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Page 53-64

Separation theorem for fuzzy normed space

Noori F.Al Mayahi , Dunea A . Abd alsaheb

(Department of Mathematics , College of Computer Science and Mathematics , University of AL- Qadissya)

Afm 2005@yahoo.com

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ABSTRACT. The aim of this paper is to study the generalization of the fuzzy normed space . we discussed some theorems in fuzzy normed space and we have some new results on separation theorems .

Mathematics Subject Classification:46A22

1. INTRODUCTION

The theory of fuzzy sets was introduced byZadeh [1] in 1965. After the pioneer work of Zadeh, many researchers have extended this concept in various branches of mathematics and introduced new theories like fuzzy group theory, fuzzy differential equation, fuzzy topology, fuzzy normed spaces etc. We are especially interested in theory of fuzzy normed spaces and their generalizations. In this paper , we have studied the continuity and boundedness in fuzzy normed spaces . T.K. Samanta and SumitMohinta [2] have introduced the concept of intuitionistic fuzzy ψ-normed space and discussed continuity and boundedness in this structure.We prove some separation theorem in fuzzy normed space.

Keyword:t-norm, fuzzynormed space , fuzzy continuity , fuzzy boundedness, separation theorem .

2. PRELIMINARIES

<u>Definition</u>2.1([3]). Let $*$ abinary operation on the set $I = [0,1]$, i.e . $*: I \times I \longrightarrow I$ is a function. $*$ is said to be t-norm continuous (triangular-norm) on the set I if the following axioms are satisfied :

(1) $a * 1 = a$ for all $a \in I$.

(2) $a * b = b * a$ for all $a, b \in I$,(commutative).

(3) If $b, c \in I$ such that $b \leq c$ then $a * b \leq a * c$ for all $a \in I$ (monotone).

(4)$a * (b * c) = (a * b) * c$ for all $a, b, c \in I$,(associative)

(5) $*$ is continuous .

<u>Remark</u>2.2 ([3]).

(a) for any $r_1, r_2 \in (0,1)$ with $r_1 > r_2$, there exist $r_3 \in (0,1)$ such that

$$r_1 * r_3 \geq r_2$$

(b) for any $r_4 \in (0,1)$,there exist $r_5 \in (0,1)$such that

$$r_5 * r_5 \geq r_4 .$$

<u>Definition</u> 2.3([3]). Let X be a vector space over F,$*$ be a continuous t- norm on $I = [0,1]$,a function $\aleph: X \times (0, \infty) \longrightarrow [0,1]$is called fuzzy normif satisfies the following axiom :

For all $x, y \in X, t > 0$,

(1)$\aleph(x, t) > 0$,

(2) $\aleph(x, t) = 1 \Leftrightarrow x = 0$,

(3) $\aleph(cx, t) = \aleph\left(x, \frac{t}{|c|}\right)$for all $c \neq 0$,

(4) $\aleph(x, t) * \aleph(y, s) \leq \aleph(x + y, t + s)$,

(5) $\aleph(x, \cdot): (0, \infty) \longrightarrow [0,1]$ is continuous,

(6) $lim_{t \to \infty} \aleph(x, t) = 1$.

$(X, \aleph, *)$ is called fuzzynormed space.

<u>Lemma</u>2.4([4]). let $(X, \aleph, *)$ $befuzzynormedspace$, then :

$(i)\aleph(x, \cdot)$is non-decreasing with respect to t for each $x \in X$.

$(ii)\aleph(-x, t) = \aleph(x, t)$hence $\aleph(x - y, t) = \aleph(y - x, t)$.

Example 2.5([1]). let $X = R$ define $a * b = a.b$ for all $a, b \in I$,then

$$\aleph(x,t) = \left[e^{\frac{|x|}{t}}\right]^{-1}$$

is fuzzy normed space.

Remark 2.6. let $(X, \aleph, *)$ be fuzzy normed space and let $x, y \in X, t > 0$,
$0 < r < 1$, then if $\aleph(x,t) > 1 - r$ we can find t_0 with $0 < t_0 < 1$, such that

$$\aleph(x, t_0) > 1 - r$$

Definition 2.7.([4]). let $(X, \aleph, *)$ be a fuzzy normed space .We define the $openball$ B$(x, r, t) andtheclosedballB[x, r, t]$ with center $x \in X$and radius $0 < r < 1$,as follows : For $t > 0$

$$B(x, r, t) = \{y \in X : \ \aleph(x - y, t) > 1 - r\}$$

$$B[x, r, t] = \{y \in X : \ \aleph(x - y, t) \geq 1 - r\}.$$

Definition 2.8.[5] let $(X, \aleph, *) beafuzzynormedspace$. A subset A of X is said to be open set , if for all$x \in A$, $\exists\, r \in (0,1)$, $t \in (0, \infty)$such that $B(x, r, t) \subset A$

Theorem 2.9. In fuzzy normed space the intersection finite numbers of open sets is open .

Proof : let $(X, \aleph, *)$ be a fuzzy normed space and let $\{B_i : i = 1,2, \ldots, n\}$ be a finite collection of open set in fuzzy normed space ,let $H = \cap \{B_i : i = 1,2, \ldots, n\}$we must to prove H is open set. let $x \in H \Longrightarrow x \in B_i \forall i = 1,2, \ldots, n$

$\because B_i$open set $\forall i \Longrightarrow \exists r_i \in (0,1) and t_i > 0$

$$\ni B(x, r_i, t_i) \subset B_i, i = 1, \ldots, n$$

Let $t_k = max\{t_1, t_2, \ldots, t_n\}\, and\; r_k = min\{r_1, r_2, \ldots, r_n\}$

$$\Longrightarrow B(x, r_k, t_k) \subset B_i foralli = 1,2,3, \ldots, n$$

$$\Longrightarrow B(x, r_k, t_k) \subset \cap B_i$$

$$\Longrightarrow B(x, r_k, t_k) \subset H$$

$$\therefore Hisopenset$$

Theorem 2.10. In fuzzy normed space, the union of an arbitrary collection of open sets is open .

Proof: let $(X, \aleph, *)$befuzzy normed space and let$\{G_\lambda : \lambda \in \Lambda\}$ be arbitrarycollection of open set in X.let $G = \cup \{G_\lambda : \lambda \in \Lambda\}$we must to prove G is open, now let $x \in G$ then $x \in G_\lambda$ for some $\lambda \in \Lambda$since G_λ is open set then there exist $r \in (0,1), t > 0$ such that $B(x, r, t) \subset G_\lambda$ and since $G_\lambda \subset G$ Then $B(x, r, t) \subset G \Longrightarrow$ G is open set

<u>Theorem</u> 2.11. let $(X, \aleph)$ be fuzzy normed space if A is open set in vector space X and $B \subset X$ then $A + B$ is open set in X

Proof : let $x \in X$ and $a \in A$ since A is open set then there exist $r \in (0,1)$such that

$$B(a, r, t) \subset A \Longrightarrow B(a, r, t) + x \subset A + x$$

$$= B(a + x, r, t) \subset A + x$$

$\Longrightarrow A + x$ is open set in X for all $x \in X$

and since$A + B = \cup \{A + b : b \in B\}$

$\Longrightarrow A + B$ is open set in X

<u>Theorem</u> 2.12. Every open ball in fuzzy normed space $(X, \aleph, *)$ is open set .

Proof : let $B(x, r, t)$ is open ball and $y \in B(x, r, t)$implies that

$$\aleph(x - y, t) > 1 - r .$$

Since $\aleph(x - y, t) > 1 - r$ by remark (2.6) we can find $0 < t_0 < t$ such that

$$\aleph(x - y, t_0) > 1 - r$$

Let $r_0 = \aleph(x - y, t_0) > 1 - r$since $r_0 > 1 - r$ we can find $0 < s < 1$,such that

$$r_0 > 1 - s > 1 - r$$

Now for given r_0 and s such that $r_0 > 1 - s$ we can find r_1 , $0 < r_1 < 1$, such that

$$r_0 * r_1 \geq 1 - s$$

Now consider the ball $B(y, 1 - r_1, t - t_0)$we claim $B(y, 1 - r_1, t - t_0) \subset B(x, r, t)$

Let $z \in B(y, 1 - r_1, t - t_0)$then $\aleph(y - z, t - t_0) > r_1$ therefore

$$\aleph(x - z, t) \geq \aleph(x - y + y - z, t - t_0 + t_0)$$

$$\geq \aleph(x - y, t_0) * \aleph(y - z, t - t_0)$$

$$> r_0 * r_1 \geq 1 - s > 1 - r$$

Therefore $z \in B(x, r, t)$ and hence $B(y, 1 - r_1, t - t_0) \subset B(x, r, t)$

<u>Definition</u>2.13.([3]). Let $(X, \aleph , *)$ be fuzzy normed space , then :

(a) A sequence $\{x_n\}$in X is said to be converges to x in X if for each $\varepsilon \in (0,1)$and each $t > 0$, there exists $k \in Z^+$ suchthat

$$\aleph(x_n - x, t) > 1 - \varepsilon \text{ for all } n \geq k$$

(Or equivalently $limt_{n \to \infty} \aleph(x_n - x, t)$ =1)

(b) A sequence $\{x_n\}$in X is said to be Cauchyif for each $\varepsilon \in (0,1)$andeach t> 0, there exists $k \in Z^+$ such that

$\aleph(x_n - x_m, t) > 1 - \varepsilon$ for all $n, m \geq k$

(Or equivalently $limt_{n,m\to\infty}\aleph(x_n - x_m, t) = 1$).

(c) A fuzzy normed space in which every Cauchy sequence convergent is said to be complete(fuzzy Banach space).

Definition 2.14.([3]) let $(X, \aleph, *)$ and $(Y, \aleph, *)$be two fuzzy normed spaces The function $f : X \longrightarrow Y$ is said to be continuous at $x_0 \in X$ if for all $\varepsilon \in (0,1)$ and all $t > 0$ there exist $\delta \in (0,1)$ and $s > 0$ such that for all $x \in X$

$\aleph(x - x_0, s) > 1 - \delta$ implies $\aleph(f(x) - f(x_0), t) > 1 - \varepsilon$.

The function f is called continuous function, if it continuous at every point of X.

Theorem 2.15.([3]). Let X be a fuzzy normed space over F . Then the functions $f : X \times X \longrightarrow X$, $f(x, y) = x + y$ and

$g: F \times X \longrightarrow X$, $g(\lambda, x) = \lambda x$ are continuous functions .

Definition 2.16. let $(X, \aleph, *)$ be a fuzzy normed space and A is subset of X. A is said to be bounded set if there exist r, $0 < r < 1$, $t > 0$ such that

$$\aleph(x, t) > 1 - r \ \forall x \in A.$$

Definition 2.17.([6]). let $(X, \aleph, *)$ and $(Y, \aleph, *)$be two fuzzy normed spaces the linear functional $f: X \to Y$ is said to be bounded if $f(A)$ bounded set in Y for each A bounded set in X $i.e$

$$\{\forall\ A\ bounded\ set\ in\ X \Longrightarrow f(A) bounded\ set\ in\ Y\}$$

Theorem 2.18. let f be linear functional of fuzzy normed linear space X into another fuzzy normed linear space Y Then the following statements are equivalent

1- f is continuous

2- f is continuous at origin

3- f is bounded

Proof:1→ 2

Let f continuous and let $\{x_n\}$ any sequence in X convergent to 0

By continuity of $f \Longrightarrow f(x_n)$ converge to 0

$\Longrightarrow f$ is continuous at origin

2 → 1

Let$\{x_n\}$ any sequence in X such that $x_n \to x$, $x \in X$

Since f is continuous at 0 then

$$\forall\, \varepsilon \in (0,1) \;, t > 0 \quad \exists \;\; \delta \in (0,1) \,, \;\; s > 0, \qquad (x_n - x) \in X$$

$$\aleph\,((x_n - x) - 0\,, s) > 1 - \delta \Longrightarrow \aleph\,(f(x_n - x) - f(0))\,, t) > 1 - \varepsilon$$

$$\aleph\,((x_n - x)\,, s) > 1 - \delta \Longrightarrow \aleph\,(f(x_n) - f(x)) - 0\,, t) > 1 - \varepsilon$$

$$\aleph\,((x_n - x)\,, s) > 1 - \delta \Longrightarrow \aleph\,(f(x_n) - f(x))\,, t) > 1 - \varepsilon$$

$\Longrightarrow f(x_n) \longrightarrow f(x) \Longrightarrow f$ is continuous at x

Since x is arbitrary point $\Longrightarrow f$ is continuous

2→ 3

Let f is continuous at 0 $\Longrightarrow \forall\, \varepsilon \in (0,1) \;, t > 0 \quad \exists \;\; \delta \in (0,1) \;, \;\; s > 0$

$$\aleph\,(x, s) > 1 - \delta \Longrightarrow \aleph(f(x), t) > 1 - \varepsilon$$

let f is not bounded $\Longrightarrow$ there exist A is bounded set in X such that$f(A)$ is not bounded in Y then for all $k \in (0,1)\,, t > 0$ such that $\aleph(x,t) > 1 - k$ for all $x \in A$ and $\aleph(f(x),t) \leq 1 - k$ for all $f(x) \in f(A)$ then f is not continuous for 0 that's contradiction

$\therefore f$ is bounded

3→ 2

Let $x \in A$ and A is bounded set in X then $x \in X$,let f is not continuous at 0

$$\Longrightarrow \forall\, \varepsilon \in (0,1), t > 0 \;\exists\;\; \delta \in (0,1), s > 0$$

$$\ni \; \aleph\,(x - 0, s) > 1 - \delta \Longrightarrow \aleph(f(x) - f(0), t) \leq 1 - \varepsilon$$

$$\Longrightarrow \aleph\,(x, s) > 1 - \delta \Longrightarrow \aleph(f(x), t) \leq 1 - \varepsilon$$

$\Longrightarrow f(A)$ is not bounded in Y since $\aleph(f(x),t) \leq 1 - k$ for all $k \in (0,1)$ and $f(x) \in f(A)$that's contradiction

$\therefore f$ is continuous at 0.

Definition 2.19.([3]) let (X, d) be a metric space. Let $a * b = a.b$ for all a ,b ∈ [0,1] and let M_d be fuzzy set on $X^2 \times (0, \infty)$defined as follows :

$M_d(x, y, t) = \frac{t}{t+d(x,y)}$, then (X,$M_d$,$*$) is a fuzzy metric space . We call this fuzzy metric induced by a metric d as the standard intuitionist fuzzy metric .

Definition 2.20.([3]) let (X,M_d,$*$) be fuzzy metric space. $A, B \subseteq X$ and $A \neq \varphi$, $B \neq \varphi$, the diameter of set A with respect to t is represented by $\delta(A, t)$ and is defined by :

$$\delta(A, t) = \{M_d(x, y, t): x, y \in A, t > 0\}.$$

The degree of nearness from p to A with respect t is represented by$M_d(p, A, t)$ and is defined by :

$$M_d(p, A, t) = \sup\{ M_d(p, x, t) : x \in A, t > 0\} .$$

The degree of nearness from A to B with respect t is represented by$M_d(A, B, t)$ and is defined by :

$$M_d(A, B, t) = \sup\{M_d(x, y, t) : x \in A , y \in B, t > 0\}.$$

Remark 2.21.([3])if $A \subseteq X$ and $p \in A$ then $M_d(p, A, t) = 1$ for all $t > 0$.

3. Main result

Definition 3.1.([6]) Let X be a vector space over F . The function $P : X \longrightarrow R$ is called Sub-linear functional on X if

(1) $P(x + y) \leq P(x) + P(y)$for all $x, y \in X$

(2) $P(\lambda x) = \lambda\, P(x)$ for all $x \in X$ $and\ for\ all\ \lambda \geq 0$

Theorem3.2. let $(X, \aleph ,*)$ be a fuzzy normed space we further assume that

$$(x_i) a * a = a \quad \forall\ a \in [0,1]$$

Define $P_\alpha(x) = \inf\{t : \aleph(x, t) > \alpha \}$, $\alpha \in (0,1)$,$t \in (0, \infty)$ then $\{P_\alpha : \alpha \in (0,1)\}$ is sub-linear functional on X.

Proof :

1-$P_\alpha(x) + P_\alpha(y) = \inf\{ t > 0 : \aleph(x,t) > \alpha\} + \inf\{ s > 0 : \aleph(y,s) > \alpha\}$

$$t, s \in (0,\infty) \quad and \quad \alpha \in (0,1)$$

$$= \inf\{t + s : \aleph(x,t) > \alpha, \aleph(y,s) > \alpha\}$$

$$= \inf\{t + s : \aleph(x,t) * \aleph(y,s) > \alpha * \alpha\}$$

$$\geq \inf\{ t + s : \aleph(x + y, t + s) > \alpha \}$$

$$= P_\alpha(x + y)$$

2- if $c = 0 \Longrightarrow P_\alpha(cx) = P_\alpha(0) = \inf\{t : \aleph(0,t) > \alpha\}$

$$\because N(0,t) = 1 > \alpha \quad \forall\, t \in (0,\infty) \ \ and \quad \forall \ \alpha \in (0,1)$$

$$\Longrightarrow \inf\{(0,\infty) : \aleph(0,t) > \alpha\} = 0 = c\, P_\alpha(x)$$

if $c \neq 0 \Longrightarrow P_\alpha(cx) = \inf\{s : \aleph(cx,s) > \alpha\}$

$$= inf\left\{s : \aleph\left(x, \frac{s}{|c|}\right) > \alpha\right\}$$

Let t $= \frac{s}{|c|} \Longrightarrow \quad P_\alpha(cx) = \inf\{|c|t : \aleph(x,t) > \alpha\}$

$$= |c| \inf\{t : \aleph(x,t) > \alpha\}$$

$= |c| P_\alpha(x) = cP_\alpha(x)$ (since c$> 0 \Longrightarrow |c| = c$)

$\therefore P_\alpha$is sub-linear functional

<u>Theorem</u>3.3. If A is open set in fuzzy normed space $(X, \aleph, *)$ satisfying the condition (x_i) then

$$A = \{x \in X : P_\alpha(x) < t\}$$

Proof : let B= $\{x \in X : P_\alpha(x) < t\}$

Let $x \in A \Longrightarrow \exists\, r \in (0,1), t \in (0,\infty) \ni B(x, r, t) \subset A$

$$\Longrightarrow B(x,r,t) = \{y \in X : \aleph(x - y, t) > 1 - r\} \subset A$$

if $y = 0 \in X$ (since X vector space)

$$\Longrightarrow B(x,r,t) = \{0 \in X : \aleph(x,t) > 1 - r\} \subset A$$

Let $\alpha \leq 1 - r \Longrightarrow \alpha \in (0,1)$

$$\Longrightarrow \aleph(x,t) > \alpha$$

$$\therefore P_\alpha(x) < t$$

$$\therefore x \in B$$

If $y \neq 0$

$$\aleph(x,t) = \aleph(x-y+y, t_1+t_2) \qquad such\ that\ \ t_1+t_2=t$$

$$\geq \aleph(x-y,t_1) * \aleph(y,t_2)$$

$$> 1-r * \aleph(y,t_2)$$

Since $y \neq 0$ then $\aleph(y,t_2) \neq 1$ and $\aleph(y,t_2) > 0$

Then $\aleph(y,t_2) \in (0,1)$

Let $r' = \aleph(y,t_2) \Rightarrow r' \in (0,1)$

$$\Rightarrow \aleph(x,t) > 1-r * r' = \alpha \in (0,1) \qquad such\ that\ \alpha = 1-r * r'$$

$$\therefore A \subset B$$

$$\Leftarrow let\ x \in B \Rightarrow P_\alpha(x) < t \Rightarrow \aleph(x,t) > \alpha\ ,\ \ \alpha \in (0,1)$$

Let r =1- α $\Rightarrow$ $r \in (0,1)$ $\Rightarrow 1-r = \alpha$

$$\aleph(x,t) > 1-r = \aleph(x-0,t) > 1-r \ \Rightarrow B(x,r,t) \subset A$$

$$\therefore x \in A$$

$$\therefore A = \{x \in X : P_\alpha(x) < t\}$$

Theorem 3.4. let $(X, \aleph, *)$ *be* a fuzzy normed space satisfying the condition (x_i) , and $x_0 \in X$, *if* A is open set such that $x_0 \notin A$, then there exists $f \in X^*$such that $f(x_0) = 1$ *and* $f(x) < 1\ \forall x \in A$.

Proof :if $0 \in A$,since $x_0 \notin A$ then $x_0 \neq 0$

let $P_\alpha : X \to R$

$$\ni P_\alpha(x) = inf\{t > 0 : \aleph(x,t) > \alpha\ , \qquad \alpha \in (0,1)\}\ \ \forall x \in X$$

$$\Rightarrow P_\alpha \text{ is sub-linear and } P_\alpha(x) \geq 0\ \ \forall x \in X$$

Let M = $[x_0]$ $\Rightarrow$ $M = \{tx_0\ ,\ t \in R\} \Rightarrow M$ is subspace of X

Define $g : M \longrightarrow R \ \ \ni g(tx_0) = t$

$\Rightarrow$ $g \in M'$ and $g(x) \leq P_\alpha(x) \forall x \in M$

By Hahn Banach theorem $\exists f \in X' \ \ \ni \ f(x) = g(x)\ \ \forall x \in M$

$$\Rightarrow f(x) \leq P_\alpha(x) \quad \forall x \in X$$

$$\because x_0 = 1.x_0 \in M \ \Rightarrow\ f(x_0) = g(x_0) = g(1.x_0) = 1$$

and since A is open set $\Rightarrow$ A =$\{x \in X \ \ni\ P_\alpha(x) < t\}$

if $t = 1 \implies f(x) \le P_\alpha(x) < 1 \ \forall x \in A$.

we must to prove f is continuous that's equivalent to prove f is bounded

let $x \in X \implies f(x) \le P_\alpha(x)$

if $x \in A \implies P_\alpha(x) < 1 \implies f(x) < 1$

if $x \in -A \implies -x \in A \implies f(-x) < 1 \implies f(x) > -1$

$$\therefore -1 < f(x) < 1 \quad \forall x \in D = A \cap -A$$

$\therefore f$ is bounded on the set D and D is open set ,$0 \in D$

$\implies f$ is bounded .

If $0 \notin A$, take

$$A_1 = A - x_0 \ , \quad x_0 \in A$$

$$\implies 0 \in A_1$$

And Can prove same that last method .

Theorem3.5. let A and B disjoint ,nonempty, convex set in fuzzy normed space $(X, \aleph, t)$.If A is open set in X ,then there is $f \in X^*$and $\lambda \in R$ such that $f(x) < \lambda \le f(y)$ for all $x \in A$ and $y \in B$.

Proof:let$x_0 = b - a$ where $a \in A$, $b \in B$

Let $D = A - B + x_0 \implies D = (A - a) - (B - b)$

Since A , B convex $\implies D$ is convex and since A is open then D is open

$$\because a \in A \implies 0 = a - a \in A - a$$

$$0 = b - b \in B - b$$

$$\implies 0 \in D$$

we must to prove $x_0 \notin D$

Let $x_0 \in D \implies x_0 \in A - B + x_0 \implies 0 = x_0 - x_0 \in A - B$

$$\implies 0 = (b - a) - (b - a) \in A - B \ , \qquad a \in A \ , b \in B$$

$$0 = (a - a) - (b - b) \implies a = b \implies a \in A \cap B \ and \ b \in A \cap B$$

$\implies A \cap B \neq \emptyset$ that's contradiction

By theorem (3.4) there exist $f \in X^* \ni f(x_0) = 1$ and $f(x) < 1 \quad \forall x \in D$

Now $\forall a \in A$ and $\forall b \in B$

$$a - b + x_0 \in D \implies f(a - b + x_0) < 1$$

$$\implies f(a) - f(b) + f(x_0) < 1$$

$$\implies f(a) < f(b)$$

Set $\lambda = \sup\{f(a) : a \in A\}$,then it holds that

$$f(a) \leq \lambda \leq f(b) \quad \forall a \in A, b \in B.$$

To show that the first is strict , assume there is $a' \in A$ such that $f(a') = \lambda$, take $k \in X$ such that $f(k) \neq 0$.

Since A open set then $a' + k \in A$, then $\lambda \geq f(a' + k) = f(a') + f(k)$,

Which is contradiction (since $f(k) \neq 0$)

$$\Rightarrow f(a) < \lambda \leq f(b) \qquad \forall a \in A, b \in B.$$

Theorem 3.6. let $(X, \aleph)$ be a locally convex fuzzy normed space and A , B convex , non empty, and disjoint subsets of X, if A is compact , B is closed ,then there exists a continuous linear functional $f \in X^*$and $\lambda \in R$ such that

$$f(a) < \lambda < f(b) \qquad \forall a \in A, \qquad b \in B$$

Proof : let V isa neighborhood of 0 then

$C = A + V$ is open , convex set , and still disjoint from B

Now by theorem (3.5) there exists $f \in X^*$and $\alpha \in R$ such that

$$f(c) < \alpha \leq f(b) \qquad \forall c \in C, b \in B$$

Since f is continuous and$A$ is compact ,$c = a + v$ $(a \in A, v \in V)$ then

$$f(a + v) < \alpha \leq f(b) \Rightarrow f(a) + f(v) < \alpha \leq f(b)$$

$$f(a) < \alpha - f(v) \leq f(b)$$

Put $\lambda = \alpha - f(v) < \alpha$

$$f(a) < \lambda < f(b) \qquad \forall a \in A, b \in B$$

Theorem3.7. let A is convex , closed subset of fuzzy normed space $(X, \aleph)$ and let $x_0 \in X$, $x_0 \notin A$ then there exist $f \in X^*$ and $\lambda \in R$ such that

$$f(x_0) < \lambda \leq f(x) \ \forall x \in A$$

Proof : since $x_0 \notin A$ then put $r = M_d(x_0, r, t) \Rightarrow r \in (0,1)$

if$B(x_0, r, t)$ is open ball with centerx_0 and radius r then $B(x_0, r, t)$ is open and convex set and $B(x_0, r, t) \cap A = \emptyset$

By theorem (3.5) there exists $f \in X^*$and $\lambda \in R$ such that

$$f(y) < \lambda \leq f(x) \quad \forall x \in A, y \in B(x_0, r, t)$$

In particular $\quad f(x_0) < \lambda \leq f(x)$

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مبرهنات الفصل في الفضاء المعياري الضبابي

أ.م.د. نوري فرحان المياحي ، دنيا أمين عبد الصاحب
(قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة القادسية)

الملخص

نقدم في هذه البحث تعميم في الفضاء المعياري الضبابي ،حيث ناقشنا بعض مبرهنات الفضاء المعياري الضبابي وحصلنا على بعض النتائج الجديدة حول مبرهنات الفصل في الفضاءات المعيارية الضبابية .