

An Alternative Approach for Angle of Arrival Estimation

Dr. Bassim Sayed Mohammed

Electrical Engineering Departmen, University of Technology/Baghdad.

E-mail: bassim_sayed@yahoo.com

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ABSTRACT:

The Multiple Signal Classification (MUSIC) algorithm is the most popular algorithm to estimate the Angle of Arrival (AOA) of the received signals. This algorithm is based on the calculation of thermal noise Eigen values and their corresponding Eigen vectors and then using them in a matrix form in the MUSIC direction finding (DF) formula. All researches in this field assumed isotropic sensors for array antennas. But for practical application in V/UHF, a practical antenna is to be used like half-wave dipole or any type of wired antenna. When MUSIC algorithm is tested with a well-known half-wave dipole which has a directive pattern in E-plane, a false reading is raised from the two angles coincide with the dipole element axis. It is found that the MUSIC algorithm does not take into consideration the nulls that result from the pattern multiplication between array factor AF (θ) and element pattern $f(\theta)$. This paper suggests an alternative approach for angle of arrival estimation. A new DF formula is derived. The suggested approach is based on the use of modified array processor in conjunction with minimum noise variance constrain algorithm as an angle of arrival estimator in spite of interference canceller as it is used for. The results show that the alternative approach gives a quite good result without any false reading or any degradation in the performance of AOA estimation.

Keywords; Angle of Arrival Estimation; MUSIC; Array Antenna System.

طريقه بديله لتخمين زاوية الوصول باستخدام مصفوفة الهوائيات

الخلاصة:

خوارزمية التصنيف المتعدد للاشارة والتي مختصرها (Music Algorithm) هي من اشهر الطرق الحديثه في تخمين زاوية الوصول. تعتمد هذه الطريقه على حساب قيم (الايكن وقيم متجه الايكن) المرتبطه بمصفوفة الاشارات المستلمه وبعدها يتم استخدام المتجهات الخاصه بقيم الضوضاء الحراريه المتولده في قنوات الاستلام وفق معادله محدهه لاجداد قيمة زاوية الوصول. لاحظنا ان هذه الخوارزميه تعمل بشكل جيد وبدون اخطاء اذا فرضنا ان مصفوفة الهوائيات تتشكل من عناصر بث (isotropic) وعندما تم فحصها مع اشهر هوائي عملي وهو (Half wave dipole) تبين انها تعطي قراءات خاطئه من الزوايا التي تتطابق مع الازهار الموجوده في شكل نموذج الاشعاع للهوائي الدايبول علما ان جميع البحوث المنشوره في هذا المجال تقترض ان مصفوفة الهوائيات تتكون من مجموعه من الهوائيات نوع (Isotropic). ولغرض حل هذه المشكله تم اقتراح استخدام طريقه بديله تتكون من منظومه هوائيات متكيفه باستخدام الهوائي (Half wave dipole) لتخمين زاوية الاتجاه بعد عمل نموذج رياضي له واشتقاق

معادله جديده لتخمين الاتجاه. بعد فحص النموذج البديل وجدنا انه تغلب على مشكلة القراءات الخاطئة التي برزت في طريقة (MUSIC) بشكل نهائي مع اعطاء نتائج تتطابق ودقة نتائج خوارزمية (MUSIC).

INTRUDUCTION

Over the last decade, the smart antenna system took a great deal of in radar navigation and communication systems in all frequency bands. This type of array antenna used to improve the signal to noise plus interference ratio (SNIR) at the output system by using beam-former network to create nulls in the array pattern in front of the directions of all interference sources. The Angle of Arrival (AOA) estimation has been studied and verified by many researches. Many techniques were used depending on the amplitude compression (Watson Watt) technique [1] or on the difference of the time of arrival of the received signal to a set of antennas distributed in a certain shape and on the phase difference between signals from antenna sets used in AOA system [2, 3,4]. The accuracy of estimating angle of arrival is found depending on many factors, like the size of antenna system aperture ,the operating frequency band, the propagating manner of electromagnetic wave at this band of frequency and finally on the site of operation. The MUSIC algorithm is considered as a most recent popular algorithm which is used for estimating the (AOA) of received signals [5, 6, 7].

The adaptive antenna system shown in Fig.(1) formed from a set of N-sensors followed by front end down converters to convert the frequency band of arrived RF signal to IF frequency and then fed to a digital signal processing stage through A/D converter. In the processor the optimum adapted weight vector W is calculated according to the applied algorithm. Then the received signal vector X by all array sensors is multiplied by the optimum weight vector W and then summed to form the scalar output $y(t)$. The main function of the processor and the applied algorithm is to calculate an optimum weight vector in the way of all feeding channels in order to make a readjustment of the beam pattern to give a maximum directivity in front of all received signals rather than the other directions (i.e. work as spatial optimum matched filter).

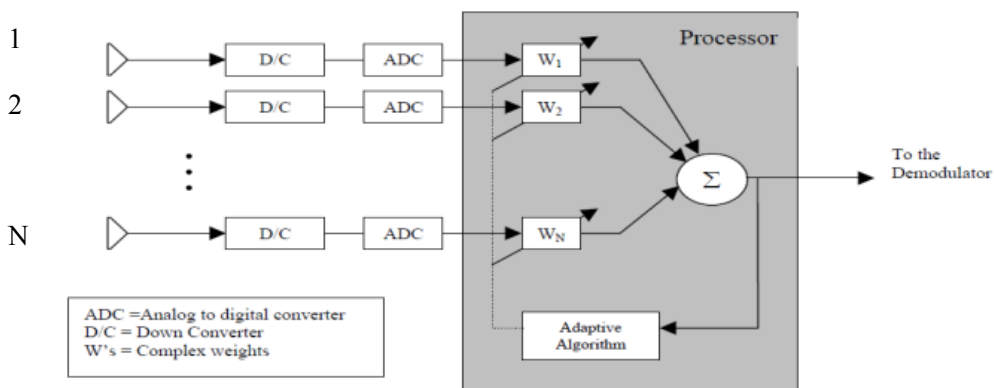


Figure (1): Adapted Phased Array Receiving System

Mathematical Formulation

The k snapshot received signals vector $\mathbf{X}(k)$ is given by

$$\mathbf{X}(k) = \sum_{i=0}^D \mathbf{X}_{d(i)}(k) + \mathbf{X}_n(k) \quad \dots (1)$$

where

$\mathbf{X}_{d(i)}(k)$, \mathbf{X}_n are the i th received signal and thermal noise components of size $(N \times 1)$ vectors respectively and D is the number of received signals under angle of arrival estimating process. If the received signal is incident from angle θ_d , then received signal in the j th element is

$$x_{aj(i)}(k) = A_d e^{j[\omega_0 k + \Psi_{d(i)}]} \quad \dots (2)$$

In vector form as

$$\mathbf{X}_{d(i)}(k) = A_{d(i)} e^{j[\omega_0 k + \Psi_{d(i)}]} \mathbf{U}_{d(i)} \quad \dots (3)$$

where A_d is the signal amplitude, ω_0 is the center angular frequency, Ψ_d is arbitrary carrier phase angle distributed on $(0, 2\pi)$, and \mathbf{U}_d is a desired signal array vector given by.

$$\mathbf{U}_{d(i)} = [g_z(\theta_{d(i)}) e^{-jz\beta d \cos(\theta_{d(i)})}]^T \quad z = 0, 1, \dots, \quad \dots (4)$$

Where $g_z(\theta_{d(i)})$ the element field pattern in the direction of i th desired signal and T denotes the vector transpose.

The thermal noise voltages of the array elements are considered to be independent random signals with zero mean and variance σ^2 . The array thermal noise voltage components are assumed to be mutually uncorrelated since they are random signals and so they can be expressed as

$$\mathbf{X}_n(k) = [n_1(k), n_2(k), \dots, n_N(k)]^T \quad \dots (5)$$

The adaptive array output signal can be written as [1]

$$y(k) = \sum_{j=1}^N w_j x_j(k) \quad \dots (6)$$

Eq. (6) in a vector form is

$$y(k) = \mathbf{W}^T \mathbf{X} = \mathbf{X}^T \mathbf{W} \quad \dots (7)$$

Where

(\mathbf{W}) is a weight vector notation and \mathbf{X} is the received signals vector and they are given by

$$\mathbf{W} = [w_1, w_2, \dots, w_N]^T \quad \dots (8)$$

$$\mathbf{X} = [x_1, x_2, \dots, x_N]^T \quad \dots (9)$$

Signal and noise in adaptive array systems may be described in terms of their statistical properties, this enables the system to be evaluated in terms of its statistical average $E[.]$. The evaluation of average leads directly to interested quantities related to the second

statistical moment such as covariance matrix which is closely related to correlation matrix for stationary signals.

The covariance matrix of a received signal vector of array antenna is defined as

$$Cov[\mathbf{X}\mathbf{X}] = E([\mathbf{X} - E(\mathbf{X})][\mathbf{X} - E(\mathbf{X})]^T) \quad \dots (10)$$

Where

E is statistical expected value. Since $\mathbf{X}(t)$ is zero mean stationary process, then

$$Cov[\mathbf{X}\mathbf{X}] = E[\mathbf{X}^*\mathbf{X}^T] = \mathbf{R}_{xx} \quad \dots (11)$$

Where

\mathbf{R}_{xx} is $(N \times N)$ autocorrelation matrix of a received signal vector $\mathbf{X}(t)$ and for N -element array antenna it may be written in the following form [5]

$$\mathbf{R}_{xx} = E[\mathbf{X}^*\mathbf{X}^T] = \overline{[\mathbf{X}^*\mathbf{X}^T]} \quad \dots (12)$$

The autocorrelation matrix \mathbf{R}_{xx} is Hermitian matrix (i.e. $\mathbf{R}_{xx} = \mathbf{R}_{xx}^{*T}$).

Eigen Vector Decomposition for Covariance Matrix

From theory of matrices, a positive definite Hermitian matrix \mathbf{R}_{xx} can be diagonalized by a nonsingular orthonormal transformation matrix \mathbf{Q} which is formed by eigenvectors of \mathbf{R}_{xx} as follows:

$$\mathbf{Q}^*\mathbf{R}_{xx}\mathbf{Q}^T = \mathbf{A} \quad \dots (13)$$

Eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_N$, and the corresponding eigenvectors are;

$$\mathbf{Q} = [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_N] \quad \dots (14)$$

Where

\mathbf{e}_i is $(N \times 1)$ eigenvector corresponds to eigenvalue λ_i .

Corresponding to each Eigenvalue there is an associated Eigenvector \mathbf{e}_i that satisfies

$$\mathbf{R}_{xx}\mathbf{e}_i = \lambda_i\mathbf{e}_i \quad \dots (15)$$

Since,

$\mathbf{R}_{xx} = E\{\mathbf{X}^*\mathbf{X}^T\}$ it follows that Eq. (13) may be written as [8]

$$[\mathbf{Q}^*\mathbf{R}_{xx}\mathbf{Q}^T] = \overline{[\mathbf{Q}^*\overline{\mathbf{X}^*}\overline{\mathbf{X}^T}\mathbf{Q}^T]} = \overline{[\mathbf{X}^{*\sim}\mathbf{X}^{T\sim}]} \quad \dots (16)$$

Where

the over bar () is referred to the expected value E and

$$\mathbf{X}^{*\sim} = \mathbf{Q}^*\mathbf{X}^* \quad \dots (17)$$

Where:

\mathbf{A} Is $(N \times N)$ diagonal matrix, its diagonal elements are a real

The \mathbf{Q}^* matrix transforms the complex vector \mathbf{X}^* into the orthogonal vector $\mathbf{X}^{*\sim}$. The components of $\mathbf{X}^{*\sim}$ are determined by the eigenvectors of \mathbf{R}_{xx} , as

$$x_i^{*\sim} = \mathbf{e}_i^\dagger \cdot \mathbf{X}^* \text{ for } i = 1, 2, \dots, N \quad \dots (18)$$

Where

\dagger is a conjugate transpose.

The orthogonal vector components $x_i^{*\sim}$ have two specific characteristic, they are uncorrelated and their amplitudes are given by the square root of the corresponding Eigenvalue, so that the array correlation matrix has N eigenvalues $(\lambda_1, \lambda_2, \dots, \lambda_N)$ along with N associated eigenvectors $\mathbf{Q} = [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_N]$ where N is the number of sensors

If the Eigenvalues are sorted from smallest to largest, matrix Q can be divided into two sub matrices such that $Q = [Q_n, Q_s]$.

The first sub matrix Q_n is called the noise sub matrix and it is composed from $(N - D)$ eigenvectors associated to the noise channels Eigenvalues, the Eigenvalues are given as $\lambda_1 = \lambda_2 = \dots = \lambda_{N-D} = \sigma_n^2$ where D is the number of received signals and (σ_n^2) is the variance of noise componen. The second sub matrix Q_s is called the signal sub matrix and it is composed from D Eigen vectors associated with the D Eigen values $(\lambda_{N-D+1}, \dots, \lambda_N)$ with the magnitudes depend on the power level of received signals [9].

Music Algorithm

The MUSIC algorithm is a simple, popular, high resolution and efficient Eigen structure method [5].The MUSIC spatial spectrum can be expressed as follows [10]

$$DF(\theta) = \frac{1}{c^T(\theta(s)) Q_n Q_n^H c^*(\theta(s))} \dots (19)$$

Where

$C(\theta(s))$ is a spatial vector given by

$$C(\theta(s)) = [g_z(\theta_s) e^{-jz*\beta d \cos(\theta_s)}]^T, z = 1, \dots N \dots (20)$$

and $\theta(s)$ is a spatial angle from 0 to 2π .

The MUSIC flow graph can be summarized in Fig. (2)

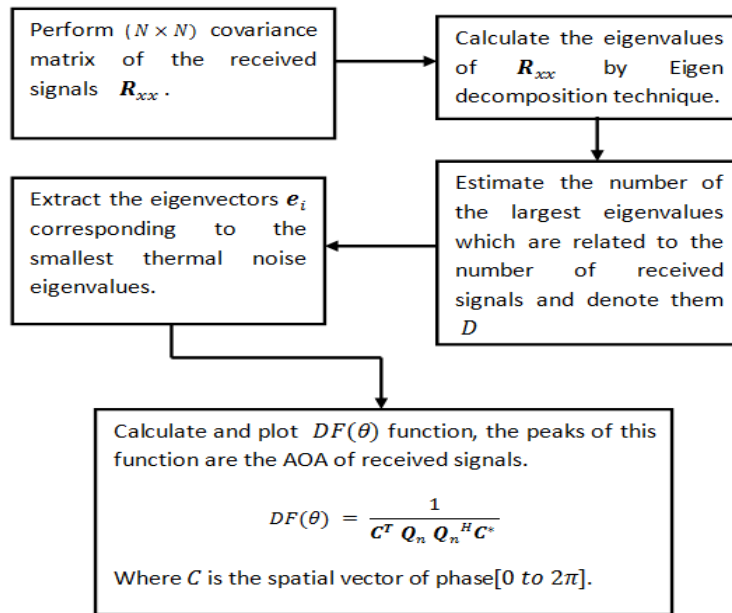


Figure (2): Flow Graph of MUSIC Technique.

Mathematical Model for Alternative Approach.

In this approach an adaptive array system is considered in conjunction with constraints on a weight vector **W** as follows:-

If **R_{xx}** is a covariance matrix of received signals plus thermal noise, the output power of the system will be [11]

$$P = \mathbf{W}^H \mathbf{R}_{xx} \mathbf{W} \quad \dots (21)$$

and if the system is subjected to a constraint

$$\|\mathbf{W}\| = \mathbf{W}^H \mathbf{W} = 1 \quad \dots (22)$$

(This is Euclidean norm constraint, any non-zero value can be chosen).

The so called cost function of the system can be written as [12]: -

$$H(\mathbf{w}) = \mathbf{W}^H \mathbf{R}_{xx} \mathbf{W} + \lambda(1 - \mathbf{W}^T \mathbf{W}) \quad \dots (23)$$

The aim is to minimize this function after incorporating the constraint with the undetermined Lagrange multiplier λ and then differentiating the function on a complex weight vector by using (complex gradient operator)

$$\nabla_{\mathbf{w}}^* H(\mathbf{w}) = \mathbf{R}_{xx} \mathbf{W} - \lambda \mathbf{W} \quad \dots (24)$$

and then putting it equal to zero leads to

$$\mathbf{R}_{xx} \mathbf{W} = \lambda \mathbf{W} \quad \dots (25)$$

It can be seen that the stationary value occurs when **W** is an eigenvectors of **R_{xx}**. When **W** is one of the normalized eigenvectors **e_i** with corresponding eigenvalue λ_i . It can also be seen that the value of the output is λ_i , so the lowest eigenvalue is corresponding to the minimum output power which are relating to the channel thermal noise powers (σ_n^2). In general if there are *N* elements and *D* source in the environment at the frequency of interest, when [*N* > *D*] there will be [*N* - *D*] eigenvalues corresponding to the receiver thermal noise powers.

Now the array gain in the constraint **C** direction (i.e. spatial vector in a look direction) with the weight vector **W** is **CW^T**, where **C** is a set of element gain in the direction of $\theta(s)$.

So the normalized gain will be $\|\mathbf{C}\mathbf{W}^T\| / (\|\mathbf{C}\| \|\mathbf{W}\|)$ and this gain has a maximum value of unity when **W=C*** (or any multiple of **C***).

For each look direction we can expand **C*** in terms of eigenvalues of **R_{xx}** (which are forming a complete orthogonal set of weight vector in to look direction).

$$\mathbf{C}^* = \sum_{i=1}^N g_i \mathbf{e}_i \quad \text{where } g_i = \mathbf{C}^* \mathbf{e}_i^H = (\mathbf{C} \mathbf{e}_i^T)^* \quad \dots (26)$$

But now we restrict **W** to be a component of **C*** which is made up of only the eigenvectors which give receiver noise at the array output.

$$\mathbf{W} = \sum_{i=1}^{N-D} g_i \mathbf{e}_i = \sum_{i=1}^{N-D} (\mathbf{C} \mathbf{e}_i^T)^* \mathbf{e}_i \quad \dots (27)$$

Where

i < *N* - *D* corresponds to the noise level eigenvectors. In this case normalized gain is generally slightly less than unity.

However in the signals directions all the noise eigenvectors produce (in principle) zero array gain, and so will any linear combination of them.

The DF function plotted is in fact the reciprocal normalized gain, give peaks in the signal direction, and becomes with this choice of \mathbf{W} .

$$DF(\theta) = \|\mathbf{C}\| / \sum_{i=1}^{N-D} |\mathbf{e}_i^T \mathbf{C}|^2 \quad \dots (28)$$

The alternative approach can be summarized as shown in the flowing diagram Fig. (3)

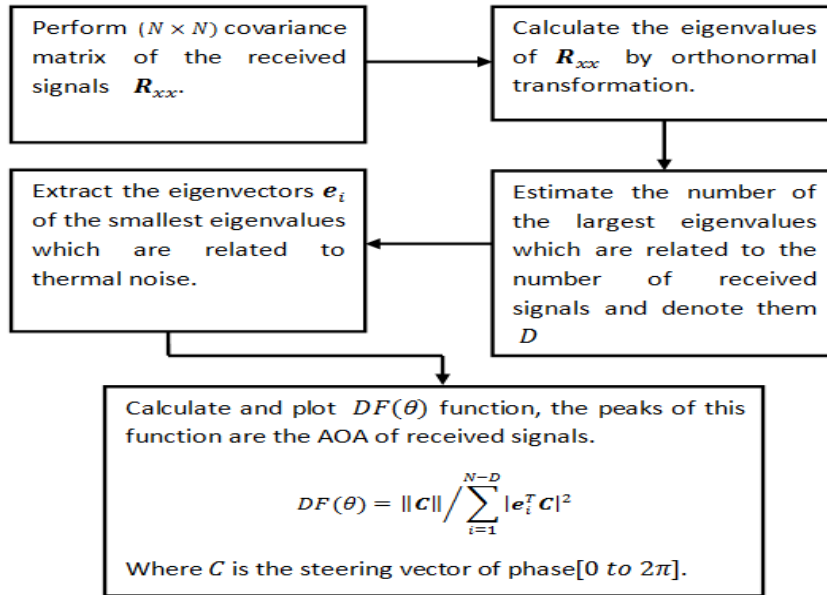


Figure (3): Flow Graph of Alternative Approach

Simulation and Results

All the simulation programs were written by MathLab7.1 and the following assumptions are considered

- Linear array antenna with six elements (N=6) distributed along z-axes.
- The inter element spacing equal to $0.5\lambda_0$.
- The input SNR = 0dB.
- Three types of antenna element were tested in this simulation.

Isotropic elements:

Six isotropic elements with 0.5λ inter element spacing, single source arrived from angle (100°) with SNR 0 dB and four sources arrived from angles ($20^\circ, 40^\circ, 60^\circ, 80^\circ$) with SNRs 0 dB are assumed to be received by a linear array antenna with MUSIC DF function.

Figs (4, 5) show the MUSIC DF plots give an accurate AOA estimation to the single and multi-sources. It can be seen that the system does not suffer any problem to track a multiple sources with the use of isotropic array elements.

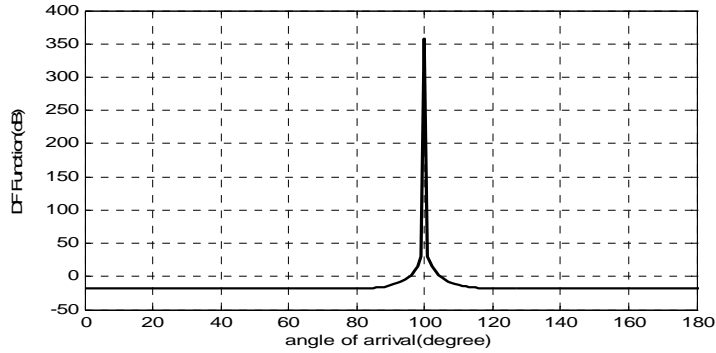


Figure (4): DF Function Plots for MUSIC Technique with Six Arrays Isotropic Elements for Signal Coming From 100°

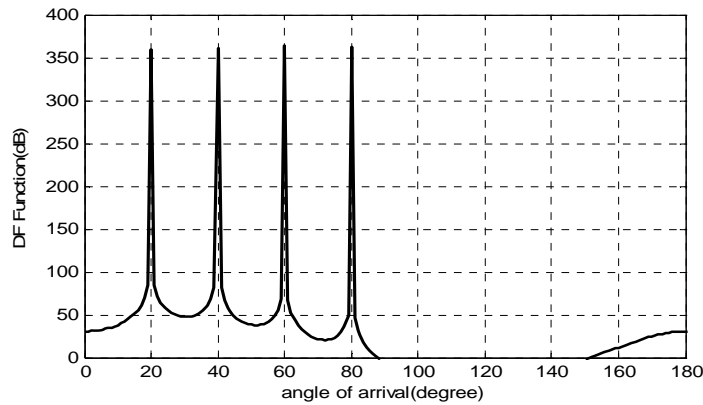


Figure (5): DF Function Plots for MUSIC Technique with Six Isotropic Array Elements for Signals Coming From (20°, 40°, 60°, 80°)

B- $\lambda/2$ Dipole Elements:

Six $\lambda/2$ dipoles array elements with 0.5λ inter element spacing is assumed with a single source come from (100°).

Fig. (6) Shows that the MUSIC DF interpreted three sources from [0°,100°,180°] rather than the actual single source come from (100°). This means that the MUSIC DF experienced false reading from angles [0°,180°] due to the use of $\lambda/2$ dipole elements. The false reading is due to the existence of true nulls from [0°,180°] in the original pattern of half wave length dipole element as shown in figure (7). The MUSIC DF function is mainly depending on creating orthogonal nulls in the final pattern of the array in the direction of received sources and then interprets these nulls plus the nulls in the original pattern of half wave dipole (the elements of array) by the formula given by Eq. (19) as a peaks in the arrival directions of these sources which leads to this wrong results.

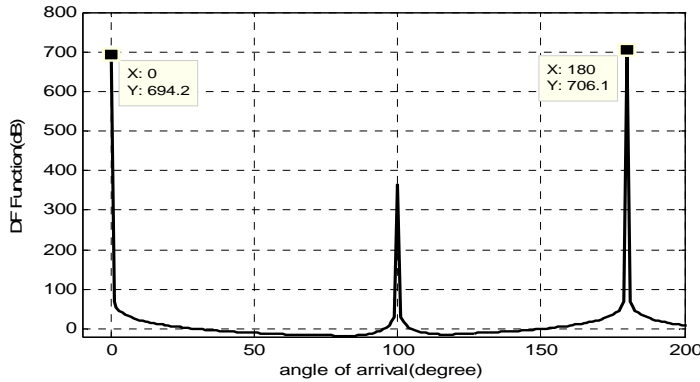


Figure (6): DF Function Plots for MUSIC Technique with Six $\lambda/2$ Dipole array elements for Signal Coming From (100°)

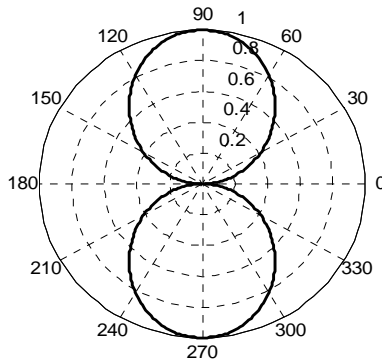


Figure (7): E-Plane Pattern Polar Plot for $\lambda/2$ Dipole Element

C- Test Results of Alternative Approach

The alternative approach with the formula given by Eq. (28) is tested under the same assumption given in case B.

Fig.(8) shows that the final DF pattern exhibit a direction to a single source from $\theta = 100^\circ$ and nothing from $(0^\circ, 180^\circ)$ as the case of MUSIC.

To make sure that the alternative approach gives a correct response when a real sources are arriving from angels $(0^\circ, 180^\circ)$ in addition to other sources coming from $(40^\circ, 80^\circ, 120^\circ)$, the system is tested under this assumption and the result is shown in figure (9). It can be seen that the system gives a correct AOA estimation for all sources even for the two sources come from angels $(0^\circ, 180^\circ)$ which means that the alternative approach is overcoming the problem raised in MUSIC when it is used with a practical half wave dipole array elements.

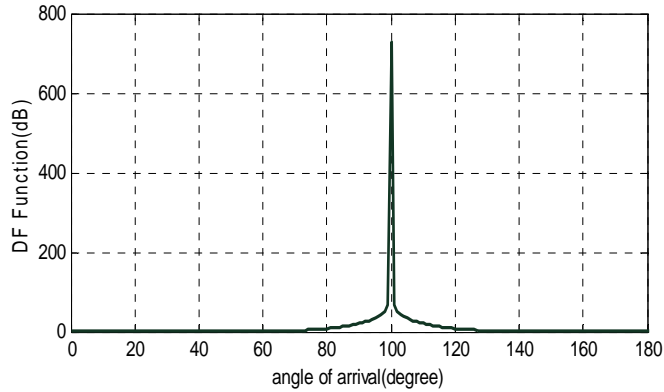


Figure (8): DF Function Plot for Alternative Approach with Six $\lambda/2$ Array Elements Dipoles for Signal Coming From 100°

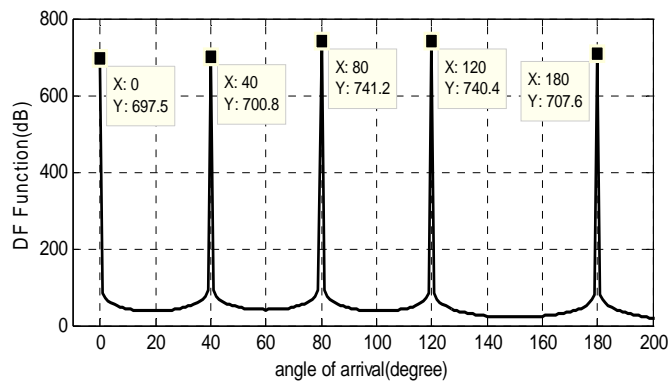


Figure (9): DF Function Plot for Alternative Approach with Six Array Elements Dipole for Signal Coming From $(0^\circ, 40^\circ, 80^\circ, 120^\circ, 180^\circ)$

CONCLUSIONS

Angle of arrival (AOA) estimation based on MUSIC algorithm with isotropic and $\lambda/2$ dipole array elements is investigated and the results show the following.

With isotropic array elements the MUSIC DF can simultaneously estimate the directions to the single and multiple sources with a good accuracy even for those sources which are placed at angles $[0^\circ \text{ or } 180^\circ]$.

When

$\lambda/2$ dipole is used as an array element, it has been found that the MUSIC DF system exhibit a false direction reading from angles $[0^\circ, 180^\circ]$ because of these directions are coincide with the nulls in the element pattern.

The alternative approach with the formula given by Eq.(28) is tested with $\lambda/2$ dipole element to verify its ability to overcome the problem raised in MUSIC Technique and the results shows that the new formula is completely solved this problem and it is work well even when, the arriving signals are coming from angles $[0^\circ, 180^\circ]$ and it did not suffer any problem to deal with single or multi sources coming from any directions by using different type of array elements (isotropic, $\lambda/2$ dipole and any dipole length).

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