

Adaptive OFDMA Resource Allocation using Modified Multi-Dimension Genetic Algorithm

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Abstract A considerable work has been conducted to cope with orthogonal frequency division multiple access (OFDMA) resource allocation with using different algorithms and methods. However, most of the available studies deal with optimizing the system for one or two parameters with simple practical condition/constraints. This paper presents analyses and simulation of dynamic OFDMA resource allocation implementation with Modified Multi-Dimension Genetic Algorithm (MDGA) which is an extension for the standard algorithm. MDGA models the resource allocation problem to find the optimal or near optimal solution for both subcarrier and power allocation for OFDMA. It takes into account the power and subcarrier constraints, channel and noise distributions, distance between user's equipment (UE) and base stations (BS), user priority weight – to approximate the most effective parameters that encounter in OFDMA systems. In the same time multi dimension genetic algorithm is used to allow exploring the solution space of resource allocation problem effectively with its different evolutionary operators: multi dimension crossover, multi dimension mutation. Four important cases are addressed and analyzed for resource allocation of OFDMA system under specific operation scenarios to meet the standard specifications for different advanced communication systems. The obtained results demonstrate that MDGA is an effective algorithm in finding the optimal or near optimal solution for both of subcarrier and power allocation of OFDMA resource allocation.

Index Terms - Communication Systems, OFDMA, Resource Allocation, multi-dimension Genetic Algorithm, optimization.

I. INTRODUCTION

In recent years, the orthogonal frequency division multiplexing (OFDM) and orthogonal frequency division multiple access (OFDMA) are turn out to be the essential transmission methods in broadband mobile systems. Most fourth-generation (4G) systems use OFDM and OFDMA [1], including Mobile WiMAX [2], Long Term Evolution (LTE) [3], and LTE-Advanced [4].

In OFDMA, the subcarriers are shared by different users so that multiple users can be arranged to receive data at the same time. The advantage of OFDMA is that it's structurally robustness against multipath fading and Inter-symbol Interference (ISI) [1, 5]. In addition, Time Division Multiple Access (TDMA) may

be combined with OFDMA to provide division of radio resources in the time-frequency plane to form time-frequency blocks. For example, such time-frequency blocks in LTE system are known as Resource Blocks (RBs) [5]. In OFDM and OFDMA systems, the accomplished throughput by each user depends on several factors such as the number of assigned subcarriers and the channel quality of those subcarriers. The number of subcarriers assigned and their allocation organization is critical in maximize the system performance and overall capacity [6]. So, finding a suitable or an optimal subcarrier (or subchannel) allocation and power allocation (which known as resource allocation problem) is one of the hot issues. It

has drawn a great attention of many researchers due to its crucial role in the efficiency of such systems. For example, Wonjong Rhee and John M. Cioffi [7] introduced an analytic algorithm to solve suboptimal multiuser subchannel allocation problem in the downlink of OFDM systems. Jiho Jang and Kwang Bok Lee [8] have suggested an analytic transmit power adaptation method to maximize the total data rate of multiuser OFDM systems in a downlink transmission. Yenumula B. Reddy and Nandigam Gajendar [9] have proposed a genetic algorithm approach for subcarrier and bit allocation to minimize the overall transmit power in the downlink transmission. Atta-ur-Rahman, et al. [10] have presented an adaptive resource allocation schemes and investigated it for OFDM systems: one scheme is based on standard GA and Fuzzy Rule and the other is Water-Filling and Fuzzy Rule. Hai-Lin Liu and Qiang Wang [11] worked on a hybrid algorithm for OFDM resource allocation by combining evolutionary algorithm (EA) with Karush-Kuhn-Tucker conditions.

This work considers the previous conscious efforts and extends it by using a modified multi dimension genetic algorithm to find the optimal or near optimal solution for OFDMA resource allocation. Four cases are considered to demonstrate the effectiveness of the modified multi-dimension genetic algorithm under specific operation scenarios that meet the most requirements of the next generation of mobile systems (such as, LTE and LTE-advanced) with taking into account the most involved factors is such problems. In the first case, equal number of users and subcarriers (16 each) are chosen to find the best subcarrier and power allocation through multi runs (10 runs). Doubling the number of users and subcarriers that are considered in the first case (32 each) to explore the strength of the algorithm as the number of users and subcarriers are increased. While, both second and third case represent special yet important more practical scenarios; in the second case, the number of users (16 users) is half the number of available subcarriers (32

subcarriers) as an example of situation when the number of users is less than the available subcarriers and case three represents the contrary situation, in which subcarriers are less than the number of users: 32 users and 16 subcarriers. Case three is important example of resource sharing of the limited resource which needs a time-frequency sharing plan to provide a reasonable data rate to each user with respect to the channel conditions.

Based on the obtained results from this work, the modified multi-dimension genetic algorithm is an effective algorithm in finding the optimal or near optimal solution for both subcarrier and power allocation for the OFDMA resource allocation. The rest of the paper is organized as follows: section 2 presents a theoretical background for the resource allocation and the algorithms are based in this work. Section 3 illustrates the achieved simulation results. Then, the paper is wrapped up with section 4 which concludes the main achieved results.

II. THEORY

Consider a single cell uplink OFDMA system with a centralized scheduling scheme with K users and N subcarriers to be allocated. Also, assume that all users have a variable bit rate (VBR) with error-free data throughput and a proper coding for the given assignment of subcarriers to the user. So, the channel gain-to-noise ratio (CNR) is given by [7,12] as:

$$g_{k,i} = \frac{H_{k,i}}{\sigma_{k,i}^2}, \text{ for } k = 1, \dots, K, i = 1, \dots, N \quad (1)$$

where $H_{k,i}$ is the channel gain and $\sigma_{k,i}^2$ is the total noise power for each user k and subcarrier i .

Now, if denote $\alpha_{k,i}$ as the binary decision variable of subcarrier allocation, then:

$$\alpha_{k,i} = \begin{cases} 1, & \text{if subcarrier } i \text{ is assigned to user } k \\ 0, & \text{if subcarrier } i \text{ is not assigned to user } k \end{cases} \quad (2)$$

Given that each subcarrier is only assigned to a single user, this leads to:

$$\sum_{k=1}^K \alpha_{k,i} \leq 1, \text{ for } i = 1, \dots, N \quad (3)$$

to compactly refer to the sequence $\alpha_{k,i}$ takes 1 or 0 only, where the zero value indicates that the subcarrier is not assigned to any user. Also, for notational convenience, we use the matrix (\mathbf{A}) with $(K \times N)$ dimensions and indices $(\alpha_{k,i})$:

$$\mathbf{A} = \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1N} \\ \vdots & \ddots & \vdots \\ \alpha_{K1} & \cdots & \alpha_{KN} \end{bmatrix} \quad (4)$$

to refer to the channel allocation. On the other hand, the power $P_{k,i}$ is allocated to a subcarrier i by user k . The power assigned to a specific user overall its allocated subcarriers should not exceeded allowable maximum transmission power $P_{k,max}$ for a specific user; i.e.:

$$\sum_{i=1}^N P_{k,i} \leq P_{k,max}, \quad \text{for } k = 1, \dots, K \quad (5)$$

in addition to that:

$$P_{k,i} \geq 0, \quad \text{for } k = 1, \dots, K \quad (6)$$

Likewise, \mathbf{P} is a $(K \times N)$ matrix of allocated powers $P_{k,i}$ and expressed as:

$$\mathbf{P} = \begin{bmatrix} P_{11} & \cdots & P_{1N} \\ \vdots & \ddots & \vdots \\ P_{K1} & \cdots & P_{KN} \end{bmatrix} \quad (7)$$

As a consequence, the total rate of user k is:

$$R_k = \sum_{i=1}^N \alpha_{k,i} \log_2(1 + P_{k,i} g_{k,i}) \quad (8)$$

and the total system rate is given by:

$$R(\mathbf{A}, \mathbf{P}) = \sum_{k=1}^K \sum_{i=1}^N \alpha_{k,i} \log_2(1 + P_{k,i} g_{k,i}) \quad (9)$$

So, for the OFDMA resource allocation problem the maximization of the weighted ergodic sum-rate can be formulated as:

$$\max E_g \left\{ \sum_{k=1}^K \pi_k \sum_{i=1}^N \alpha_{k,i} \log_2(1 + P_{k,i} g_{k,i}) \right\} \quad (10)$$

which is subject to:

$$E_g \left\{ \sum_{i=1}^N P_{k,i} \leq P_{k,max} \right\}, \quad \text{for } k \text{ user} \quad (11)$$

and the user rate must be greater or at least equal to its allowable (or desired) minimum data rate $R_{k,min}$:

$$E_g \left\{ \sum_{i=1}^N \alpha_{k,i} \log_2(1 + P_{k,i} g_{k,i}) \right\} \geq R_{k,min}, \quad \text{for } k \text{ user} \quad (12)$$

where $E\{\cdot\}$ is the expectation operator and π_k is the weight given to the rate of specific user k .

The weights given to the users' rates are chosen such that:

$$\sum_{k=1}^K \pi_k = 1 \quad (13)$$

The resource allocation problem, due to the discrete set of values of $\alpha_{k,i}$, referring to Eq. (10) is a non-convex. It becomes a convex when relaxing the condition of $\alpha_{k,i}$ by permitting them to take any value in the interval $[0, 1]$. This is equivalent to allowing time-sharing of a single subcarrier between different users. In this way, during a given scheduling interval a number of users can transmit on the specified subcarrier alone in a portion of the interval. Moreover, with assuming that $f_{k,i} = \alpha_{k,i} P_{k,i}$ the resource allocation problem can be rewritten as:

$$\max E_g \left\{ \sum_{k=1}^K \pi_k \sum_{i=1}^N \alpha_{k,i} \log_2(1 + \frac{f_{k,i}}{\alpha_{k,i}} g_{k,i}) \right\} \quad (14)$$

which is subject to:

$$E_g \left\{ \sum_{i=1}^N f_{k,i} \leq P_{k,max} \right\}, \quad \text{for } k \text{ user} \quad (15)$$

or,

$$E_g \left\{ \sum_{k=1}^K \sum_{i=1}^N \alpha_{k,i} P_{k,i} \leq P_{total} \right\}, \quad \text{for all users} \quad (16)$$

and,

$$E_g \left\{ \sum_{i=1}^N \alpha_{k,i} \log_2 \left(1 + \frac{f_{k,i}}{\alpha_{k,i}} g_{k,i} \right) \right\} \geq R_{k,min} ,$$

for k user (17)

where Eq. (14) is convex since the expectation conserve convexity and the $\log_2(1 + b/a)$ is recognized as concave function forms. So, the gained advantage is that the problem can then be solved reliably and efficiently [12,13]. It should be noted that the resource allocation problem is subject to constraints in Eq. (3), Eq. (5), Eq. (6) and Eq. (13) in addition to Eqs. (15-17).

In this work, modified multi-dimension Genetic algorithm (MDGA) which will illustrate shortly is used for finding the optimal or near optimal solution (subcarrier and power allocation). The basic idea with using Multi Dimension Genetic Algorithm (MD-GA) is inspired by multi dimension aspect of the problem itself. It is generalized to be able to solve other hard multi-element multi-dimension optimization problems. Each individual consists of several chromosomes. While each chromosome is in turn include several genes. Such population is represented in Fig. (1).

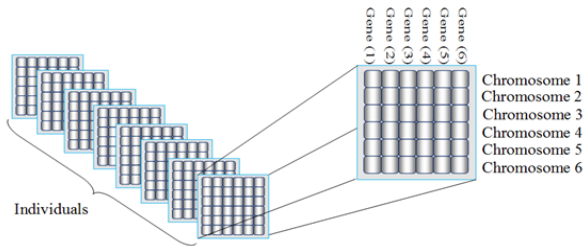


Fig. 1 an example of multi-dimension population.

The multi-dimension operators depicted in Fig. 1 have the following main features:

A) Multi dimension crossover: is possible to apply any type of standard crossover (single point, multi point, or uniform crossover) but with two levels:

1) Local crossover (same individual): either perform crossover between the chromosomes in the same individual (horizontal) or perform crossover between a column of genes in some

vertical position (for all chromosomes) with a column of genes in another vertical position that both within the same individual (vertical). This can be seen as rearranging of the genes' position within some of the chromosomes. Figure (2) shows both types of local crossover.

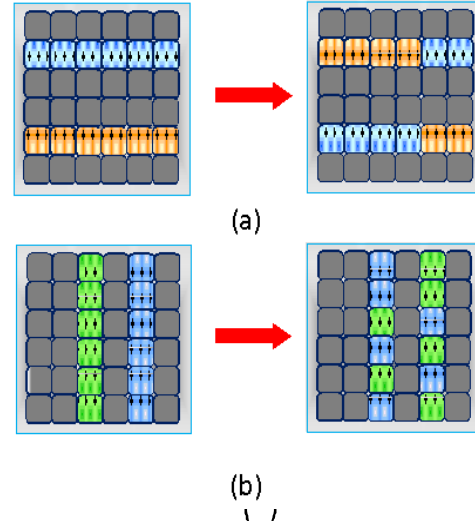
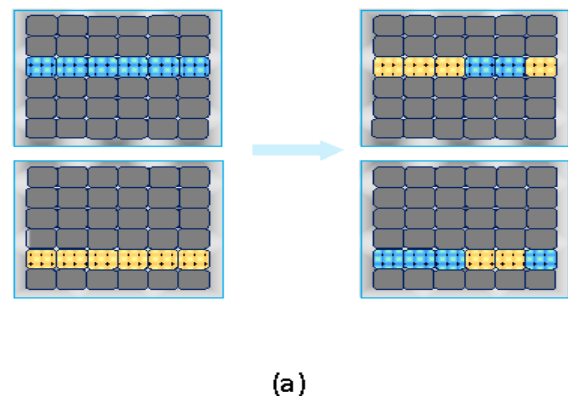


Fig. 2 local crossover: a) horizontal, b) vertical.

2) Global crossover (two individual): either perform crossover between chromosomes belong to different individuals (horizontal) or perform crossover between a column of genes in some vertical position (for all chromosomes) in some individual with a column of genes in another vertical position within another individual (vertical). Figure (3) illustrates both types of global crossover. The crossover rate can be set between [0,1] (usually 0.5-1) with presence of local-global rate (for simplicity can be set equal to 0.5).



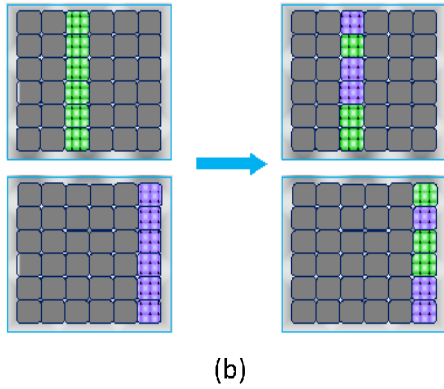


Fig. 4 global crossover: a) horizontal, b) vertical.

B) Multi dimension mutation: the mutation will also work on several dimensions (not only single one in case of standard GA). In multi dimension mutation, it's possible to apply any type of standard mutation (single point or multi point mutation). However, it should be used with different levels by randomly mutate the value of either genes, column of genes (vertical), chromosomes (horizontal) or/and even the entire individual in some special cases to avoid losing of the population diversity, as shown in Fig. (4). The mutation rate can be set between [0,1] (usually low values) with presence of mutation level rate.

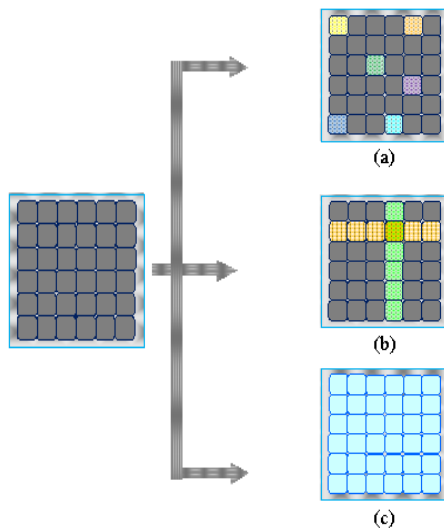


Fig. 4 mutation types: a) gene mutation, b) horizontal and vertical mutation, c) individual mutation.

A brief pseudo code for MDGA can be rewritten as:

```
% Multi Dimension Genetic Algorithm
(MD-GA)
Define the problem related information.
Define MD-GA parameters, variables and
multi-level selected.
Generate initial population of individuals
(solutions).
Evaluate individuals and find best one.
While counter ≤ Generationmax
    Perform selection, elitism, and worst
    elimination.
        Perform multi-level crossover.
        Perform multi-level mutation.
        Evaluate individuals and find best one
    If (any of termination conditions are
    satisfied)
        Terminate evolution loop.
    End If
End While
Output results and required visualization.
```

III. Implementation and Simulation results

In this work, MATLAB m-file is used to compute and simulate the resource allocation. The modified multi dimension Genetic Algorithm to find the optimal or near optimal subcarrier and power allocation matrixes to maximize the weighted ergodic sum-rate of OFDMA system (measured in bit/sec/Hz).

Assuming a single cell OFDMA system with subcarriers subject to Rayleigh fading distribution (1000 channel realizations are used). The propagation loss is modeled using the path loss model given as:

$$L_p = cD_k^{-u} \quad (18)$$

where c is the path loss constant (selected to be -128.1 dB), D_k is the distance in km from the user k to the base station (BS), and u is the path loss exponent ($= 3.76$ for urban environments) [12,14]. All users are assumed to be equally distanced away from the BS. The minimum rate allowed is selected to be

equal to zero for all users and the noise level is -16.9 dbm, the weights are equal (weight of any user is equal to reciprocal of the number of users to satisfy the condition of Eq. (13)). Also, the maximum allowed transmitted power for each user, $P_{k,max}$, set to be within the allowable range for each simulated case. These assumptions are considered in order to simplify the simulation while the m-file program is designed to allow different distances, minimum data rates, and/or weights for each user. The simulation discusses several cases as summarized in Table I.

TABLE I
SUMMARIZATION FOR THE
CONSIDERED CASES.

Case No.	Users (K)	Subcarriers (N)	$P_{k,max}$ (in watt)	Distance (in meter)	Generations	Individuals (I)	Crossover rate	Mutation rate
1 st	16	16	0.8	400	1000	100	0.9	0.2
2 nd	16	32	0.8	300	1000	100	0.9	0.2
3 rd	32	16	0.8	300	1000	100	0.9	0.2
4 th	32	32	0.8	400	1000	100	0.9	0.2

The individuals of GA's population solutions represent the legal solutions of the subcarrier and power allocation. Each individual has a pair of chromosomes groups, first group is the subcarrier allocation matrix **A** ($K \times N$) and the second one is the power allocation matrix **P** ($K \times N$), where K users represent the number of chromosomes and N subcarriers is the number of genes as depicted

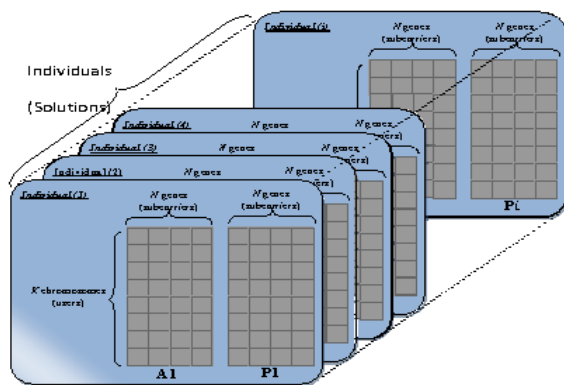


Fig. 5 GA representation of resource allocation problem.

The real numbers values are used to represent the individuals with the **A** elements values lies in [0,1] while **P** elements can take any value between 0 to the value of maximum power of a specific user. This representation is done in a way that guarantees the subcarrier and power allocation constraints are both satisfied and all the individuals (solutions) are lie in the legal area of the search space. Roulette wheel method is utilized to perform the selection operator with using of elitism method to ensure of survival of the best individual (solution) of the current generation to be reproduced in the next generation.

It should be noted that the m-files is also designed to allow selection of different values for all GA factors and parameters (such as crossover rate, mutation rate, generations, and/or individuals). The termination condition of the evolution is selected to be depending on the number of generation to explore the full evolution progress of the solutions and to avoid premature-termination. The following results are detailed for each of the cases listed in table (1) with the best results which are selected for over 10 runs (each run has 1000 generation). Each run is used the same channel conditions and resource allocation operation scenario assumptions. The best run is selected depending on final iteration results (best sum of rates which related to best solution (individual) that contain best **A** and **P**).

Case1: In this case 16 users over 16 subcarriers are considered and the best sum of rates for 10 runs is shown in Fig. 6. It's noticeable that there is a rapid increasing of the targeted best sum of rates (related to improvement of individuals) during the range of the first 200 generations with obvious small differences in values between runs. As the generation number increases the obtained best sum of rates improving and tending to slow down with much less difference in values between the different runs.

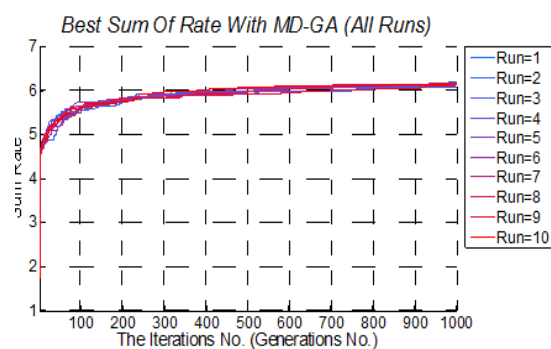


Fig. 6 Best sum of rates of 10 runs using MD-GA (users = 16, subcarriers = 16).

The best resulted run (run no. = 1) is shown in Fig. 7. The best sum of rates is increased in very fast manner starting from its initial generation to just before the generation 100 and continues its increasing behavior in slower manner after that.

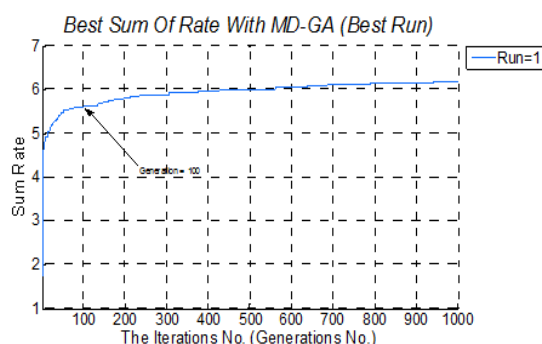


Fig. 7 Best sum of rates of 1st run (users=16, subcarriers=16).

Both best and worst sum of rates of each generation of run 1 is listed in Fig. 8. It is obvious that the large difference gaps between them is increased at early generations and tend to slightly increase afterword. This can be explained that with keeping in mind the genetic evolution process principles the multi dimension genetic operators (crossover and mutation) produce better solution (individual) with variance of less optimum (or worst) solutions while elitism keep introduce best solution as a part of the next generation to allow for further improvement.

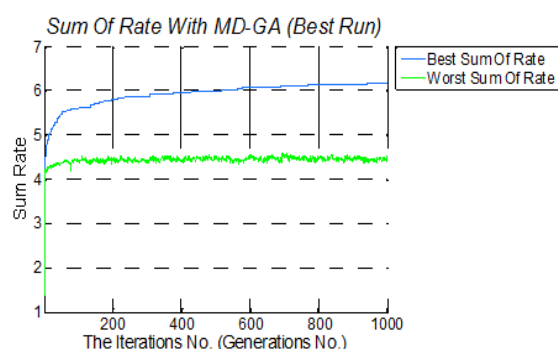


Fig. 8 Best and worst sum of rates of 1st run.

Figure 9 shows the subcarrier and power allocation of the best individual (solution) of run 1. In despite of scale difference, it is clear that both evolved subcarrier and power allocation distributions of the best solution (individual) are very similar but not identical (depending on genetic evolution process).

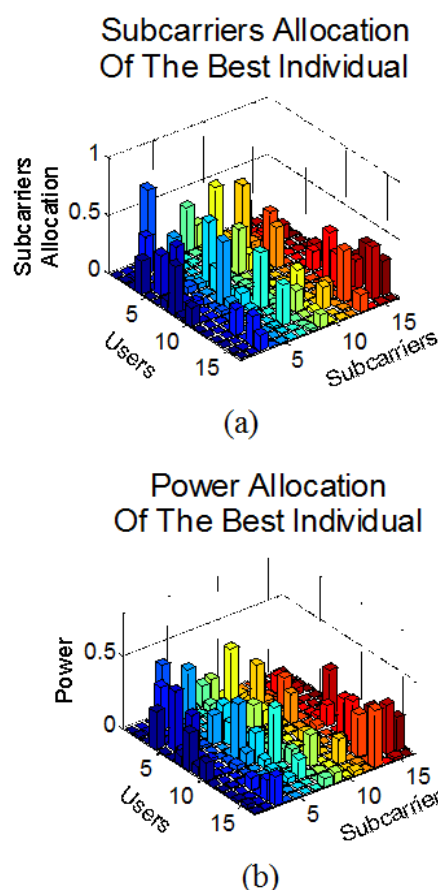
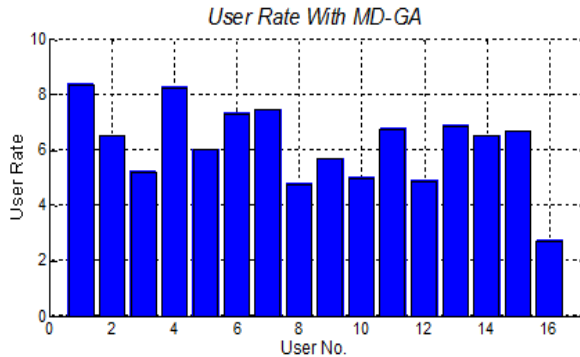


Fig. 9 Best solution of 1st run: a) subcarrier allocation, b) power allocation.

For run 1, the users' rates are shown in Fig. 10, which is depending on the subcarrier and power allocation of the best individual (best

solution) and the channel condition. It shows that all the users have a data rate greater than the minimum value (which was set to zero).



Case 2: In this case, we run the simulation for 16 users over 32 subcarriers. The best sum of rates was for 10 runs. It noticed that, as in case1, there is a rapid increasing of the targeted best sum of rates (related to improvement of individuals) during the range of the first 300 generations with obvious differences in values between runs. Then, as the generation number increases the targeted best sum of rates is tending to slow down with difference in values between the runs. Both best and worst sum of rates of each generation of run 9 is shown in Fig. 11. It's notable that the large difference gap between them is highly increased at the early 300 generations and tends to slowly increase afterword. This can be explained easily by keeping in mind principles of the genetic evolution process. The genetic operators (crossover and mutation) produce better solution (individual) with variance of less optimum (or worst) solutions while elitism keep introduce the best one as a part of the next generation to allow for further improvement.

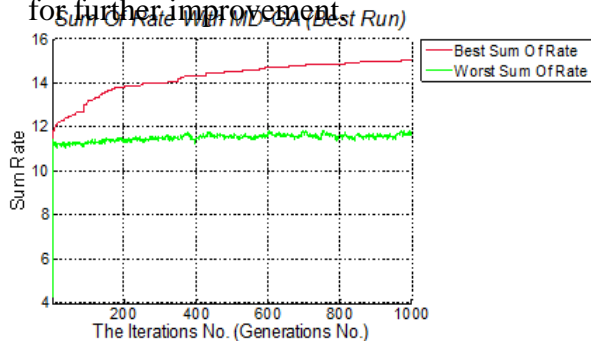


Fig. 11 Best and worst sum of rates of 9th run.

The subcarrier and power allocation of the best individual (solution) of run 9 is given in Fig. 12. Even with scale difference, it obvious that both evolved subcarrier and power allocation distributions of the best solution (individual) are having some similarity (depending on genetic evolution process).

For run 9, the users' rates are shown in Fig. 13, which is depending on the subcarrier and power allocation of the best individual (best solution) and the channel condition. All the users have a data rate greater than the minimum data rate.

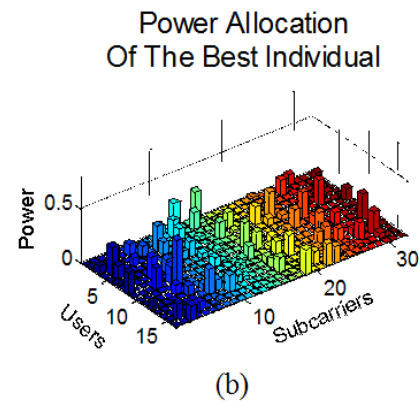
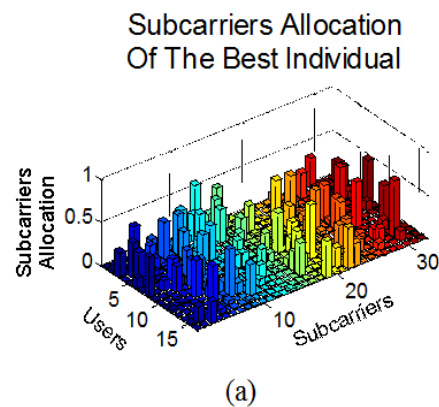


Fig. 12 Best solution of 9th run: a) subcarrier allocation, b) power allocation.

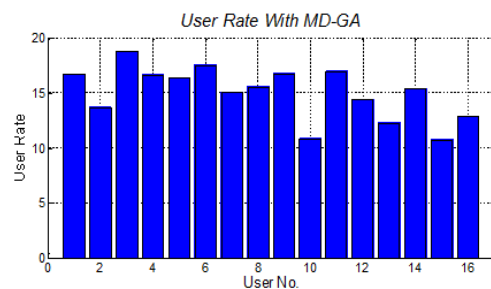


Fig. 13 Users' rates of 9th run.

Case 3: In this case, 32 users over 16 subcarriers is considered and the results are presented for the best sum for 10 runs. During the range of the first 200 generations there is a fast increase of the targeted best sum of rates (solution or individuals), while in the following generations, the value of the targeted best sum of rates is tending to increase in slow manner comparatively. Also, it's noticeable that there is a small (yet obvious) difference in values between runs at the range of the first 400 generations. Both best and worst sum of rates of each generation of run 7 are shown in Fig. 14 which shows that the large difference gap between them is highly increased at early generations and tend to slightly increase afterword. This is due to that the genetic operators (crossover and mutation) produce better solution (individual) with variance of less optimum or worst solutions while elitism keep introduce best solution as a part of the next generation to allow for further improvement.

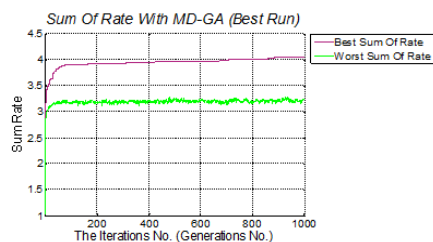


Fig. 14 Best and worst sum of rates of 7th run.

Figure 15 shows that the subcarrier and power allocation of the best individual (best solution) of run 7. With keeping in mind the scale difference the evolved subcarrier and power allocation distributions of the best solution are less similar and there are differences between them. This is related to effect of genetic operators and parameters and ability of convergence toward optimal solution.

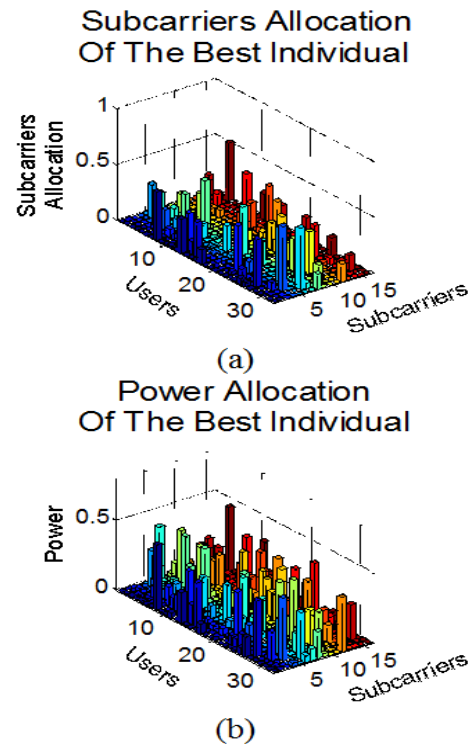


Fig. 15 Best solution of 7th run: a) subcarrier allocation, b) power allocation.

For run 7, the users' rates are illustrated in Fig. 16, which is depending on the subcarrier and power allocation of the best solution (best individual) and the channel condition. As the above cases, all the users have a data rate greater than the specified minimum value.

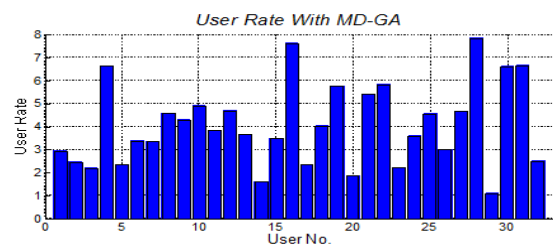


Fig. 16 Users' rates of 7th run.

Case 4: In this case, the best sum of rates is considered for 10 runs with 32 users over 32 subcarriers. Both best and worst sum of rates of each generation of run 7 is given in Fig. 17. The large difference gap between them is highly increased at early generations and tend

to slightly increase afterword. This is due to the genetic operators (crossover and mutation) produce better solution (individual) with variance of less optimum (or worst) solutions while elitism keep introduce best solution as a part of the next generation to allow for further improvement.

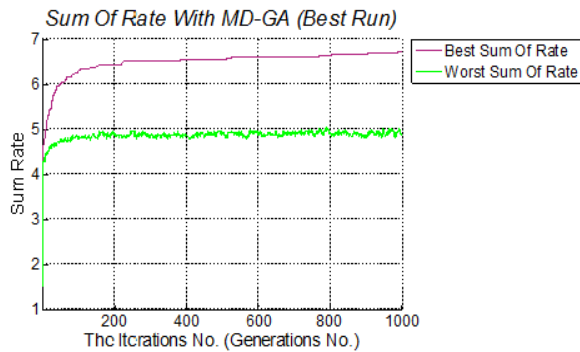
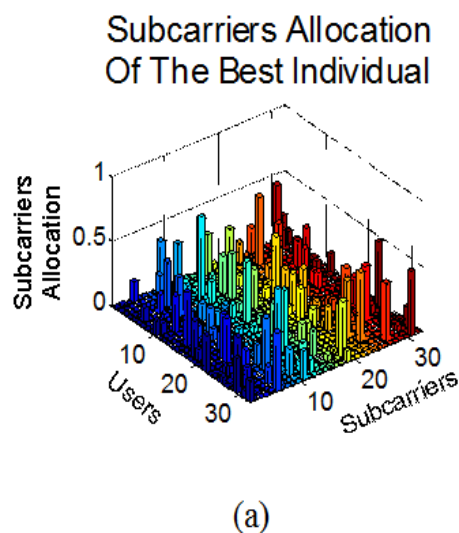


Fig. 17 Best and worst sum of rates of 7th run.

Figure 18 showcase the subcarrier and power allocation of the best individual (solution) of run 7. In spite of scale difference the both evolved subcarrier and power allocation distributions of the best solution (individual) are alike but not identical (depending on genetic algorithm evolution process).



Power Allocation
Of The Best Individual

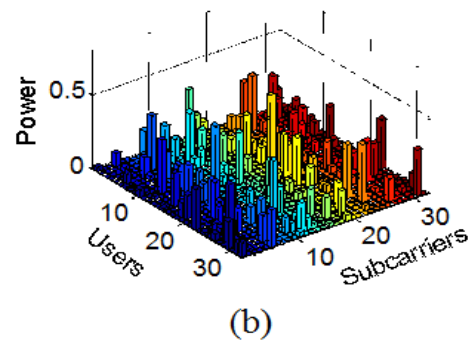


Fig. 18 Best solution of 7th run: a) subcarrier allocation, b) power allocation.

For run 7, the users' rates are listed in Fig. 19, which is depending on the subcarrier and power allocation of the best individual and the channel condition. As that in the previous cases, all the users have data rate are greater than the minimum data rate (which set to zero).

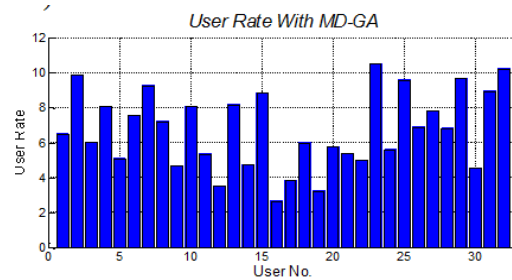


Fig. 19 Users' rates of 7th run.

IV. Conclusion

The obtained results demonstrate that the proposed modified multi-dimension genetic algorithm (MDGA) is an effective algorithm in finding optimal or near optimal solution for both subcarrier and power allocation for OFDMA resource allocation. The improved multi-dimension crossover and mutation give a good balance between local and global search, respectively, with a moderate converging speed.

In all the analyzed cases, the results show that modified multi-dimension genetic algorithm is capable to reach the suitable or acceptable solution with ability to continually evolving toward better solutions. Also, using multi run (10 runs) for each case show that there are notable differences between the runs

and these differences tend to decrease as the number of generation increases especially after the first 300 generations. This might give an illustration of a good number of generation (100-300 generation) but must be noted that it might also depend on other GA parameters values such as multi-dimension crossover rate, multi-dimension mutation rate, the targeted problem in hand and also on the balance between level of best result required in one side and the processing cost and time on the other side.

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