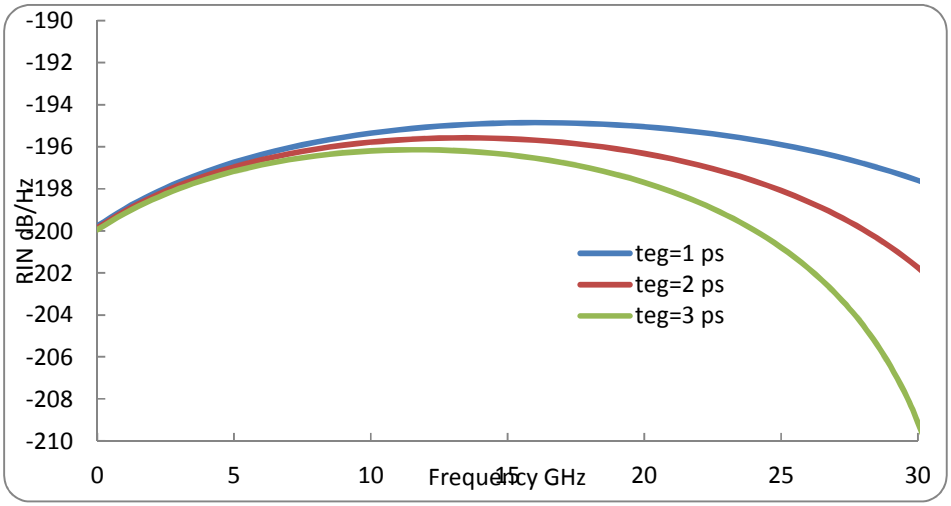
Figure(8) Effect of τ_d and τ_{cap}on the relative intensity noise.





Figure(9) Effect of τ_{sR} and τ_{esc}on the relative intensity noise.

In escape time case,we have one curve, this is because of the small contribution for this time in noise equation, see eq. (11a).Noise has a constant value at different escape time values when the occupation probability value for the wetting layer close to zero.



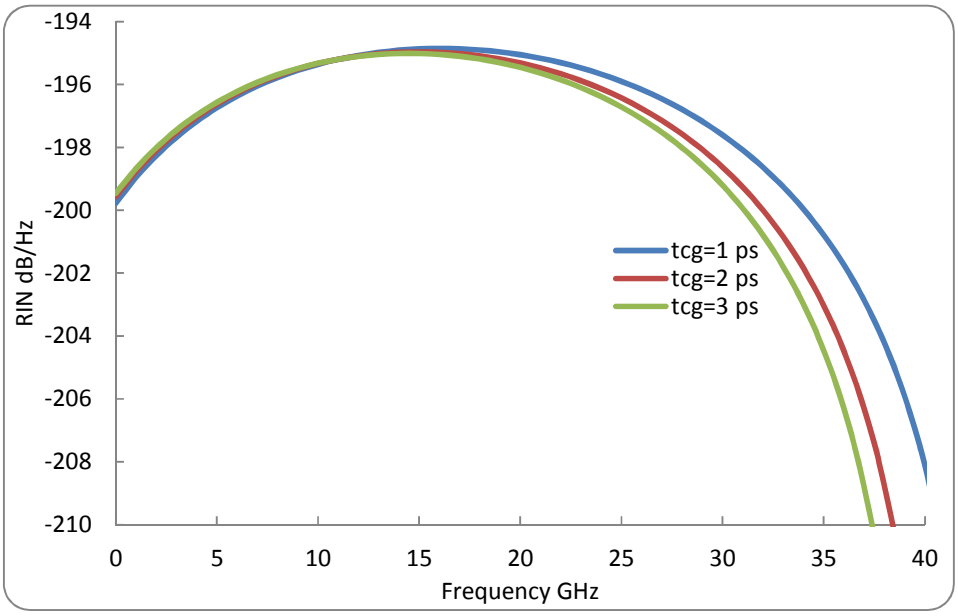
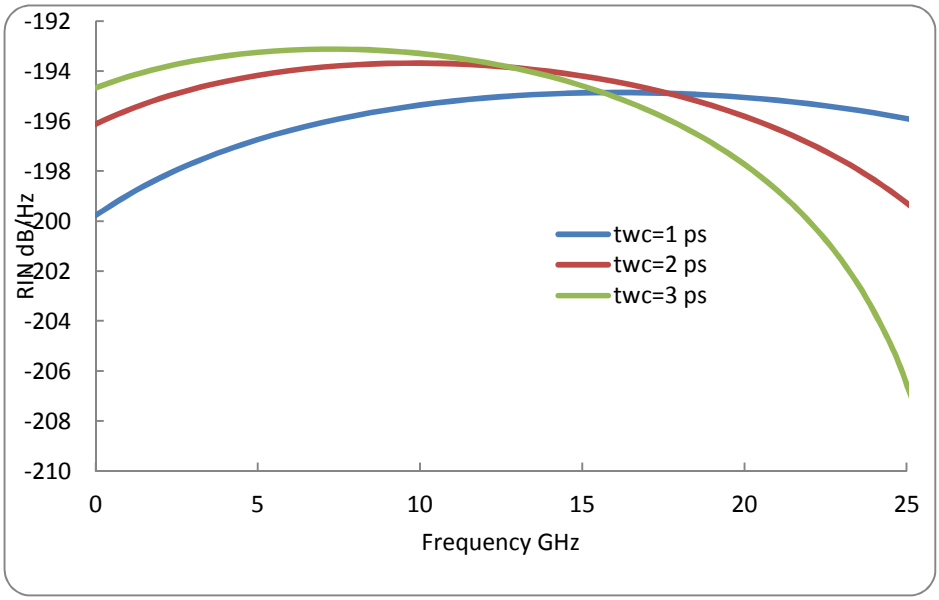


Figure (10) Effect of τ_{eg} and τ_{cg}on the relative intensity noise.



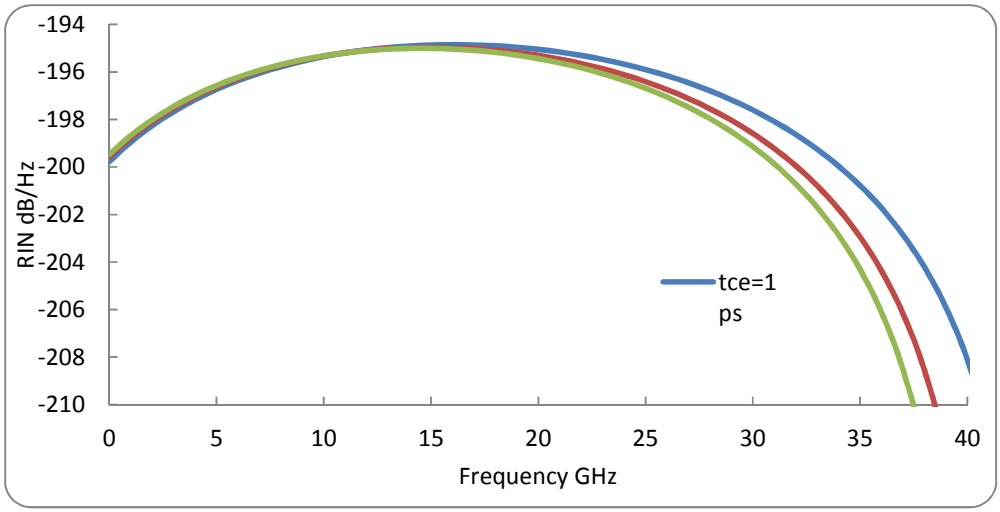


Figure (11) Effect of τ_{wc} and τ_{ce} on the relative intensity noise

CONCLUSIONS

Putting things altogether, we can conclude that: the mathematical analysis indicates that the nonlinearities not affect on the laser noise in a similar way to its effect on the modulation response and vary according to carrier transport and relaxation/escape lifetimes outside or inside the quantum dots in long wavelength (low photon density) quantum dot lasers. photons density in present paper is about 104 cm3. So, it is not necessary to calculate the impact of this factor on the behavior of semiconductor lasers as sources of light in communication network systems because of weakness of the suggested nonlinearity formula in the present analysis to detect the noise behavior. Therefore, the present formula for representing the effect of nonlinear gain parameter ε is needed to improve by modifying ε to include injection heating, non stimulated recombination heating, stimulated recombination heating, and free-carrier absorption heating separately. The carrier transport (outside dots) and time relaxation (inside dots) has a strong effect on the noise calculation and modulation response. So, the carriers transport must be improved to increase the modulation response bandwidth and decrease the noise value In this study, the present model of the rate equations includes many affecting factors on the noise properties, which used to be are neglected in previous theoretical papers, therefore, this is the origin of the difference between the present results and the other ones.

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