Specific Chaotic System and its Implementation in Robotic Field

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ABSTRACT

This paper presented a three-dimensional continuous autonomous chaotic system, modified from the Arnold system which has a single product term in each equation of the system; therefore it's different from Arnold system and other existing system. This system has nine parameter (i.e. A,B,C,D,E, and F) which gives a more flexibility in generation chaotic behavior throughout these parameters. Basic properties of the presented system were analyzed by means of Lyapunov exponent spectrum, Poincar'e mapping, fractal dimension, power spectrum and chaotic behaviors. Theoretical and numerical analysis prove that the system shows chaotic behavior. Furthermore the cited chaotic system was implemented in robotics field for coverage area purposes, where it's used to generate chaotic motion for mobile robot that's guarantee of scanning the whole connected workspace as an example of advantage of this system.
Keywords: Arnold equation, chaos, mobile robot.

اقتراح نظام فوضوي محدد واستخدامه في مجال الروبوت

الخلاصة

البحث الحالي يقدم مقترحا لنظام فوضوي ثلاثي الابعاد مستمر ومستقل ذاتيا تم اشتقاقه من نظام ارنولد حيث طور النظام القديم ليحتوي على تسع برامترات بالاضافة الى حد واحد في كل معادلة ناتج من حاصل ضرب متغيرين وبذلك فأنه يختلف عن نظام ارنولد الأصلي او أي نظام اخر معروف حاليا، كما انه يتيح امكانية أكبر لتوليد سلوك فوضوي من خلال التحكم بهذه البرامترات. الخصائص الاساسية لهذا للنظام تم تحليلها بالاستعانة بالاس الطيفي لـ ليابونوف، البعد الكسري، طيف الطاقة والسلوك الفوضوي. اكدت نتائج التحليل النظري والعددي ان النظام المقترح يظهر سلوك فوضوي، وقد وضف هذا السلوك في مجال الروبوتات لتحسين تغطية المساحات (مثلا في عملية تنظيف الغرف او البحث عن متفجرات في منطقة معينة) عن طريق توليد مسار فوضوي يتبعه الروبوت يضمن مرور الروبوت بكافة المساحة المستهدفة.

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List of Symbol

Symbol	Definition
A	System parameter,(1/s)
A_t	Total Work Space (m2)
A_u	Coverage Area (m2)
B,C,D,E, and F	System parameters,(1/s)
c	MR Platform Reference Point
D_L	Fraction Dimension
E_1,E_2,E_3, and E_4	System Equilibrium Points
J	Jacobian Matrix
K	Performance index
L_1,L_2, and L_3	Lyapunov Rotes
2ℓ	Distance between Driving Wheels (m)
r	Wheels Radius (m)
[X,Y,Z]	World Coordinates System (m)
$\dot{X}$ and $\dot{Y}$	Mobile Robot Velocity in X and Y Direction Respectively
θ	Mobile Robot Orientation (rad)
λ1, λ2, and λ3	Eigenvalues of system
ν	Mobile Robot Linear Velocity (m/s)
ω	Mobile Robot Angular Velocity (rad/s)

List of Abbreviation

Abbreviation	Definition
MR	Mobile Robot
WMR	Wheeled Mobile Robot

INTRODUCTION

Chaos is found to be useful or has great potential application in many disciplines. Recently, it has been noticed that purposefully creating chaos can be a key issue in many technological application such as communication, encryption, information storage, mobile robot, ect. Since Lorenz found the first chaotic attractor in a three-dimensional (3-D) autonomous system in 1963[1]. Lorenz system has become a paradigm for chaos research, and many new Lorenz-like chaos systems have been constructed [2-6]. Arnold equations describe chaotic behaviors of noncompressive perfect fluids on 3-D space [7].

In finding of a new system, one can construct and determine the system parameter values such that the system becomes chaotic following some basic ideas of chaotification [8], namely

1- It is dissipative

2- There exist some unstable equlibria, especially some saddle points.

3- There are some cross-product terms, so there are dynamical influences between different variables.

4- The system orbits are all bounded.

This motivates the present study on the problem of generating new chaotic equations. Under these guidelines, though not sufficient, a new chaotic system is generated by modifying from Arnold system.

A robot following a chaotic path, generated by the Arnold's equation, was introduced for the first time by Y.Nakamara and A. Sekiguchi [9]. The main objective in exploiting chaotic signals for an autonomous mobile is to increase and to take advantage of coverage are desirable for many applications such as robots designed for scanning of unknown workspaces with borders and barriers of unknown shape, as in patrol or cleaning purpose. In this paper the cited chaotic system implemented to generate chaotic path, for scanning unknown workspace with borders and barriers and it will be proved that the present model is better than classical Arnold's model by 7.67% at period 7000 (sec) throughout performance index K, which is represent the ratio of areas that the trajectory pass through over the total working area. Not only large coverage areas are desirable in certain applications of mobile robot, but also coverage speed and eventually the shortest path traveled by the robot. The complexity of chaotic motion is increased by the multiple reflections of the robot trajectory on the workspace boundaries and obstacles.

The New Chaotic System and Its Properties.

The new chaotic system introduced in this paper is described as the following autonomy differential equations:

$$\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{pmatrix} = \begin{pmatrix} A\sin z & C\cos y & D\cos z\sin y\\ B\sin x & A\cos z & E\cos x\sin z\\ C\sin y & B\cos x & F\cos y\sin x\end{pmatrix} \quad \ldots (1)$$

Where

A, B, C, D, E, and F are constant parameters of the system. Taken A=0.5; B=0.25; C=0.135; D = 2; E = 0.85; F = 1.5, system (1) is chaotic and each equation contain a single quadratic cross-product term which different form the Arnold system, and therefore it is a new Arnold- system like but not equivalent chaotic behavior where it is known that the Arnold equation shows periodic motion when one of the constant (i.e. A, B, and C) , for example C, is 0 or small and show chaotic motion when C is large [10], this behavior does not hold any more in the new chaotic system.

Note: system parameters have been chosen throughout trial and error.

System Equilibrium

The equilibria of system (1) are found by evaluating the three equations $\dot{x} = \dot{y} = \dot{z} = 0$, and it's very complicated so that using numerical evaluation by means of **NEWTON METHOD FOR A SYSTEM OF NONLINEAR EQUATION** [11] is very helpful to reach approximated equilibrium points, which are respectively described as follows:

E_1 (-34.4,-15.8,52.8); E_2 (-18.7,-84.6,-8.9);E_3 (-34.4,-15.8,90.5); E_4 (-34.4,21.9,90.5).

The Jacobian matrix J of system (1) is:

$$\boldsymbol{J} = \begin{bmatrix} \frac{\partial f_1(X)}{\partial x} & \frac{\partial f_1(X)}{\partial y} & \frac{\partial f_1(X)}{\partial z}\\ \frac{\partial f_2(X)}{\partial x} & \frac{\partial f_2(X)}{\partial y} & \frac{\partial f_2(X)}{\partial z}\\ \frac{\partial f_3(X)}{\partial x} & \frac{\partial f_3(X)}{\partial y} & \frac{\partial f_3(X)}{\partial z}\end{bmatrix} \quad \ldots(2)$$

$$J = \begin{bmatrix} 0 & -0.135\sin y + 2\cos z\cos y & 0.5\cos z - 2\sin y\sin z\\ 0.25\cos x - 0.85\sin z\sin x & 0 & -0.5\sin z + 0.85\cos x\cos z\\ -0.25\sin x + 1.5\cos y\cos x & 0.135\cos y - 1.5\sin y\sin x & 0\end{bmatrix} \ldots(3)$$

For equilibrium point E1(-34.4,-15.8,52.8), the Jacobain matrix has the following results:

$$J_{E_1} = \begin{bmatrix} 0 & 1.623 & -0.515 \\ -0.732 & 0 & 0.4 \\ 1.514 & -0.113 & 0 \end{bmatrix} \qquad \ldots(4)$$

To gain it's eigenvalues, let $|J_{E_1} - \lambda I| = 0$, applying the same procedure for other equilibrium points, then the eigenvalues that corresponding to each equilibrium points are shown in table (1):

Table (1) Eigenvalues corresponding to the system equilibrium points

	λ_1	λ_2	λ_3
E_1	0.6459	$-0.323 + 1.1887i$	$-0.323 - 1.1887i$
E_2	1.5991	$-0.7595 + 0.4733i$	$-0.7595 - 0.4733i$
E_3	0.6459	$-0.323 + 1.1887i$	$-0.323 - 1.1887i$
E_4	0.6459	$-0.323 + 1.1887i$	$-0.323 - 1.1887i$

For each of the four equilibrium points E_1, E_2, E_3, and E_4 the results show that $\lambda_1$is positive real number, λ_2 and λ_3 become a pair of complex conjugate eigenvalues with negative real parts. Therefore, equilibrium points E_1, E_2, E_3, and E_4 are all saddle-focus points, so, these equilibrium points are all unstable.

Observation of Chaotic and Complex Dynamics

The initial values of the system are selected as (3.5, 4, 0). Using MATLAB program, the numerical simulation have been completed. This nonlinear system exhibits the complex and aboundant chaotic dynamics behaviors, Figure (1) show trajectories of the proposed chaotic system. The waveforms of x(t) in time domains are shown in Figure (2). The waveforms of x(t) is aperiodic.

The Poincare section for three dimensional trajectory shows a certain type of organization can be an infinity of points (in contrast with the finite number of points for the periodic and quasi-periodic case) irregularly scattering of the proposed system is shown in Figure (3), its structure becomes better and better defined in time after the accumulation of points.

As is well known, the Lyapunov exponents measure the exponential rates of divergence or convergence of nearby trajectories in phase space, according to the numerical as well as theoretical analysis, the largest value of position Lyapunov exponents of this chaotic system is obtained as L_1= 0.4132. It is related to the expanding nature of different direction in phase space.

Another one Lyapunov exponent is L_2=0. It is related to the critical nature of different direction in phase space.

While negative Lyapunov exponent is L_3=-0.421.It is related to the contracting nature of different direction in phase space. When x(0)=3.5,y(0)=4, and z(0)=0, Lyapunov spectrum of the proposed system is shown in Figure (4). It can be seen that the three Lyapunov exponents have tended to be a fixed constant along with the time evolution as mention above.

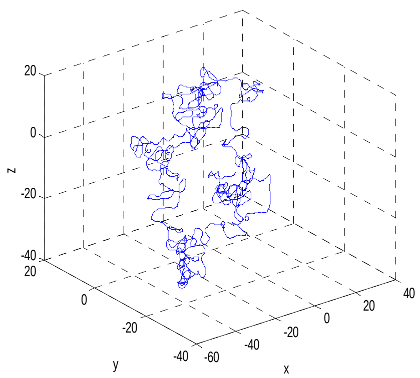
The lyapunov dimension of chaos behavior of this nonlinear system is of fraction dimension, it is described as

$$D_L = j + \frac{1}{|L_{j+1}|}\sum_{i=1}^{j} L_i = 2.974 \qquad \ldots(5)$$

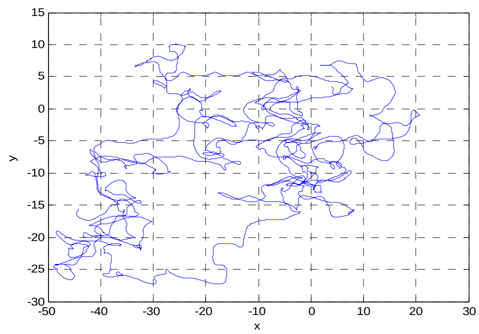
Because the system's maximum lyapunov exponent is bigger than zero, and the lyapunov dimension is fractal dimension, it can be determined that the system is a chaotic system.

Spectrum map and time domain map.

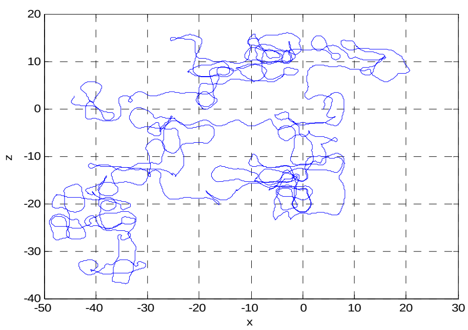
In order to discriminate between multiple periodic motions that can show also a complicated behavior and a chaotic motion, the power spectrum of the proposed system (1) is also studied, and it is exhibits a continuous broadband feature as shown in Figure (5). Figure (6) shows that the evaluation of the chaos trajectories is very sensitive to initial conditions [12]. The initial values of the system are set to $[4, 3.5, 0\,]^T$ for (a) in the system in Figure (6) and $[4.000001, 3.5,0\,]^T$ for (b) in the system in Figure (6). Figures (7-9) show system sensitivity for parameters (D, E, and F) respectively, where all parameters were changed by (0.000001). Figure (10) shows the cross-correlation for two waveforms X and Z, while Figure (11) shows the autocorrelation for waveform Z, cross and auto correlation results shows that the new system is aperiodic. While Figures (12 and 13) shows the autocorrelation and cross-correlation respectively for Arnold system and for the same parameter of A, B, and C were used by the proposed system.



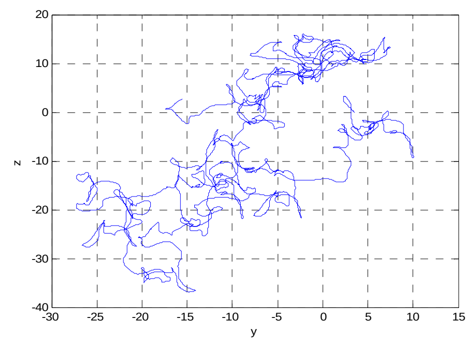
(a) x-y-z plane



(b) x-y plane



(c) x-z plane



(d) y-z plane

Figure (1) Phase portrait of the new system

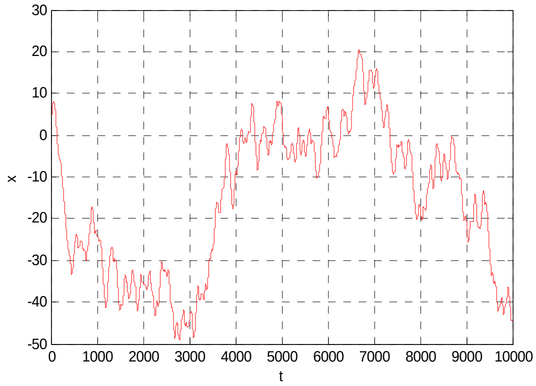


Figure (2) Waveform of the proposed system

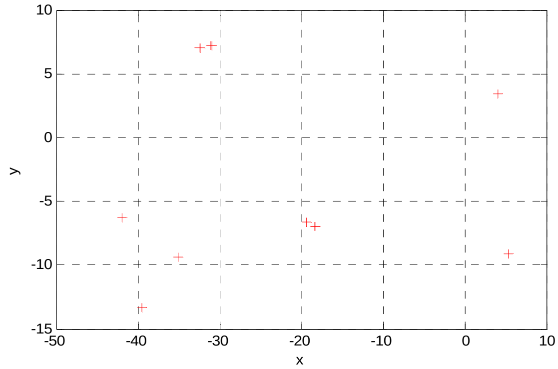


Figure (3) The Poincare map of x-y plane

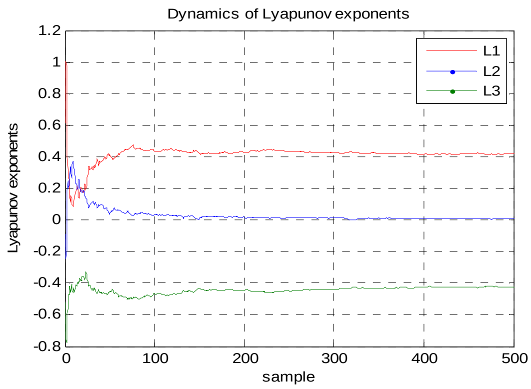


Figure (4) Lyapunov exponent spectrum of the proposed system

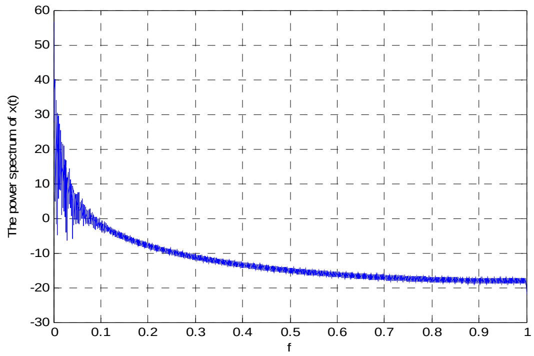


Figure (5) Power spectrum of x(t)

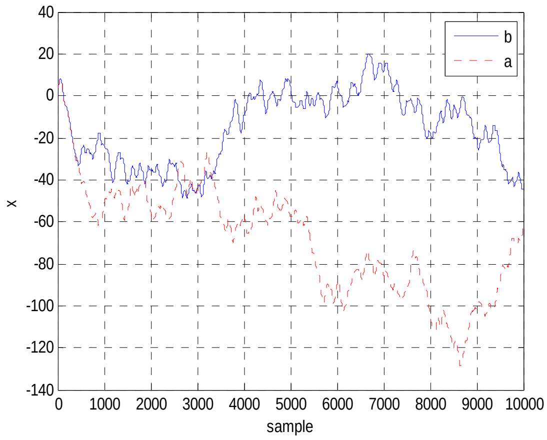


Figure (6) Sensitivity of the system for initial condition

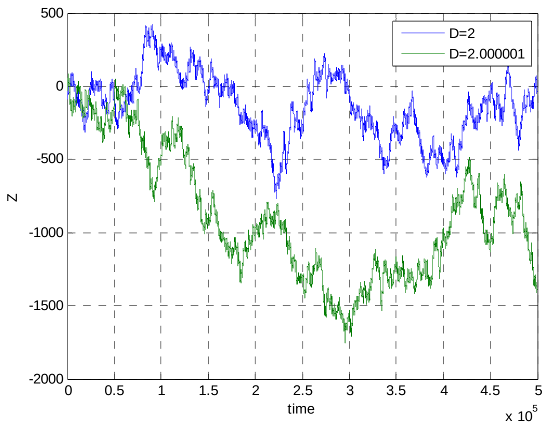


Figure (7) Sensitivity of the system for parameter (D)

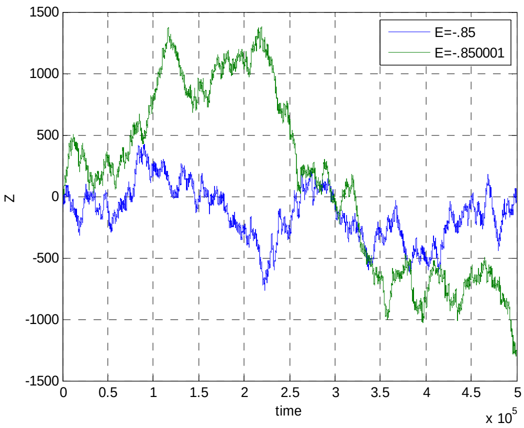


Figure (8) Sensitivity of the system for parameter (E)

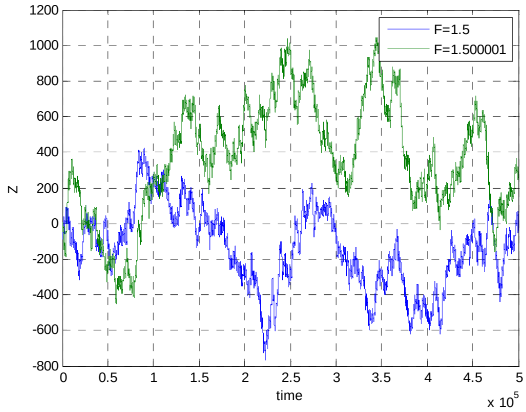


Figure (9) Sensitivity of the system for parameter (E)

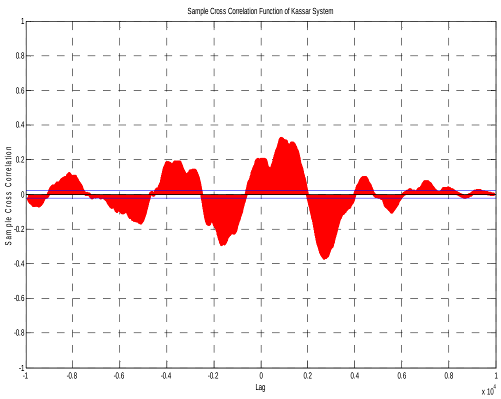


Figure (10) Cross-correlation for time series x and z

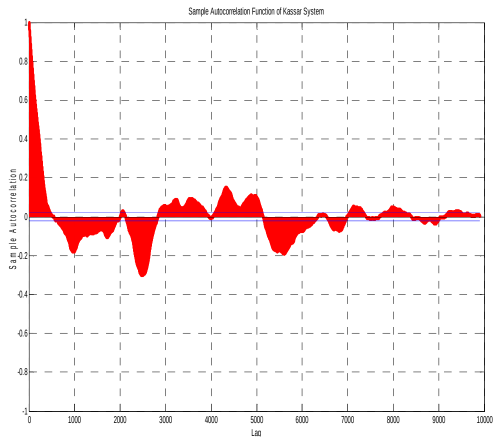


Figure (11) Auto-Correlation for time series (z)

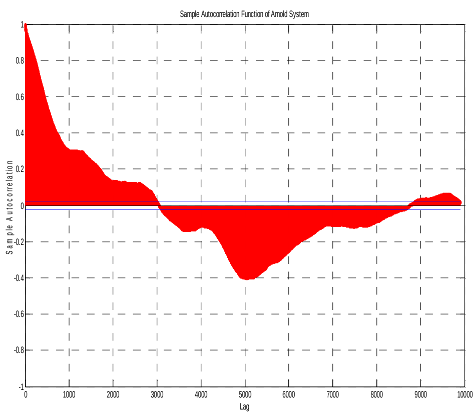


Figure (12) Auto-Correlation for time series (z) of Arnold System

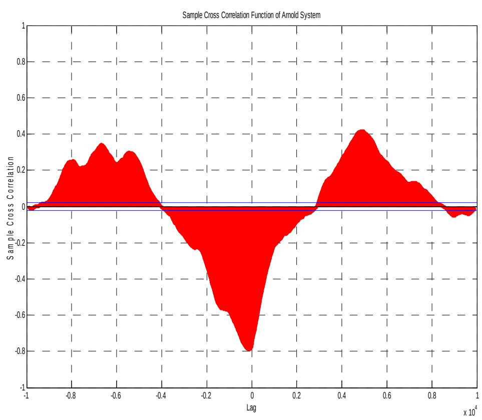


Figure (13) Cross-Correlation for time series x and z of Arnold System

Chaotic Mobile Robot

Mobile Robot

For the mathematical model it is assumed a two-wheeled mobile robot as shown in Figure 15. Let the linear velocity of the robot ν [m/s] and the angular velocity ω [rad/s] be the inputs to the system. The state equation of the mobile robot is written a follows:

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \nu \\ \omega \end{pmatrix} \quad \ldots(6)$$

Where

(X[m], Y[m]) is the position of the robot and θ [rad] is the angle rotation of the robot.

Integration of the proposed chaotic system

After integration the system (1) into the controller of the mobile robot equation (6), the state equation of the mobile robot becomes [9]:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{X} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} A\,sin\,z + C\cos y + D\cos z * \sin y \\ B\,sin\,x + A\cos z + E\cos x * \sin z \\ C\,sin\,y + B\cos x + F\cos y * \sin x \\ \nu\cos z \\ \nu\sin z \end{bmatrix} \quad \ldots(7)$$

Equation (7) includes steering system of the mobile robot. The mobile robot moves with a constant velocity and being steered by the variable z of the proposed chaotic system. In Equation (7), it is assumed that the robot moves in a smooth state space contains boundaries. However, real moves in spaces with boundaries like walls or surfaces of obstacles. To solve this problem, we use the mirror mapping [9]. In the real space, the mobile robot moves as if it is reflected by the boundary. The resultant trajectory of the mobile robot is shown in Figure (15).

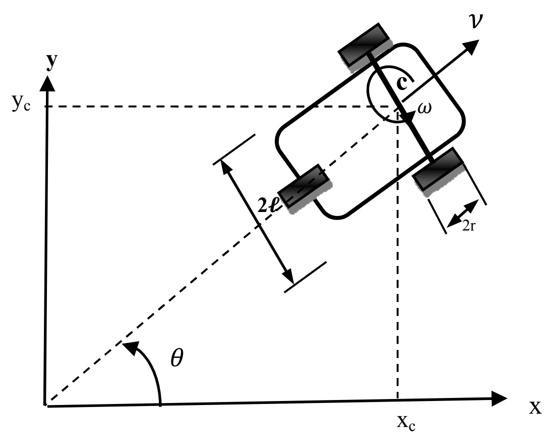


Figure (14) Geometry of robot move in x-y plane

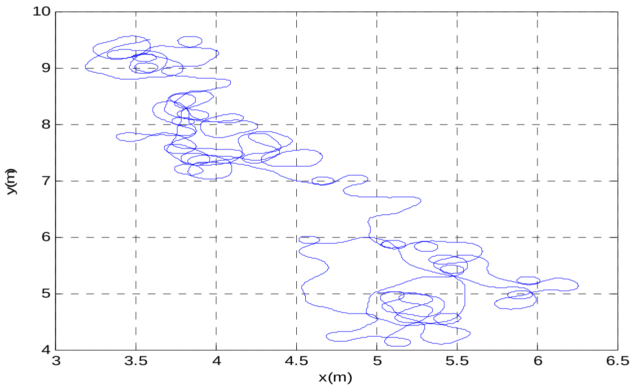


Figure (15)Trajectory of the mobile in x-y plane (v=0.1(m/s))

Evaluation Criteria

The evaluation criteria are set according to the application purpose. Since the robot is used in wandering around in the area of no maps, the chaotic trajectory should cover the entire areas of patrolling as much as possible. The best criteria to discover performance of chaotic mobile robot is used the performance index K, which is represent the ratio of areas that the trajectory passes through or used space(A_u), over the total working area (A_t).

$$K = \frac{A_u}{A_t} \quad \dots (8)$$

The used area A_u and the total area A_t can be calculated by the following algorithm:

1- Dividing the hole area into (N*M) pixels (i.e. A_t).

2- Using image processing to assign to 255 for all white pixels and 0 for all black pixels which pass through the trajectory of the robot.

3- Count the number of 255 pixels W (i.e. white pixel)

4- Then used area $A_u = A_t$ - W = N*M – W.

Numerical Analysis of the Behavior of the Robot

Investigation by numerical analysis has been achieved to show whether the mobile robot with the proposed controller actually behaves in a chaotic manner. Examples of trajectories of the robot are obtained by applying the mirror mapping on three type of environment as shown in Figures (16-19). Where the chaotic mobile robot run at following condition:

Linear velocity: $v = 0.2\ m/s$

Initial states: $[\,x\ y\ z\ X\ Y]^T = [\,4\ \ 3.5\ \ 0\ \ 0.5\ 0.5]^T$

The proposed system parameters: A=0.5, B=0.25, C=0.135, D= 2, E=0.85, F=1.5.

Session time: 4000 sec

Evaluating the chaotic controller by means of performance index K is shown in Figure 20.

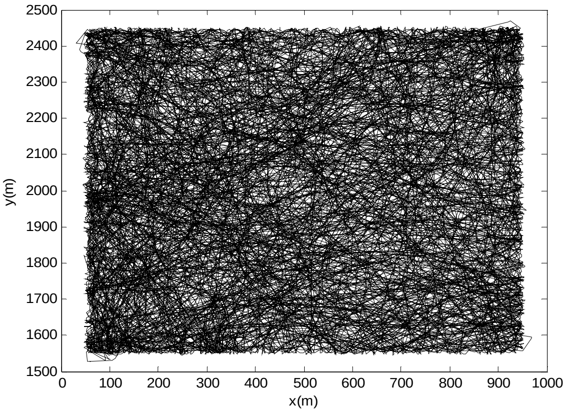


Figure (16) Simulation trajectory of the mobile robot in the x-y plane (Environment 1)

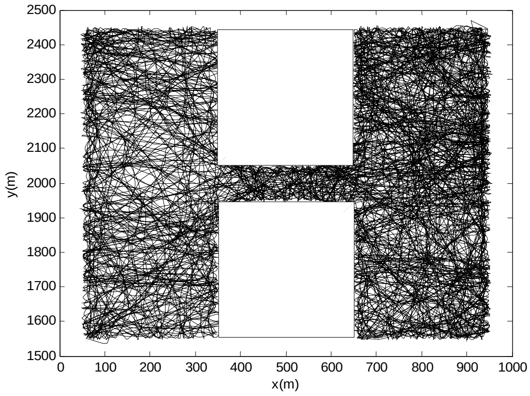


Figure (17) Simulation trajectory of the mobile robot is the x-y plane (Environment 2

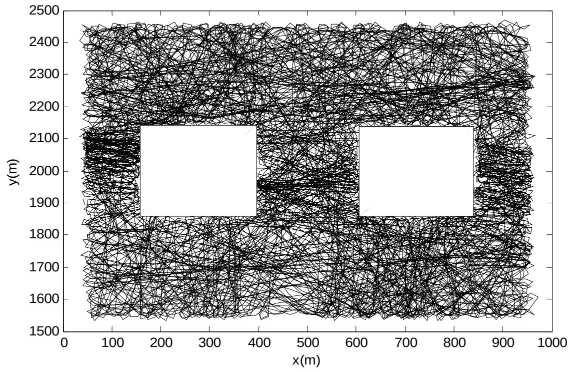


Figure (18) Simulation trajectory of the mobile robot in the x-y plane (Environment

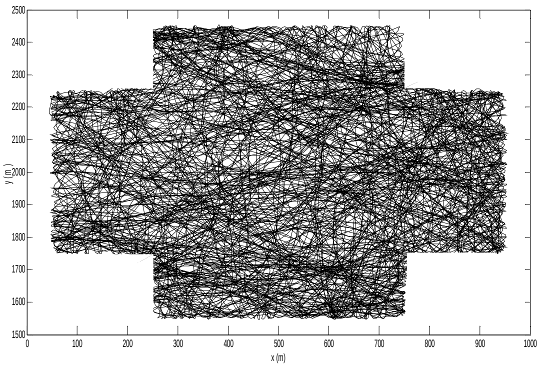


Figure (19) Simulation trajectory of the mobile robot in the x-y plane (Environment 4).

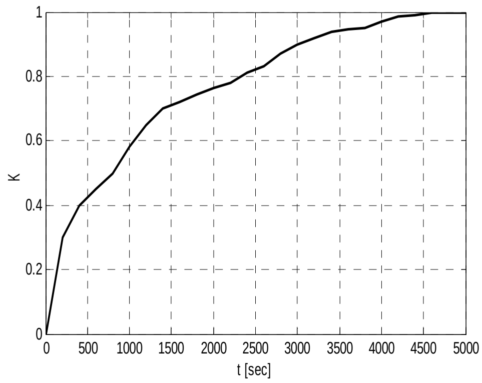


Figure (20) Performance index K of controller for environment (1)

Proposed system versus Arnold system

To prove the enhancement of coverage area by means of the proposed system that developed form Arnold system, a comparison has been done between these systems. Where the two systems is tested to coverage environment (1000mm x 1000mm) and the chaotic mobile robot ran with the following conditions:

Linear velocity: $v = 0.15\ (\frac{m}{s})$

Initial states: $[\,x\ y\ z\ X\ Y]^T = [\,4\ \ 3.5\ \ 0\ \ 0.5\ 0.5]^T$

Periods: [250 500 1000 2000 3000 4000 5000 6000 7000] (sec)

Arnold system coefficients [13]: A=0.27, B=0.135, C=0.135

The proposed system coefficients: A=0.5, B=0.25, C=0.135,D= 2, E=0.85, F=1.5.

Figures 21 and 22 shows the samples of simulation result of the proposed system and Arnold chaotic system respectively, where the simulation period was 7000 (sec). The rate (K) of coverage area results proves the progress of the proposed system on Arnold system, where the ratio (K) for the proposed and Arnold system was 83.66% and 75.99% respectively. While Figure 24 shows the rate of coverage area versus time in second for both chaotic system and it's clearly appear the enhancement in the ratio of coverage area for the proposed system.

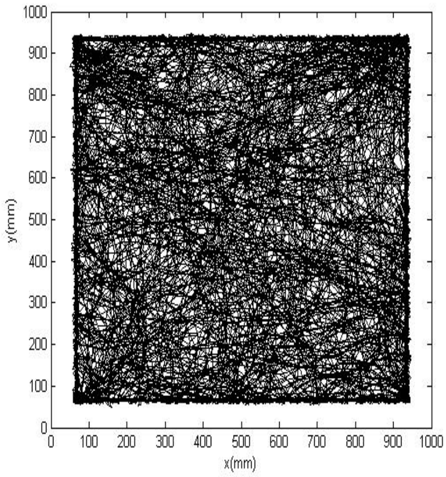


Figure (21) Simulation results of the chaotic mobile robot based on the Proposed system

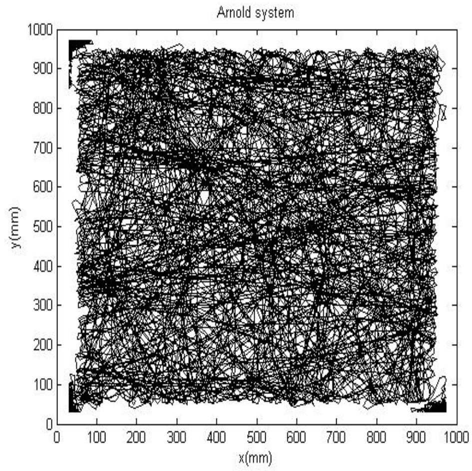


Figure (22) Simulation results of the mobile robot based on Arnold system

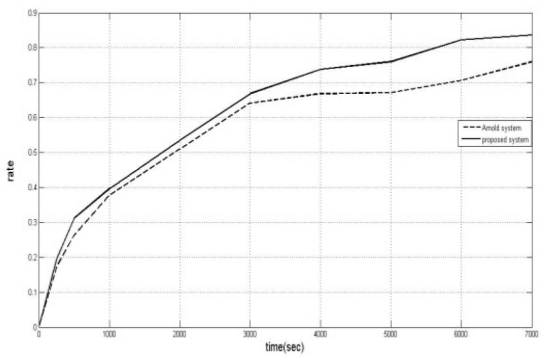


Figure (23) Rate of coverage area

Discussion and Conclusions

This paper consists of two main sections. In the first one, it has been reported and analyzed a new chaotic system, which is developed from Arnold chaotic system to contain nine parameters. The proposed system was tested by means of several criteria; system equilibria, Poincar' mapping, Lyapunov exponent, power spectrum, sensitivity for both initial condition and system parameters. All these results show that the proposed system is a chaotic system. Furthermore a comparison has been made between the proposed system and Arnold chaotic system throughout auto and cross correlation and the results show that the proposed system exhibit poor correlation more than Arnold chaotic system, which mean the proposed system have strong aperiodic behavior (i.e. chaotic behavior) [14].

The second section described the implementation of the proposed chaotic system in robotics field. Where a controller based on this system was designed to generate a chaotic trajectory for mobile robot that guarantees of coverage any unknown work space. Chaotic trajectory that generated by the proposed chaotic controller was evaluated from point of view of performance index K, which show the rate of coverage area with time of scanning operation. Four type of environment was design to test the ability of the chaotic controller from coverage whole work space. Simulation results (i.e. Figures (16-19)) show that the successes of chaotic robot from passing throughout whole connected area, where the secession time equal to 4000 second. While Figure (20) shows the evaluating of the chaotic controller using performance index K, where the work space was environment 1 (i.e. square area of 1m x 1m). Performance index K shows the successes of chaotic robot from coverage the work space in 100% by 4500 second.

To prove that the proposed chaotic system is suitable for design chaotic controller for mobile robot more than Arnold chaotic system, a comparison between both systems has been made for coverage a square work space (i.e. environment 1). Performance index K in Figure (23) shows the enhancement in coverage area by 7.67% at time 4000 second when used the proposed chaotic system.

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