Developing a Mathematical Method for Controlling the Generation of Cubic Spline Curve based on Fixed Data Points, Variable Guide Points and Weighting Factors

Dr. Abdul Monem S. Rahma
Computer Science Department, University of Technology/Baghdad
Email:monem.rahma@yahoo.com
Dr. A. M. J. Abdul Hussen
Computer Science Department, University of Technology/Baghdad
Email:dralmaalymohssen@yahoo.com
Abbas Muhammed Allawy
Computer Science Department, University of Technology/Baghdad
Email:abbas.allawy@gmail.com

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ABSTRACT:
The cubic spline interpolation method is used to design the developed curve. The method enables the designer to change the shape of the curve to the desired one without changing the original data points. That done by changing the value of the parameter t_i depending on defined guide points and weighting factors that assigned secretly by the designer, so that not easily recreate the same design by anyone else even he knows the original data points that used to design the curve. These curves can be used in many fields such as banknote design, secure printed documents, the design of cars and airplanes structures, and Ornament. This paper modifies a mathematical technique to design complex curves, which are not easy to recreate. The results of this research had compared with other related methods since the improved models of this research is used the negative values of the guide to compute the parametric rather than using the only positive ones; the efficiency of the modified technique had proved by several comparative examples.
Keywords: Cubic spline, negative parametric, weighting factor, Area Parameterization.

تطوير طريقة رياضية للسيطرة على توليد تطابق المنحني المكعب بالاعتماد على نقاط بيانات ثابتة و نقاط الاستدلال متغيره ومعاملات الاوزان

الخلاصة:
الطريقة تمكن المصمم من تغير شكل المنحنى الى الشكل المرغوب فيه من دون الحاجة الى تغير نقاط البيانات الأصلية للمنحنى. ويتم ذلك من خلال تغيير قيمة المتغير (t_i) بالاعتماد على نقاط استدلال ومعاملات اوزان توضع بسرية تامة من قبل المصمم، بحيث ليس من السهل لأي شخص اخر تقليد المنحنى حتى في حالة كشف نقاط البيانات الاصلية المستخدمة في تصميم المنحنى. هذه المنحنيات يمكن ان تستخدم في الكثير من المجالات ومنها تصميم العملات النقدية، تأمين الوثائق المطبوعة، تصميم هياكل السيارات

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والطائرات، والزخرفة. البحث طور أسلوب لتصميم منحنيات معقدة ليس من السهل تقليدها. تم استخدام الـCubic Spline في توليد المنحنى المطور. تمت مقارنة نتائج هذا البحث مع الطرق المماثلة وتم اثبات كفاءة الطريقة المطورة نظراً لكون الطريقه في هذا البحث تم تتطويرها كي ترسم المنحنيات مع القيم السالبه إضافةً الى الرسم بالقيم الموجبه; مقارنة النتائج تمت باستخدام امثلة متعددة.

INTRODUCTION

The simplest kind of cubic spline consists of one cubic polynomial, which has degree at most three, with prescribed values and prescribed slopes at two points. A general cubic spline consists of several pieces of cubic polynomials, joined to form a twice-differentiable function. Thus, the inclusion of a few exercises with cubic splines in calculus reinforces the idea that one function need not consist of a single formula, but may involve several algebraic formulae and logical tests, that some problems involve not curve sketching, but building a function subject to geometric specifications, and that such functions and problems have practical applications [1, 2].

Many previous researches discussed in cubic spline area; Rahma [1] provided the parameterization method for controlling the design by using only one guide point. This research had developed the area parameterization method for giving higher flexibility to generate and control the design by calculating the triangular area between each data point and the defined guide points with or without the use of the weighting factors ($\hat{W}_i$).

This paper presents a Modified mathematical method for controlling curves' generation by using guide points and weighting factors with the constraint of cubic spline. The method proposed for using two guide points, three guide points, two guide points with two weighting factors, three guide points with three weighting factors, the values of the parameter (t_i) had computed from the guide points and/or the weighting factors in both positive and negative cases. Spline curves are very important, it can be used in the design of cars, ships, airplanes, banknotes, passports, securities, university certificate, and other things, which are of high impotence. The curve generation algorithm should allow the designer to easily design a complex curves. For such purpose, this research had been made.

The rest of this paper is organized as follows: section 2 provides a brief description about cubic spline Preliminaries and, section 3 describes the developed area parameterization method in four cases, section 4 gives a brief description about area parameterization model, and section 5 summarizes the conclusions about the results of this research.

Cubic Spline Background

Spline theory is simple. Over *n* intervals, the routine fits *n* equations subject to the boundary conditions of *n+1* data points. The derivations of Lilley and Wheatly are used. The derivation assumes a functional form for the curve fit. This equation (Eq. (1)) form is simplified and then solved for the curve fit equation [3, 4 ,5].

$$y = a_i (x - x_i)^3 + b_i (x - x_i)^2 + c_i (x - x_i) + d_i \quad \ldots (1)$$

Where

the x, y; represent the points location on axis; The assumed form for the cubic polynomial curve fit for each segment are the (a,b,c,d)coefficients .
Where

$$\begin{bmatrix} 2(h_1+h_2) & h_2 & & \\ h_2 & 2(h_2+h_3) & \ddots & \\ & \ddots & \ddots & h_{n-2} \\ & & h_{n-2} & 2(h_{n-2}+h_{n-1}) \end{bmatrix} \begin{bmatrix} S_2 \\ S_i \\ \vdots \\ S_{n-1} \end{bmatrix} = 6 \begin{bmatrix} \frac{y_3-y_2}{h_2} - \frac{y_2-y_1}{h_1} \\ \vdots \\ \frac{y_n-y_{n-1}}{h_{n-1}} - \frac{y_{n-1}-y_{n-2}}{h_{n-2}} \end{bmatrix}$$

the spacing between successive data points is shown in Eq. (2) below:

$$h_i = x_{i+1} - x_i \qquad \ldots(2)$$

The cubic spline constrains the function value, 1st derivative $y'(x)$ and 2nd derivative $y''(x)$. The routine must ensure that y(x), $y'(x)$ and $y''(x)$ are equal at the interior node points for adjacent segments.
Substituting a variable *S* for the polynomial's second derivative reduces the number of equations from *a, b, c, d* for each segment to only *S* for each segment.
For the *i*th segment, the S governing equation shown below in Eq. (3),

$$h_{i-1} S_{i-1} + (2h_{i-1} + 2h_i)S_i + h_i S_{i+1} = 6\left(\frac{y_{i+1}-y_i}{h_i} - \frac{y_i-y_{i-1}}{h_{i-1}}\right) \qquad \ldots(3)$$

In matrix form, the governing equations reduce to a tri-diagonal form.
S_1 and S_n are zero for the *natural* spline boundary condition. If different boundary conditions are needed, the appropriate changes can be made to the governing equations.
Finally, the cubic spline properties are found by substituting into the equations shown in Eq. (4), Eq. (5), Eq. (6), and Eq. (7) . These *a, b, c* and *d* values correspond to the polynomial definition for each segment. [3, 6, 7]

$$a_i = (S_{i+1} - S_i) / 6h_i \qquad \ldots(4)$$

$$b_i = S_i / 2 \qquad \ldots(5)$$

$$ci = \frac{y_{i+1}-y_i}{h_i} - \frac{2h_iS_i + h_iS_{i+1}}{6} \qquad \ldots(6)$$

$$d_i = y_i \qquad \ldots(7)$$

Area Parameterization Model

The control of the cubic spline's shape can be done based on the area parameterization (**Fig1**) by choosing a point G = (X_G, Y_G) called the guide of parameterization [1], then we define the values of t_i to be as shown in Eq. (8)
$t_0 = 0$

$t_i=t_{i-i}+ | \delta_i |/D$ for i=l,…,N …. (8)

where

$$\delta_i = \frac{1}{2} * \begin{vmatrix} X_i - X_G & Y_i - Y_G \\ X_{i-1} - X_G & Y_{i-1} - Y_G \end{vmatrix} \quad \text{for i=1, 2, . . , N} \quad \text{…. (9)}$$

$$\text{And D } = \sum_{i=1}^{n} |\delta_{1i}| \quad \text{for i=1, 2, . . , N} \quad \text{…. (10)}$$

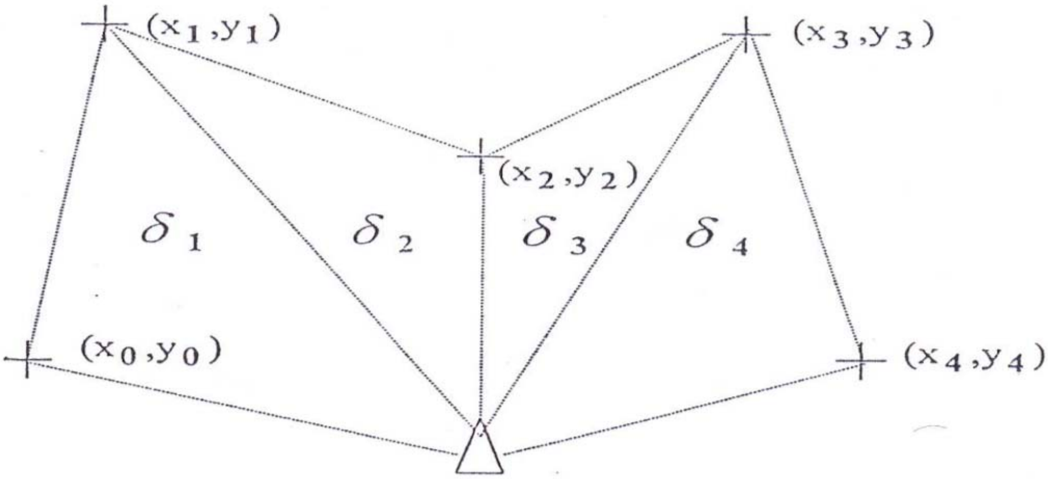


Figure (1): Guide of Parameterization

Developed Area Parameterization Model:

The area parameterization model in [1] has been tested and developed in many perspectives:

I. After programming and carefully testing the mathematical model mentioned in [1], this paper added another case which is using negative (G_x ,G_y) values to change the shape of the curve without changing the original data points values. By computing the parameter (t_i) values from negative guide points (G_x ,G_y) rather than using only the positive ones as mentioned in [1]. **Fig2** shows the result of drawing 5 data points using cubic spline interpolation with area parameterization model and one guide point with positive G(300,250), negative X, G(-300,250), negative Y, G(300,-250), and negative X,Y G(-300,-250) values.

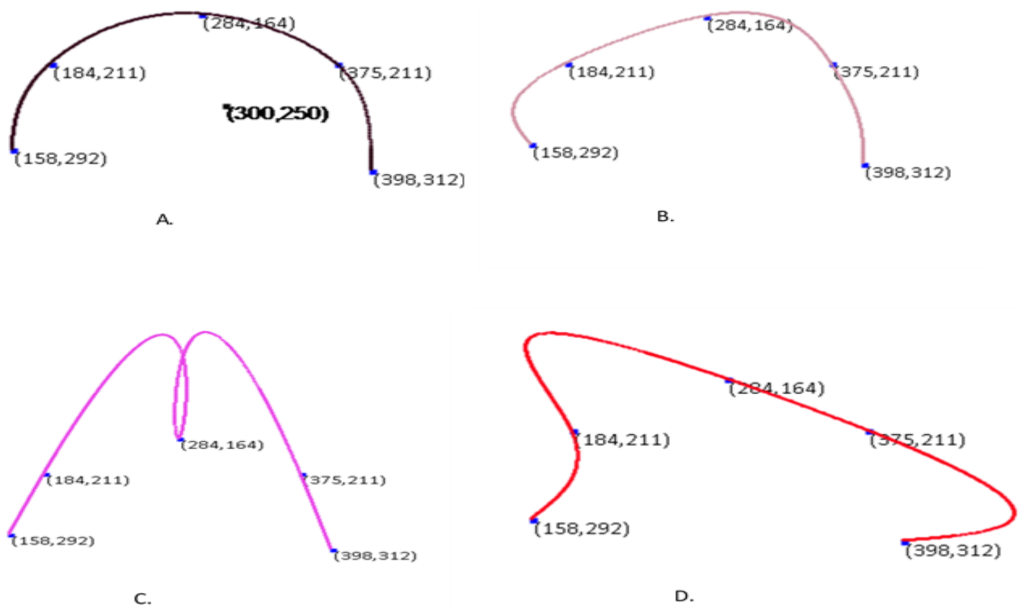


Figure (2): controlling the design of the curve using one guide Point with positive and negative values [A.G (300,250), B. G(-300,250), C.G(300,-250),D. G(-300,-250)]

II. The area parameterization model [1] has modified by using more than one guide point to control the design to the desired shape without changing the original data points. The mathematical model developed on two phases as described below:

Case1: At the first case two guide points $G_1=(G_1X,G_1Y)$ and $G_2=(G_2X,G_2Y)$ had added to control the design by calculating the parameter $\mathbf{t_i}$ values from the calculation of the triangular distances $\boldsymbol{\delta_i}$ between each guide point and all the data points (X_i,Y_i) as shown in the following mathematical models:

Model 1: Two Guide Points for area parameterization

$t_0 = \mathbf{0}$

$$t_i = t_{i-1} + \left|\frac{\delta_{1i} + \delta_{2i}}{2}\right| * \left(\frac{D_1 + D_2}{2}\right) \quad \text{for } i=1,2,..,N \qquad \ldots.(11)$$

such that:

$$\delta_{1i} = \frac{1}{2} * \begin{vmatrix} X_i - G_1 X & Y_i - G_1 Y \\ X_{i-1} - G_1 X & Y_{i-1} - G_1 Y \end{vmatrix} \quad \text{for } i=1, 2, \ldots, N \qquad \ldots.(12)$$

$$D_1 = \sum_{i=1}^{n} |\delta_{1i}| \quad \text{for } i=1, 2, \ldots, N \qquad \ldots.(13)$$

$$\delta_{2i} = \frac{1}{2} * \begin{vmatrix} X_i - G_2 X & Y_i - G_2 Y \\ X_{i-1} - G_2 X & Y_{i-1} - G_2 Y \end{vmatrix} \quad \text{for } i=1,2,..,N \qquad \ldots.(14)$$

$D_2 = \sum_{i=1}^{n} |\delta_{2i}|$ for i=1,2,..,N …. (15)

N the number of the data points

The model gives a higher flexibility than the case of using only one guide point to design the curve, the designer has more options to change the curve's shape in more than one method as described without changing the original data points.

1. Changing the design by changing one of the guide point's location or both of them in the positive or negative values or both of them, in the case of using negative values, the curve shape will be opposite to the other side with the same design. **Fig3** illustrate the case of using two guide points and five data points with mathematical model described in **case1**.
2. Changing the design without changing the location of the guide points nor changing the location of the original data points by adding a ($\hat{W}_i$) with each guide point, the $\hat{W}_i$ represents the percentage effect of the specified guide on the design of the curve. By the experiment, noticed that the design can be controlled much easier when the summation of the weight factors equal to 1 ($\sum_{i=1}^{2} \hat{W}_i = 1$). The other case (when$\sum_{i=1}^{2} \hat{W}_i \neq 1$) also changed the design, but with less flexibility. As shown in the following mathematical model

Model 2: Two guide Points (G_1,G_2)for area parameterization with 2 Weight factors percentages **(**$\hat{W}_1$ & $\hat{W}_2$) of impact on each guide point. Parameters of this model are shown below in Eq. (16), Eq. (17), Eq. (18), Eq. (19), and Eq. (20).

$t_0 = 0$

$$t_i = t_{i-1} + \left| \frac{(\delta_{1i} * \hat{W}_1) + (\delta_{2i} * \hat{W}_2)}{2} \right| * \left(\frac{D_1 + D_2}{2}\right) \quad \text{for i=1,2,..,N} \qquad …. (16)$$

such that:

$$\delta_{1i} = \frac{1}{2} * \begin{vmatrix} X_i - G_1 X & Y_i - G_1 Y \\ X_{i-1} - G_1 X & Y_{i-1} - G_1 Y \end{vmatrix} \quad \text{for i=1, 2, . . , N} \qquad …. (17)$$

$$D_1 = \sum_{i=1}^{n} |\delta_{1i}| \quad \text{for i=1, 2, . . , N} \qquad …. (18)$$

$$\delta_{2i} = \frac{1}{2} * \begin{vmatrix} X_i - G_2 X & Y_i - G_2 Y \\ X_{i-1} - G_2 X & Y_{i-1} - G_2 Y \end{vmatrix} \quad \text{for i=1,2,..,N} \qquad …. (19)$$

$$D_2 = \sum_{i=1}^{n} |\delta_{2i}| \quad \text{for i=1,2,..,N} \qquad …. (20)$$

Where

N performs number of the data points.

The value of $\hat{W}_i$ can be negative or positive integer or float numbers.

The mathematical model of **model2** allows more control over designing than the case of using two guides without the use of $\hat{W}_i$.

The design can be controlled with many options by changing one of the guide points, the weight factors, the data points, or all of them together with positive and negative values as shown in the **Fig4**, **Fig5**, **Fig6**, and **Fig7**.

Case 2: At the second case three guide points $G_1=(G_1X,G_1Y)$,$G_2= (G_2X,G_2Y)$ and $G_3= (G_3X,G_3Y)$ had added to control the design by calculating the values of the parameter $\mathbf{t_i}$ from the calculation of the triangular distances $\boldsymbol{\delta_i}$ between each guide point and all the data points (X_i,Y_i) as shown in the mathematical models.

Model 3: Three Guide Points for area parameterization, equations below (Eq. (21), Eq. (22), Eq. (23), Eq. (24), Eq. (25), Eq. (26), and Eq. (24)) performs the mathematical presentation of this model.

$t_0= 0$

$$t_i= t_{i-1} + \left|\frac{\delta_{1i} + \delta_{2i} + \delta_{3i}}{3}\right| * \left(\frac{D_1 + D_2 + D_2}{3}\right) \text{ for i=1,2,..,N} \quad .... (21)$$

such that:

$$\delta_{1i} = \frac{1}{2} * \begin{vmatrix} X_i - G_1 X & Y_i - G_1 Y \\ X_{i-1} - G_1 X & Y_{i-1} - G_1 Y \end{vmatrix} \quad \text{for i=1, 2, . . , N} \quad .... (22)$$

$$D_1 = \sum_{i=1}^{n} |\delta_{1i}| \quad \text{for i=1, 2, . . , N} \quad .... (23)$$

$$\delta_{2i} = \frac{1}{2} * \begin{vmatrix} X_i - G_2 X & Y_i - G_2 Y \\ X_{i-1} - G_2 X & Y_{i-1} - G_2 Y \end{vmatrix} \quad \text{for i=1,2,..,N} \quad .... (24)$$

$$D_2 = \sum_{i=1}^{n} |\delta_{2i}| \quad \text{for i=1,2,..,N} \quad .... (25)$$

$$\delta_{3i} = \frac{1}{2} * \begin{vmatrix} X_i - G_3 X & Y_i - G_3 Y \\ X_{i-1} - G_3 X & Y_{i-1} - G_3 Y \end{vmatrix} \quad \text{for i=1,2,..,N} \quad .... (26)$$

$$D_3 = \sum_{i=1}^{n} |\delta_{3i}| \quad \text{for i=1,2,..,N} \quad .... (27)$$

Where

N is the number of the control points.

The model allows higher degree of control over the design than the previous cases of using one or two guide's points.

In addition, the designer can use positive and negative values to control the design and modify the $\mathbf{t_i}$ value without changing the data points, **Fig8** and **Fig9** show a few samples designed using the case3 mathematical model.

Also, here the designer can control the design without changing the data point nor the values of the three guide points by adding ($\hat{\mathrm{W}}_{\mathbf{i}}$) with each guide point as shown in the mathematical model of **Model4**. By the experiment, noticed that the design can be controlled much easier when the summation of the weight factors equal to 1 ($\sum_{i=1}^{3} \hat{\mathrm{W}}_{\mathbf{i}} = 1$). While the other case (when$\sum_{i=1}^{3} \hat{\mathrm{W}}_{\mathbf{i}} \neq 1$) also changed the design, but with less flexibility.

Model 4: Three Guide Points (G_1, G_2, G_3) for area parameterization with three weight factors ($\hat{\mathrm{W}}_{\mathbf{1}}$, $\hat{\mathrm{W}}_{\mathbf{2}}$ & $\hat{\mathrm{W}}_{\mathbf{3}}$) of impact on each guide point. The mathematical model is shown below in the following equations:

$t_0= \mathbf{0}$

$$t_i = t_{i-1} + \left| \frac{(\delta_{1i} * \hat{W}_1) + (\delta_{2i} * \hat{W}_2) + (\delta_{3i} * \hat{W}_3)}{3} \right| * \left(\frac{D_1 + D_2 + D_3}{3}\right) \quad \text{for } i=1,2,..,N \qquad .... (28)$$

such that:

$$\delta_{1i} = \frac{1}{2} * \begin{vmatrix} X_i - G_1 X & Y_i - G_1 Y \\ X_{i-1} - G_1 X & Y_{i-1} - G_1 Y \end{vmatrix} \quad \text{for } i=1, 2, \ldots, N \qquad .... (29)$$

$$D_1 = \sum_{i=1}^{n} |\delta_{1i}| \quad \text{for } i=1, 2, \ldots, N \qquad .... (30)$$

$$\delta_{2i} = \frac{1}{2} * \begin{vmatrix} X_i - G_2 X & Y_i - G_2 Y \\ X_{i-1} - G_2 X & Y_{i-1} - G_2 Y \end{vmatrix} \quad \text{for } i=1,2,..,N \qquad .... (31)$$

$$D_2 = \sum_{i=1}^{n} |\delta_{2i}| \quad \text{for } i=1,2,..,N \qquad .... (32)$$

$$\delta_{3i} = \frac{1}{2} * \begin{vmatrix} X_i - G_3 X & Y_i - G_3 Y \\ X_{i-1} - G_3 X & Y_{i-1} - G_3 Y \end{vmatrix} \quad \text{for } i=1,2,..,N \qquad .... (33)$$

$$D_3 = \sum_{i=1}^{n} |\delta_{3i}| \quad \text{for } i=1,2,..,N \qquad .... (34)$$

Where
N is the number of the control points

With the Mathematical **Model4**, the designer has the largest number of design options; he can design the curve by changing one of the parameters or all of them at the same step with negative or positive values until he gets the desired design. **Fig10** and **Fig11** show a few samples designed using the mathematical **Model4**.

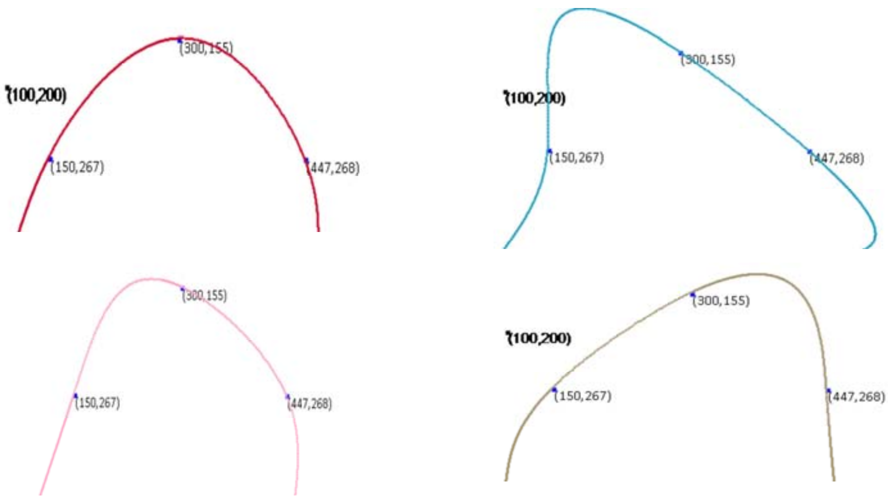
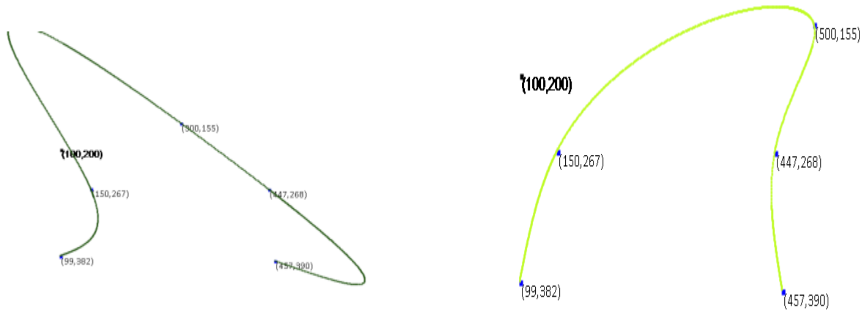


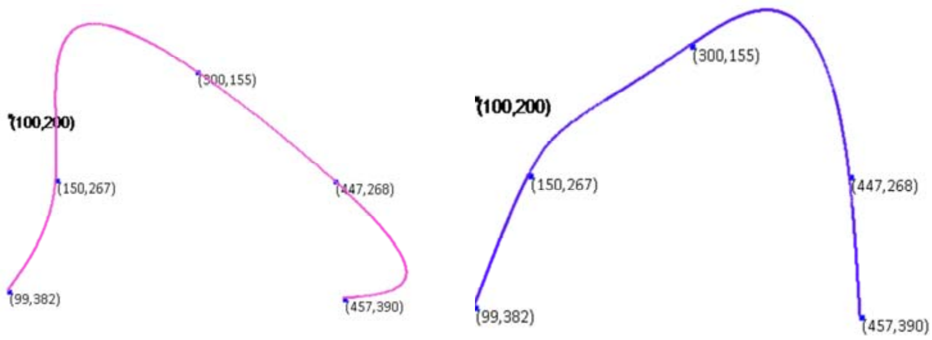
Figure (3): shows the degree of change in the curve shape with different weighting factors on the same data points and guide points.



G1=(100,200)G2=(-320,-400) with $\hat{W}_1$= 0.5 $\hat{W}_2$=0.5

G1=(100,200)G2=(-320,-400) with $\hat{W}_1$= 0.1 $\hat{W}_2$=0.9

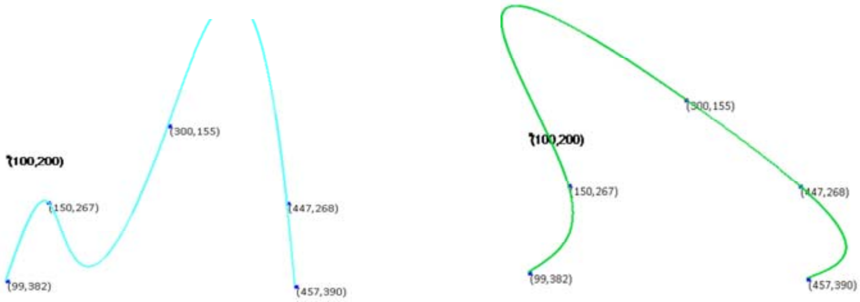
Figure (3): shows the degree of change in the curve shape with different weighting factors on the same data points and guide points.



G_1=(100,200) G_2=(-320,-400)

G_1=(100,200)G_2=(-320,-400) with $\hat{W}_1$=1.75 $\hat{W}_2$=-0.75

Figure (4): The designs with and without weighting factors here is a negative weighting factors values.



G_1=(100,200)G_2=(-320,-400) with $\hat{W}_1$= 2, $\hat{W}_2$= -1

G_1=(100,200)G_2=(-320,-400) with $\hat{W}_1$=-99, $\hat{W}_2$=100

Figure (5): The update of the design by changing the weighting factors values.

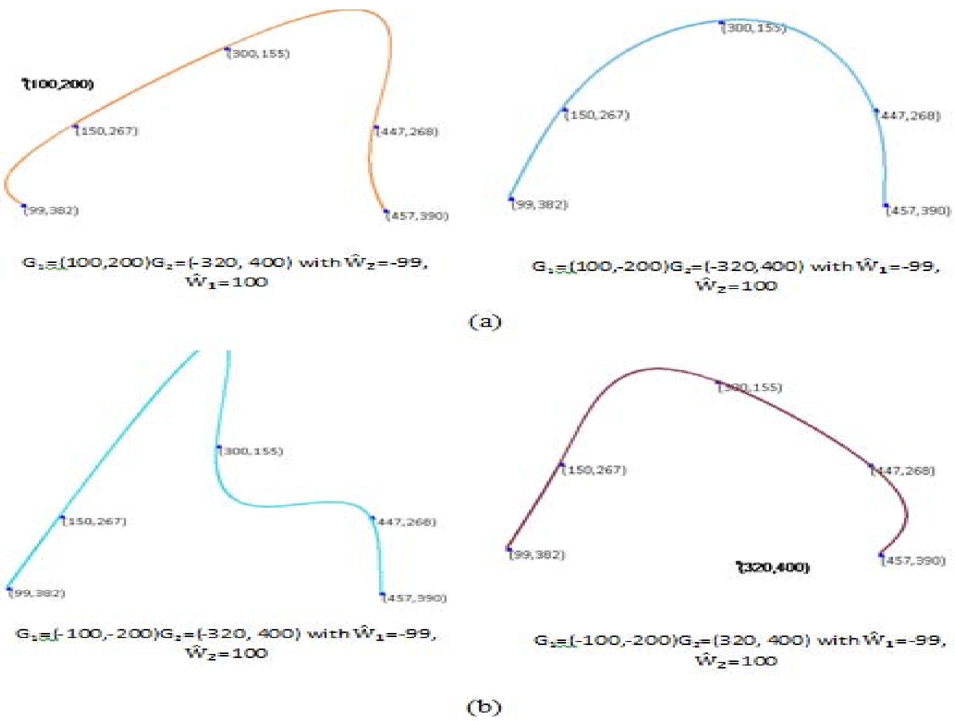


Figure (6): (a , b) The case of changing the guide point without changing the wieghting factors and the data point values.

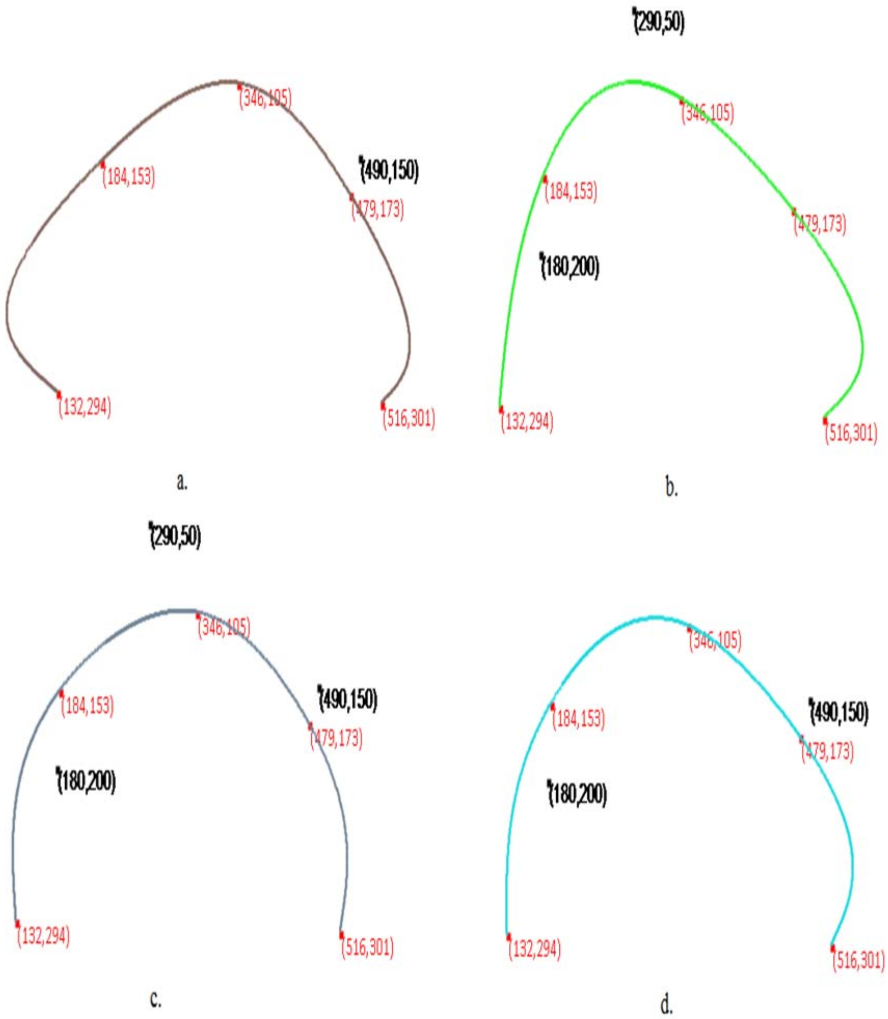


Figure (7): (a) Design curve using five data points and three guide points G_1=(180,-200), G_2=(-290,50), G_3=(490,150), (b) Design curve using five data points and three guide points G_1=(180,200), G_2=(290,50), G_3=(-490,-150), (c) Design curve using five data points and three guide points G_1=(180,200), G_2=(290,50), G_3=(490,150), (d) Design curve using five data points and three guide points G_1=(180,200), G_2=(-290,-50), G_3=(490,150).

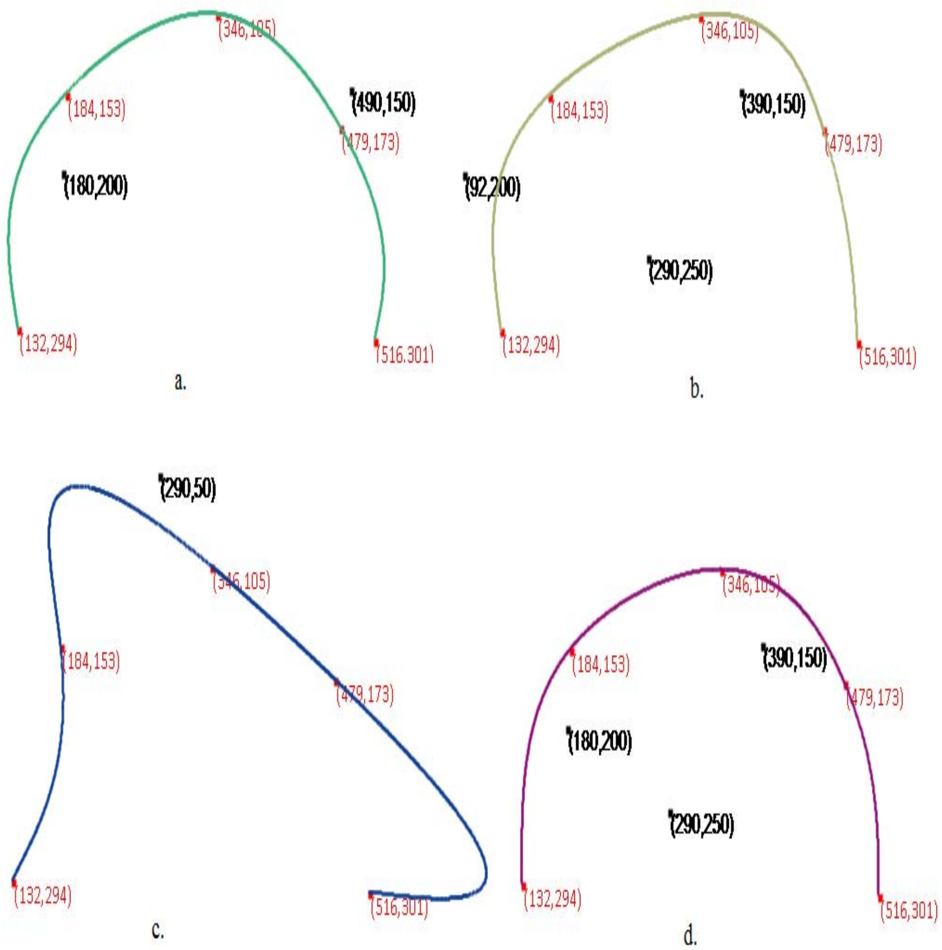


Figure (8): (a) Design curve using five data points and three guide points G_1=(180,200), G_2=(-290,50), G_3=(490,150), (b) Design curve using five data points and three guide points G_1=(92,200), G_2=(290,250), G_3=(390,150), (c) Design curve using five data points and three guide points G_1=(-180,-200), G_2=(290,50), G_3=(-490,-150), (d) Design curve using five data points and three guide points G_1=(180,200), G_2=(290,250), G_3=(390,150).

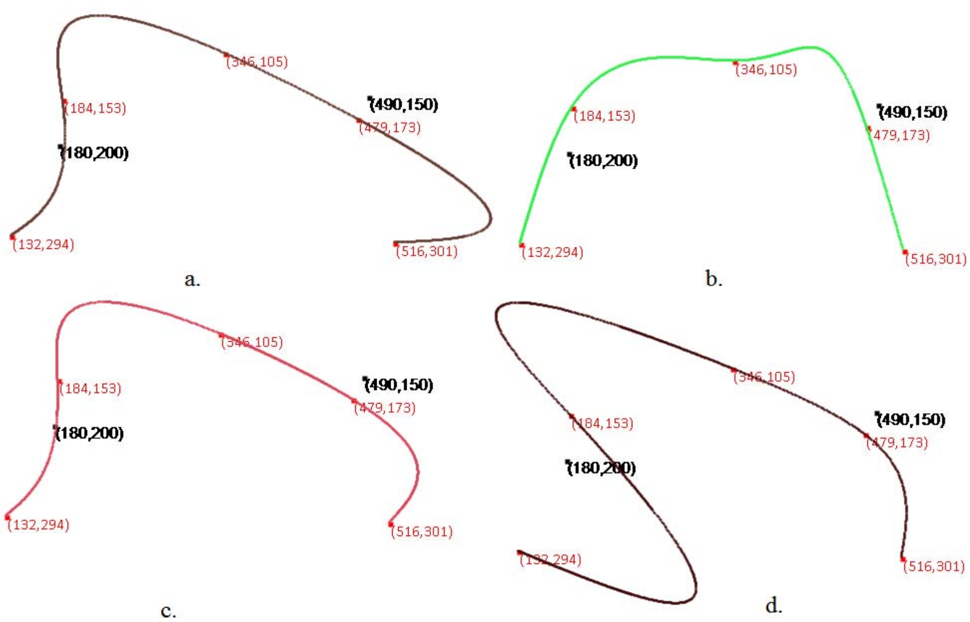


Figure (9): (a) Design a curve using five data points and three guide points plus three weight factors G_1=(180,200), G_2=(-290,-50), G_3=(490,150), $\hat{W}_1$=-0.4, $\hat{W}_2$= 0.8, $\hat{W}_3$= 0.6, (b) Design a curve using five data points and three guide points plus three weight factors G_1=(180,200), G_2=(-290,-50), G_3=(490,150), $\hat{W}_3$=0.8, $\hat{W}_2$= -0.4 $\hat{W}_3$= 0.6, (c) Design a curve using five data points and three guide points plus three weight factors G_1=(180,200), G_2=(-290,-50), G_3=(490,150), $\hat{W}_1$=9, $\hat{W}_2$= -6, $\hat{W}_3$= -2, (d) Design a curve using five data points and three guide

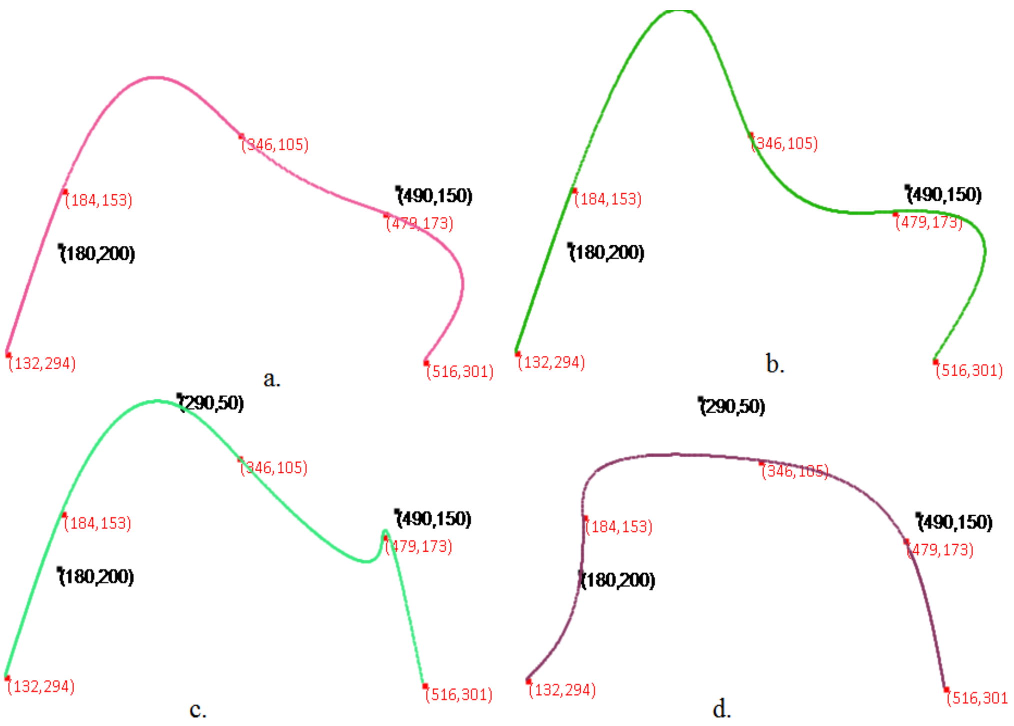


Figure (10): (a) Design a curve using five data points and three guide points plus three weight factors G1=(180,200), G2=(-290,-50), G3=(490,150), $\hat{W}_1$=-25, $\hat{W}_2$= -25, $\hat{W}_3$= 51, (b) Design a curve using five data points and three guide points plus three weight factors G1=(180,200), G2=(-290,-50), G3=(490,150), $\hat{W}_1$=-24, $\hat{W}_2$= -26 , $\hat{W}_3$= 51, (c) Design a curve using five data points and three guide points plus three weight factors G1=(180,200), G2=(-290,-50), G3=(490,150), $\hat{W}_1$=1, $\hat{W}_2$= 0, $\hat{W}_3$= 0, (d) Design a curve using five data points and three guide points plus three weight factors

CONCLUSION

Designing curve depending on extra parameters in addition to the origonal data points will increase the complexity of the curve and give the designer higher flexibilty to easily change the shape of the curve to the desired one without changing the origonal data points. In this paper the area parametarization model on multiple cases, where a new models have been added to calculate the values of t_i in the case of using only one guide at [1], the added models used the negative values of the guide to compute the parametric rather than usin, the only positive ones. In the other cases the paper developed the area parametarization using two and three guide points with positive or negative values or both of them. The paper also used two and three wieghting factors in addition to the guides also with the use of positive or negaive values or both of them for the guides and wieghting factors. When the $\sum_{i=1}^{n} \hat{\mathrm{W}}_{\mathbf{i}} = 1$the design is nicely nested more than the case of $\sum_{i=1}^{n} \hat{\mathrm{W}}_{\mathbf{i}} \neq 1$. The contribution of this paper allow the designer to easly generate curves complex design that not easy for counterfieters to recreate without knowing the extra parameters that are secretly assigned by the designer, even in the case of knowing the original data points.

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