Nonlinear Thermoviscoelastic Behavior of Composite Thin Plates

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Abstract

The nonlinear thermoviscoelastic behavior of composite thin plate from polyester reinforced with random fiber glass was investigated. The hereditary elasticity (viscoelastic) behavior described in new mathematical model predicted from experimental data from creep and relaxation tests to predict the creep compliance and relaxation modulus equations then apply that mathematical model in numerical and analytical analysis to describe the nonlinear viscoelastic behavior of thin composite plates at different loading and temperatures. The creep specimens and composite thin plate have the same volume fraction.

A very good agreement has been found between experimental, theoretical and FEM method. It is found that the deflection increases with approximate rate (50 %) at time (15 min.) and the shear stress (τ_{xy}) increases with approximate rate (58%) at time (30 min.), as a result of increasing the distributed load (q=1.934e-3 N/mm^2 to q=3.4488e-3 N/mm^2) at temperature (30 C^o) of relative dimension (a/b=0.5) and rectangular simply support plate. The increasing temperature from (30 C^o- 60 C^o) increases the deflection with approximate rate (34.6%).

Keyword: thermoviscoelastic, hereditary elasticity, creep, creep compliance, relaxation modulus.

السلوك الحراري اللزج- المرن اللاخطي لصفيحة نحيفة مركبة

الخلاصة:

تم بحث السلوك اللاخطي الحراري لصفيحة مركبة نحيفة من البولستر مدعمة بالالياف الزجاجية العشوائية الترتيب . السلوك اللزج- المرن وصف في إنموذج رياضي استنتج بالاعتماد على النتائج العملية من اختبار الزحف والاسترخاء واستنتاج معادلات خضوع الزحف ومعامل الاسترخاء ثم تطبيق ذلك الإنموذج الرياضي في التحليل العددي والنظري لوصف السلوك اللاخطي اللزج- المرن لصفيحة مركبة عند حمل منتشر مختلف ودرجات حرارة مختلفة . عينة الزحف و الصفيحة المركبة لهما نفس الكسر الحجمي.

وجد تطابق النتائج جيد جداً بين العملي , النظري و طريقة العناصر المحددة . وجد إن الانحراف يزداد بنسبة تقريبية (50%) عند زمن (15 دقيقة) وإجهاد القص يزداد بنسبة تقريبية (58%) عند (30 دقيقة) نتيجة لزيادة الحمل المنتشر (q=1.934E-3 N/mm^2) إلى q=3.4488E-3 (N/mm^2) عند درجة حرارة (T=30 C^o)، الأبعاد النسبية a/b)=0.5) والإسناد البسيط للصفيحة. ازدياد درجة الحرارة (60- 30 C^o) تزيد الانحراف بنسبة تقريبية (34.6%).

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Simple Notation

Symbol	**Definition**	**Unit**
a,b	Plate length and width	mm
A	Bounded area surface	mm^2
Am,Bm,Cm,Dm	Constants	----
D	Flexural rigidity of plate	N.m
D(t)	Creep compliance	1/MPa
E	Modulus of elasticity	N/mm^2
E(t)	Relaxation modulus	N/mm^2
n	Exponent	----
q	Distributed load	N/mm^2
t	Plate thickness	mm
T	Temperature	°C
W	Strain energy	N.m
w	Deflection of plate	mm
□	Normal strain	----
□	Normal stress	N/mm^2
□	Poisson's ratio	----
□	Shear strain	----
$□_{xy}$, □$□_{yx}$	Shear stresses	N/mm2
□	Load potential energy	N.mm
□□	Stress difference	MPa
$□_x$	Rotation about x-axis	Rad
$□_y$	Rotation about x-axis	Rad
□	Summation	----
□σ□□	Stress vector.	
□ε□□	Strain vector.	
□□□	External loads applied to the boundary surfaces.	
□□□	Vector of curvature.	
□□□	The strain matrix	
□□□	The shape functions matrix	
□u□	Nodal displacement matrix	
□□□	The stiffness matrix	
□*Q*]	Constant matrix	
□□□□ □□	Stiffness matrix as a function of strain.	
BC1	The boundary condition of simply support plate	
{f}	Node force vector.	
{□}	Vector of twelve unknown constants.	
[D]	The elasticity matrix	
[B]	The strain matrix	
[N]	The shape functions matrix	
[C]	12*12 matrix depending on nodal co-ordinate.	
{p}	Vector of plate Cartesian	

INTRODUCTION

Viscoelastic material as other metallic and non-metallic materials can be affected by various factors such as temperature, humidity, history variables, etc.

In this respect, literatures related to the viscoelastic behavior of composite materials under thermal effects are presented.

Joannie chin et. al., 2009 [1] studied thermoviscoelastic properties of two commercial ambient cure structural epoxy adhesives were analyzed and compared. The adhesives were formulated by the same manufacturer, but one system contained accelerators to shorten its cure time. B.A. Sami and H. Naima, 2009 [2] predicted the mechanical behavior of yarns under various levels of strain, by using only their technical parameters. The study of the yarn response to tensile test and relaxation test at different strain level has permitted us to propose an analytical model predicting the entire stress-strain response of yarn.

Madlina C. and Raluca M. [3] presented the results of nonlinear static analysis for beam subjected to constant axial load. They are presented the strain and stress distribution in after performing nonlinear analysis because of the nonlinearity of the material. W.S.Lin et. al. [4] the purpose of this study was to determine these creep related material constant for 60% wood flour reinforced extruded thermoplastic material and build linear and nonlinear viscoelastic behaviour of that samples. Sourabh P. Sawant et. al.[5] studied a multi-scale framework for analysis the nonlinear thermo-viscoelastic response of hybrid metal laminates (FML). FMLs are hybrid composite system alternating layers of fiber –reinforced polymer (FRP) lamina and metal sheet. The mechanical properties of each constituents (fiber, polymer and metal) of FMLs often vary with time ,stress ,temperature and moisture. FMLs have been analyzed using finite element (FE) method by generating detailed meshes for each constitutive behaviour can exhibited due to the viscoelasticity in the constituents of FML i.e. the fibres polymer matrix and metal.

The aim of this work is to find the mathematical description the nonlinear thermoviscoelastic behaviour of composite thin plate with taking into account the effect load diffuse.

Theoretical analysis

Bending of Simply Supported with Uniformly Loaded Rectangular Plates is expressed in term of displacements thus[6]:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \qquad \ldots (1)$$

The problems of bending of rectangular plates that have two opposite edges simply supported take the solution in the form of series solution as follows[6]:

$$w = \sum_{m=1}^{\infty} Y_m \sin\frac{m\pi x}{a} \qquad \ldots (2)$$

where:

Y_m: function only of the distance (y) in the y - axis.

It’s assumed that the sides (x=0 and x=a) as shown in Fig.(1) are simply supported. Hence each term of Eq.(2) satisfies the boundary conditions :

$$\left.\begin{aligned} w &= 0 \\ \frac{\partial^2 w}{\partial x^2} &= 0 \end{aligned}\right\} \text{ on these sides}$$

It remains to determine the function $\boldsymbol{Y_m}$ in such form as to satisfy the boundary conditions on the sides of y =± b/2 and the equation of the deflection surface (1). The solution of Eq.(1) for uniform distribution load. The solution of Eq.(1) for uniform load is assumed to be in the form :

$$w = w_1 + w_2$$

And letting :

$$w_1 = \frac{q}{24D}(x^4 - 2ax^3 + a^3 x)$$

i.e., w_1 represents the deflection of uniformly loaded strip parallel to the x-axis. It satisfies Eq.(1) and also the boundary conditions at the edges :

x=0 and x=a.

The expression w_2 evidently has to satisfy the equation:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = 0 \qquad \ldots(3)$$

Taking w in the form of series (2) in which, from symmetry (m=1, 3, 4…) and substituting into Eq.(3) gives :

$$\sum(Y_m^{IV} - 2\frac{m^2\pi^2}{a^2}Y''_m + \frac{m^4\pi^4}{a^4}Y_m)\sin\frac{m\pi x}{a} = 0 \qquad \ldots(4)$$

The general integral of this equation can take the form:

$$Y_m = \frac{qa^4}{D}(A_m\cosh\frac{m\pi y}{a} + B_m\frac{m\pi y}{a}\sinh\frac{m\pi y}{a} + C_m\sinh\frac{m\pi y}{a} + D_m\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}) \qquad ..(5)$$

Observe that the deflection surface of the plate is symmetrical with respect to the x-axis Fig.(1) . In expression of (5) only even functions of (y) are kept and the integration constants (Cm=Dm=0.0) are let the deflection surface Eq.(2) is then represented by the following expression:

$$w = \frac{q}{24D}(x^4 - 2ax^3 + a^3x) + \frac{qa^4}{D} * \sum_1^m (A_m\cosh\frac{m\pi y}{a} + B_m\frac{m\pi y}{a}\sinh\frac{m\pi y}{a}) * \sin\frac{m\pi x}{a} \qquad \ldots(6)$$

which satisfies Eq.(1) and also the boundary conditions at the sides (x=0.0 and x=a) . It remains now to determine the coefficients of Eq.(6) (Am and Bm) in such a manner as to satisfy the boundary conditions:

$$w = 0 \quad ; \quad \frac{\partial^2 w}{\partial y^2} = 0 \qquad \ldots(7)$$

On the sides y = ± b/2, we begin by developing expression from Eq.(6) in a trigonometric series which gives :

$$\frac{q}{24D}(x^4-2ax^3+a^3x)=\frac{4qa^4}{\pi^5 D}\sum_{m=1}^{\infty}\frac{1}{m^5}\sin\frac{m\pi x}{a}$$

where:
m=1, 3, 5 …
The deflection series Eq.(6) will be represented in the form :

$$w=\frac{qa^4}{D}\sum_{m=1}^{\infty}\left(\frac{4}{\pi^5 m^5}+Am\cosh\frac{m\pi y}{a}+Bm\frac{m\pi y}{a}\sinh\frac{m\pi y}{a}\right)\sin\frac{m\pi x}{a} \quad \dots(8)$$

Substituting the boundary conditions from Eq.(7) in the expression of Eq.(8) and using the notation give:

$$\frac{m\pi b}{2a}=\alpha m \quad \dots (9)$$

The following equation for determining the constant (*Am,Bm*) gives:

$$\frac{4}{\pi^5 m^5}+Am\cosh\alpha m+Bm\,\alpha m\sinh\alpha m=0$$
$$(Am+2Bm)\cosh\alpha m+Bm\,\alpha m\sinh\alpha m=0 \quad \dots(10)$$

from which :

$$\left.\begin{aligned} Am&=-\frac{2(\alpha m\tanh\alpha m+2)}{\pi^5 m^5\cosh\alpha m}\\ Bm&=+\frac{2}{\pi^5 m^5\cosh\alpha m}\end{aligned}\right\}$$

The boundary conditions at the sides (x=0.0 and x=a) and also (A_m and B_m) in such a manner as to satisfy the boundary conditions:

$$w=0 \quad ; \quad \frac{\partial^2 w}{\partial y^2}=0$$

Substituting these values of constants in Eq.(8) to obtain the equation of the plate surface deflection ,satisfying Eq.(1) and the boundary conditions are given in the following form :

$$w=\frac{4qa^4}{\pi^5 D}\sum_{m=1,3,5,\dots}^{\infty}\frac{1}{m^5}\left(1-\frac{\alpha m\tanh\alpha m+2}{2\cosh\alpha m}\cosh\frac{2y\alpha m}{b}+\frac{1}{2\cosh\alpha m}\frac{2y}{b}\sin\frac{2y\alpha m}{b}\right)*\sin\frac{m\pi x}{a} \quad \dots (11)$$

Numerical analysis

Consider a plate subjected to a distributed load (q) normal to its mid surface Fig.(2). The stresses and strain produce work that is stored in the system as strain energy (*W*) [7] such that:

$$W = \frac{1}{2}\int \{\sigma\}^T \{\varepsilon\}\mathrm{dv} \qquad \dots (12)$$

Where:

$\{\sigma\}^T$: transpose of stress vector .

$$\{\sigma\}^T = \{\sigma x, \sigma y, \tau xy\} \qquad \dots (13)$$

$\{\varepsilon\}$: strain vector

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon x \\ \varepsilon y \\ \gamma xy \end{Bmatrix} \qquad \dots (14)$$

v : volume of plate

The strain energy (W) may be expressed as :

$$W = \frac{1}{2}\int \{\sigma x, \sigma y, \tau xy\}\begin{Bmatrix} \varepsilon x \\ \varepsilon y \\ \gamma xy \end{Bmatrix}\mathrm{dv} = \frac{1}{2}\int (\sigma x \varepsilon x + \sigma y \varepsilon y + \tau xy \gamma xy)\mathrm{dv} \qquad \dots. (15)$$

The applied load (q) acting normal to the plate surface area produces the load potential energy (Ω), that is:

$$\Omega = -\int \{T\}^T \{u\} dA = -\int qwdA \qquad \dots (16)$$

The strain energy can be expressed as follows :

$$W = +\frac{1}{2}\int \{k\}^T [D]\{k\} dA = \qquad \dots (17)$$

$$\frac{1}{2}\int \{-kx \quad -ky \quad -2kxy\}[D]\begin{Bmatrix} -kx \\ -ky \\ -2kxy \end{Bmatrix}\mathrm{dv}$$

Where for the isotropic case, the elasticity matrix [D] can be written as in [8]:

$$[D] = \frac{E\ t^3}{12\ (1-\nu^2)}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 0.5(1-\nu) \end{bmatrix} \qquad \dots (18)$$

For the nonlinear thermoviscoelastic case *E(t)* and according to the practical tests have been described practical results of the equation coefficient relax, including the impact of all of time, strain and temperature composite material as in the following:

$$E(t,\varepsilon,T) = E(\varepsilon,T) * t^{n(\varepsilon,T)} \qquad \dots (19)$$

so that , the stress relaxation expressed as follows :

$\sigma(t,\varepsilon,T) = \sigma(\varepsilon,T) * t^{ns(\varepsilon,T)}$ ….(20)

The curvature displacement Eqs.(14) can be expressed in a matrix form as follows :

$$\{k\} = [B]w = \begin{bmatrix} -\dfrac{\partial^2}{\partial x^2} \\ -\dfrac{\partial^2}{\partial y^2} \\ -2\dfrac{\partial^2}{\partial x \partial y} \end{bmatrix} w \qquad \text{….(21)}$$

The nodal displacements of the plate element with (n=4) (four node) is shown in Fig.(3). Each node gives the following three degrees of freedom.The transverse deflection w, rotations about x-axis θx and rotations about y-axis θy.
The element deformed shape can be approximated with a suitable set of shape functions *Ni* (x,y) as shown in appendix A:

$$w(x,y) = \sum_i Ni(x,y)ui = [N]\{u\} \qquad \text{… (22)}$$

where the nodal displacements (*ui*) are:

$$\{u\}^T = \begin{Bmatrix} w_1, \theta_1 x, \theta_1 y, w_2, \theta_2 x, \theta_2 y, ... \\ ...., w_n, \theta_n x, \theta_n y \end{Bmatrix} \qquad \text{…(23)}$$

Substituting Eq.(22) and (23) into the strain nodal displacement Eq.(21) , the matrix [*B*],Eq.(21), can be written in the expanded form :

$$[B] = \begin{bmatrix} -\dfrac{\partial^2 N1}{\partial x^2} & -\dfrac{\partial^2 N2}{\partial x^2} & -\dfrac{\partial^2 N3}{\partial x^2} & .... & -\dfrac{\partial^2 Nn}{\partial x^2} \\ -\dfrac{\partial^2 N1}{\partial y^2} & -\dfrac{\partial^2 N2}{\partial y^2} & -\dfrac{\partial^2 N3}{\partial y^2} & .... & -\dfrac{\partial^2 Nn}{\partial y^2} \\ -2\dfrac{\partial^2 N1}{\partial x \partial y} & -2\dfrac{\partial^2 N2}{\partial x \partial y} & -2\dfrac{\partial^2 N3}{\partial x \partial y} & .... & -2\dfrac{\partial^2 Nn}{\partial x \partial y} \end{bmatrix} \qquad \text{… (24)}$$

Sub Eq.(22) in Eq.(21) and the result in Eq.(17) the strain energy equation can be written as :

$$W = \frac{1}{2}\int \{u\}^T [B]^T [D][B]\{u\} dA \qquad \text{…(25)}$$

where the element stiffness [*K*]e is given by :

$$[K]_e = \iint [B]^T [D][B] dxdy \qquad \text{….(26)}$$

And the vector of the equivalent nodal force {*f*}e is :

$$\{f\}_e = \iint q(x,y)[N] dA \qquad \text{…(27)}$$

Thus, the equilibrium for plate element can be expressed in the concise form as:

$$\{f\}_e = [K]_e \{u\}_e \quad \ldots(28)$$

the strain matrix [B] can be written as follows:

$$[B] = [Q][C]^{-1} \quad \ldots(29)$$

Where:

[C]: 12*12 matrix depending on nodal co-ordinate.

The element stiffness matrix can be evaluated by substituting Eq.(29) in Eq. (26), that is:

$$[K]^e = [C^{-1}]^T \left(\iint [Q]^T [D][Q] dxdy \right) [C]^{-1} \quad \ldots(30)$$

An explicit expression for stiffness matrix [K] has been evaluated [9]. A computer program was developed by employing orthotropic element for plane stress and analysis Newton Raphson method was applied in numerical analysis. A computer program was developed use Fortran language employing four nodal element and the flow chart is shown in Figure (4).

The experimental:

By depending on the creep tests for a number of specimens limited the nonlinear limit load shown in Figure (5). Depending on the creep tests are to determine load leading to the nonlinear behaviour of a sample creep as well as determine the relaxation coefficient, including the effect of time, strain and temperature based on those experimental data. The experimental properties nonlinear thermoviscoelastic of composite material (polyester with a curing rate 0.8% reinforced with random chopped) at different stresses and temperatures to predict a new creep compliance D(t,σ ,T) and the relaxation modulus E(t,ε,T) equations from creep and relaxation tests .

The new equations (Eq. (19) and Eq. (20)) applied in numerical and theoretical analysis to calculate the deflection in composite thin plate have the same volume fraction of creep specimens (0.26) under different distributed load (q) with thickness (t_k) , relative dimensions (a/b=0.5) as shown in Fig.(6) and temperatures from (30 C^o to 60 C^o). The experimental data compared with numerical and theoretical analysis as shown in Figures (5, 6, 7, and 8).

The results and discussion

The Effect of Temperatures:

The deflection of composite thin plate increases with approximate rate (34.6%) with increasing the temperature from (30 C^o-60 C^o) where that increasing rate created from the thermal strain as a result of increasing the temperature of specimen environment. It was found that increasing rate of deflection very high at the beginning (0<t<15 min.) then the rate of increase is nearly constant as shown in Figures (8, 9, 10 and 11).

The Effect of distributed load:

The deflection of composite thin plate (w):

Due to apply the uniform load on the composite specimen cause an increase in the deflection, which is growing rapidly until the period (3 minutes), starting from the application of load, then, increasing rate of the deflection is limited.

Figure(12) shows the increasing of the deflection (*w*) with the increase of approximate rate 80% as a result of increasing the distributed load from (q=1.934E-3 N/mm^2 to q=3.4488E-3 N/mm^2) at (T=30 C^o), relative dimension (a/b=0.5) and simply support plate.

Shear stress (τxy):

The shear stress concentrated at the corner of thin plate when the expected failure occurs. The shear stress distributed symmetrical on the diagonal of rectangular plate. The minimum value of shear stress (τ_{xy}) in rectangular composite thin plate is in the center of plate.

Due to increased load applied with the time leads to reduced stress relaxation composite material modulus and thus increase the deflection of specimen and increase the shear stresses generated in thin composite plate.

Figure(13) shows that the increasing distributed load increase the shear stress (τ_{xy}) with approximate rate (58%) after passing 30 min for composite simply support thin plate with relative dimension (a/b=0.5) and T=30 C^o .

Strain in x-axis εx and strain in y-axis εy:

The maximum value of (ε_x) concentrate along (b=180 mm) of simply support plate where the maximum value of (ε_y) concentrate along (a=80 mm) from simply support plate as shown in Figures (14, 15). the rate of decreasing of the stress relaxation modulus for two reasons the first, increase the uniform distribution load and the second that the temperature increase of due to increase strain in x-axis and y-axis . The strain in x-axis ($\varepsilon_{x)}$ increases with approximate rate (97.5 %) and strain in y-axis (ε_y) increases with approximate rate (61%) as a result of increasing the distributed load from (q=1.934E-3 N/mm^2 to q=3.4488E-3 N/mm^2) at (T=40 C^o), relative dimension (a/b=0.5) and simply support plate.

Rotation about X-axis θ_x and Rotation about Y-axis θ_y:

The increasing deflection of specimen supported all their ends, which resulted in increased regular load and therefore cause an increase the angle of rotation about the x-axis and y-axis. The rotation was in the central point equal to zero due to symmetry while the maximum value of the rotation about x-axis and y-axis concentrating at end of thin plate.

Figures (16) and (17) show the effect of increasing the distributed load from (q=1.934E-3 N/mm^2 to q=3.4488E-3 N/mm^2) at (T=30 C^o), time (30 min) ,relative dimension (a/b=0.5) and simply support plate where the rotation about x-axis (θ_x) and the rotation about y-axis (θ_y) increase with approximate rate (70 %).

Conclusions:

1- Prediction a new Kernel equation ε(σ,T) to describe the creep strain equation ε(t,ε,T) by using a polynomial function of fourth degree.

$\varepsilon(t,\sigma,T)=\varepsilon(\sigma,T) * t^{n(\sigma,T)}$

where:

$n(\sigma,T)=f_A(T)+ f_B(T) * \sigma+f_C(T)*\sigma^2)$

$\varepsilon(T,\sigma)=f_1(\sigma)+f_2(\sigma)*T+f_3(\sigma)*T^2+f_4(\sigma)*T^3 +f_5(\sigma)*T^4$

Where:

$f_1(\sigma) = C_1+C_2*\sigma + C_3*\sigma^2$
$f_2(\sigma) = C_4+C_5*\sigma + C_6*\sigma^2$
$f_3(\sigma) = C_7 + C_8*\sigma + C_9*\sigma^2$
$f_4(\sigma) = C_{10}+ C_{11}*\sigma + C_{12}*\sigma^2$
where*:*
$C_1, C_2, C_3\ C_4, \ldots\ldots, C_{12} =$ Constant

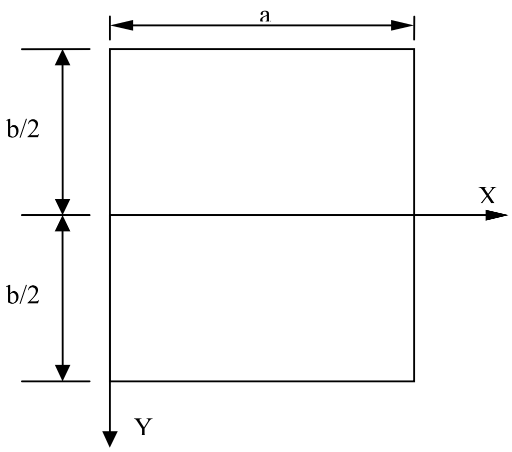
2- Suggestion of a new equation of creep compliance function D(t,σ,T) and their slope as a function of stress and temperature n(σ,T) to describe the nonlinear thermoviscoelastic behaviour of composite thin plate at different variables by using polynomial equation.

$D(t,\sigma,T)= D(\sigma,T) * t^{n(\sigma,T)}$
Where:
$D(\sigma,T)= f_{AD}(T)*\sigma + f_{BD}(T)$
Where:
$f_{AD}(T)= D_1+ D_2*T+ D_3*T^2 + D_4*T^3$
$f_{BD}(T)= D_5+ D_6*T + D_7*T^2 + D_8*T^3$
where:
$D_1, D_2, D_3\ D_4, \ldots\ldots, D_8 =$ Constant

3- Prediction a new equation of relaxation modulus E(t,ε,T) and their slope as a function of strain and temperature to describe the nonlinear thermoviscoelastic behaviour of composite thin plate at different variables by using the following polynomial equation.

$E(t,\varepsilon,T)=E(\varepsilon,T)* t^{ns(\varepsilon,T)}$
Where:
$ns(\varepsilon,T)=f_{ANS}(\varepsilon)+f_{BNS}(\varepsilon)*T+f_{CNS}(\varepsilon)*T^2 +f_{DNS}(\varepsilon)*T^3+f_{ENS}(\varepsilon)*T^4$
Were:
$f_{ANS}(\varepsilon)=E_1+ E_2*\varepsilon + E_3*\varepsilon^2$
$f_{BNS}(\varepsilon)=E_4+ E_5*\varepsilon+ E_6*\varepsilon^2$
$f_{CNS}(\varepsilon)=E_7+ E_8*\varepsilon+ E_9*\varepsilon^2$
$f_{DNS}(\varepsilon)=E_{10}+ E_{11}*\varepsilon+ E_{12}*\varepsilon^2$
$f_{ENS}(\varepsilon)=E_{13}+ E_{14}*\varepsilon+ E_{15}*\varepsilon^2$
where:
$E_1, E_2, E_3\ E_4, \ldots\ldots, E_{15} =$ Constan**t**

4- The comparison among theoretical, experimental and FEM gives a very good agreement with limited disparity percentage.



Figure(1) Simply supported plate

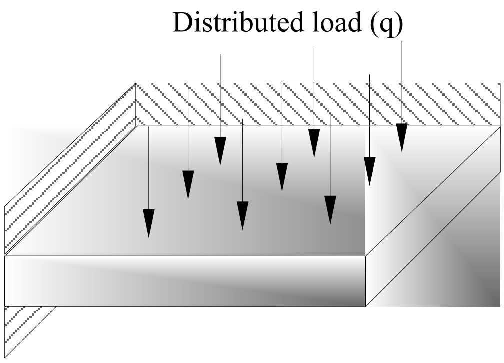


Figure.(2) Plate subjected to distribution load (q)

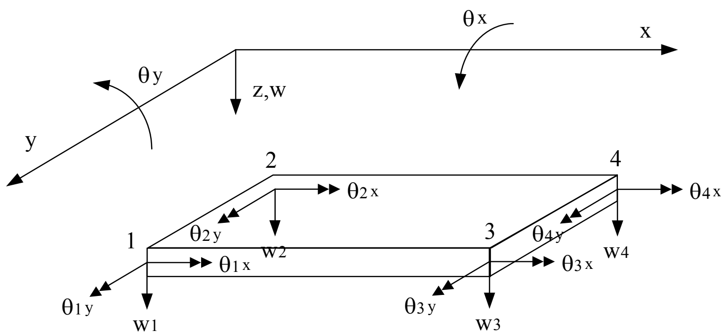


Figure.(3) Nodal displacement of plate element

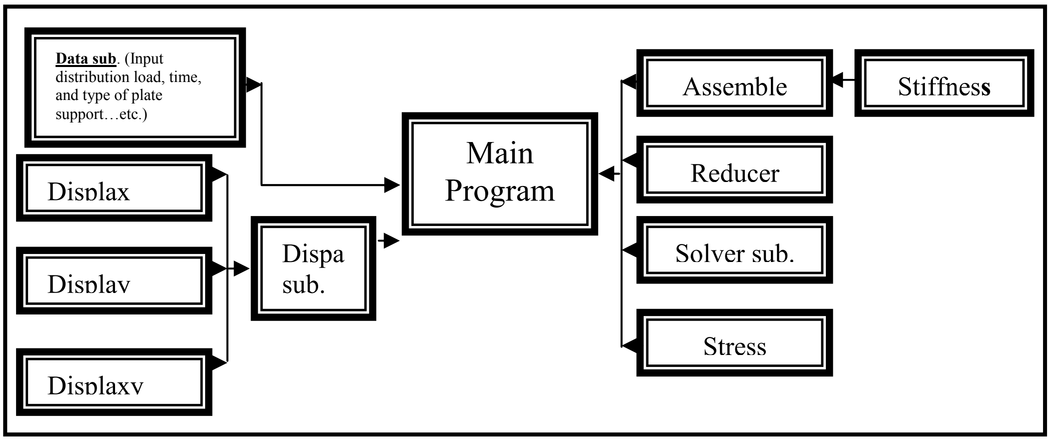


Figure. (4) Flowchart for nonlinear thermoviscoelastic behavior Program by finite element method

Figure.(5) Photograph of composite creep test specimen

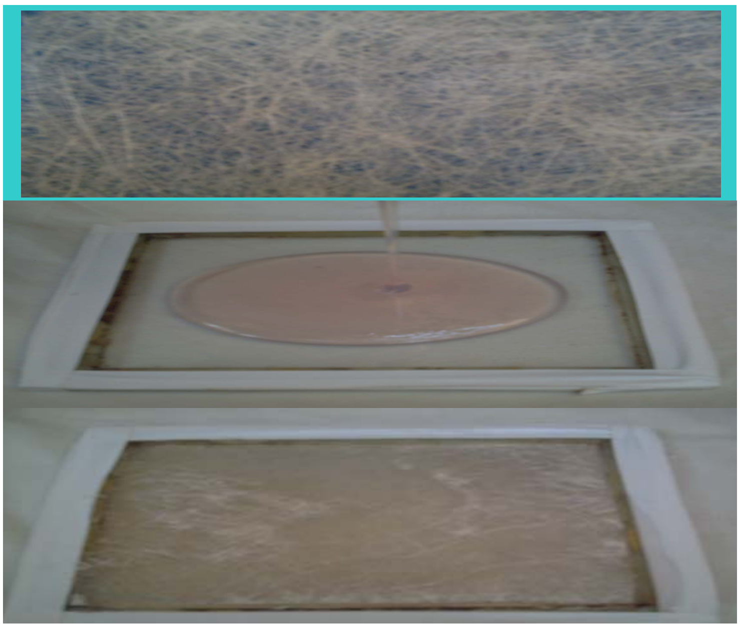
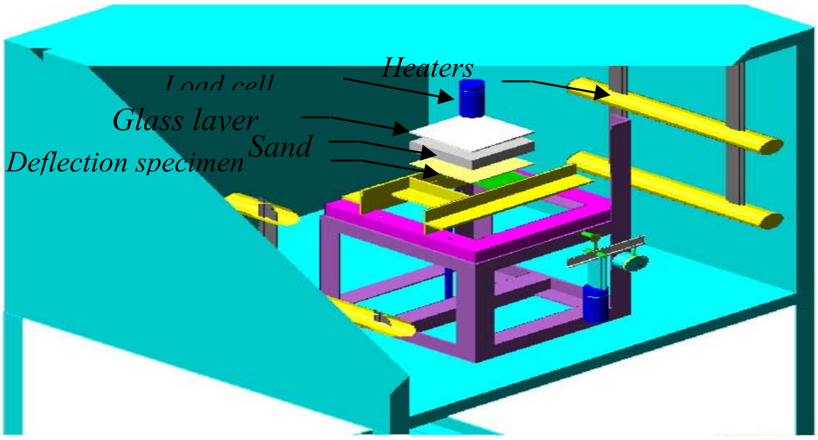
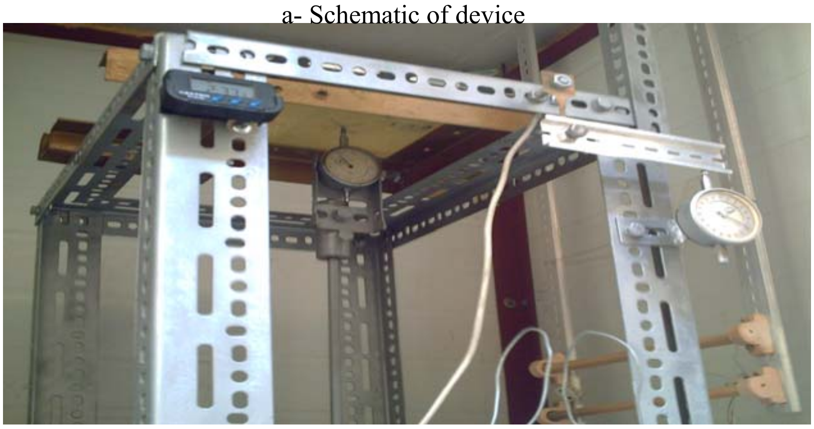


Figure.(6) Preparing deflection specimens.



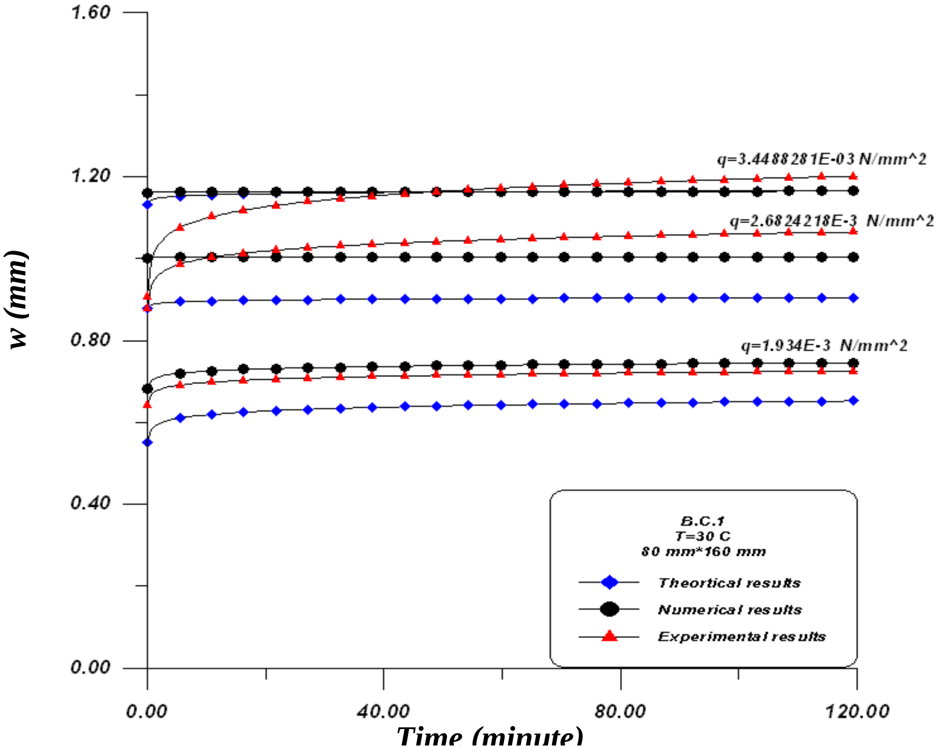
a- Schematic of device



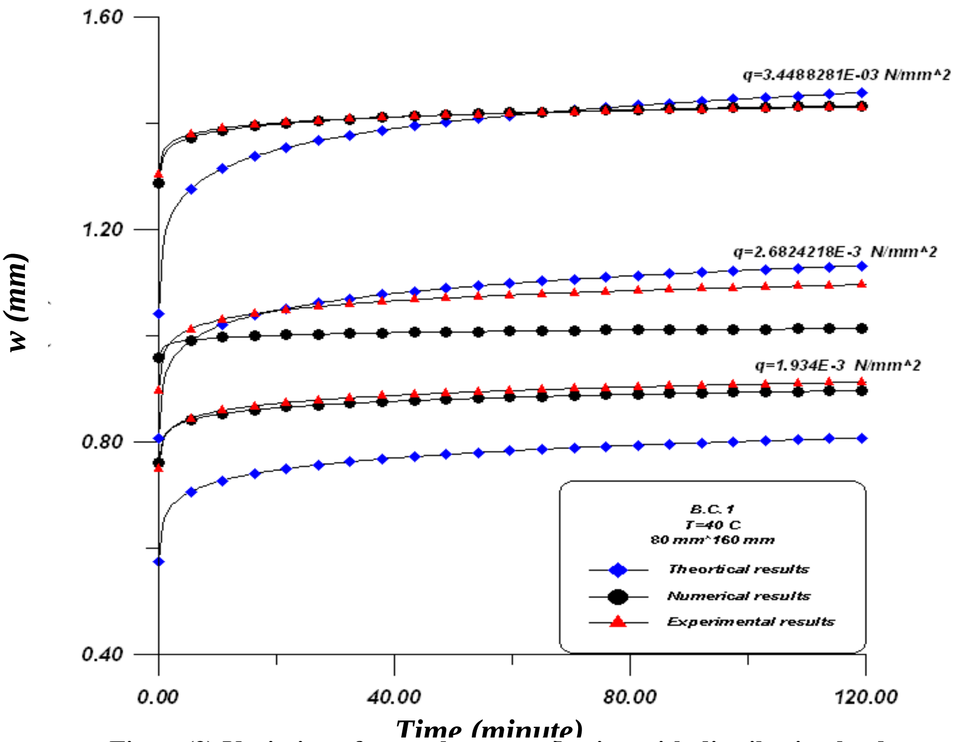
b- Photograph of equipment

c- Control board of device

Figure.(7) Deflection equipment



Figure(8) Variation of central point deflection with distribution load at T=30 C°



Figure(9) Variation of central point deflection with distribution load at T=40 C°

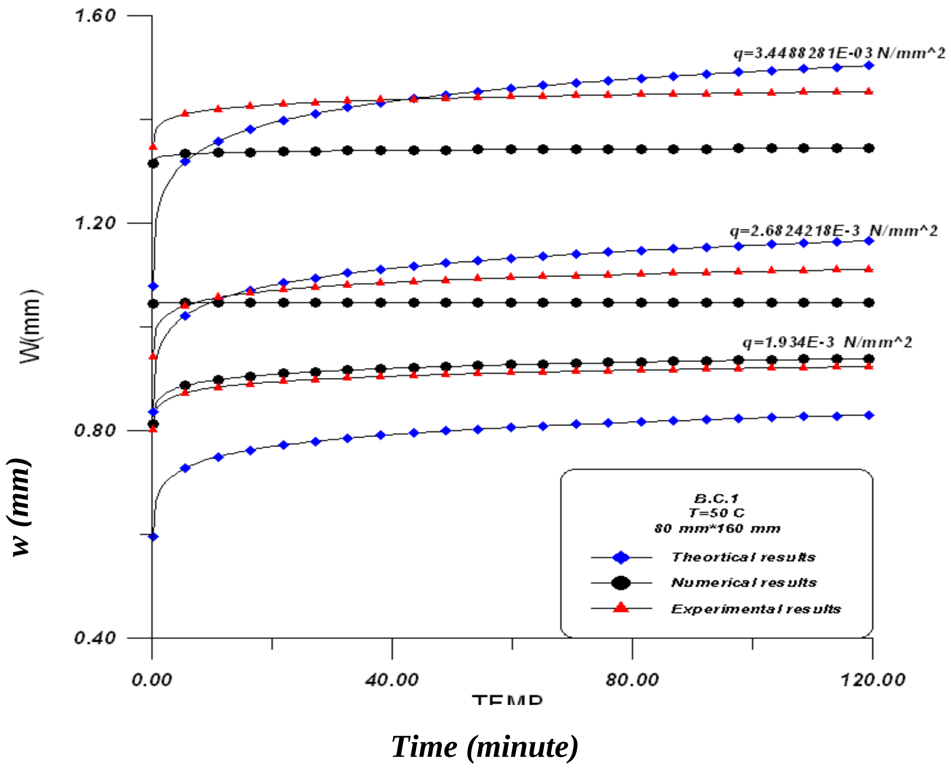
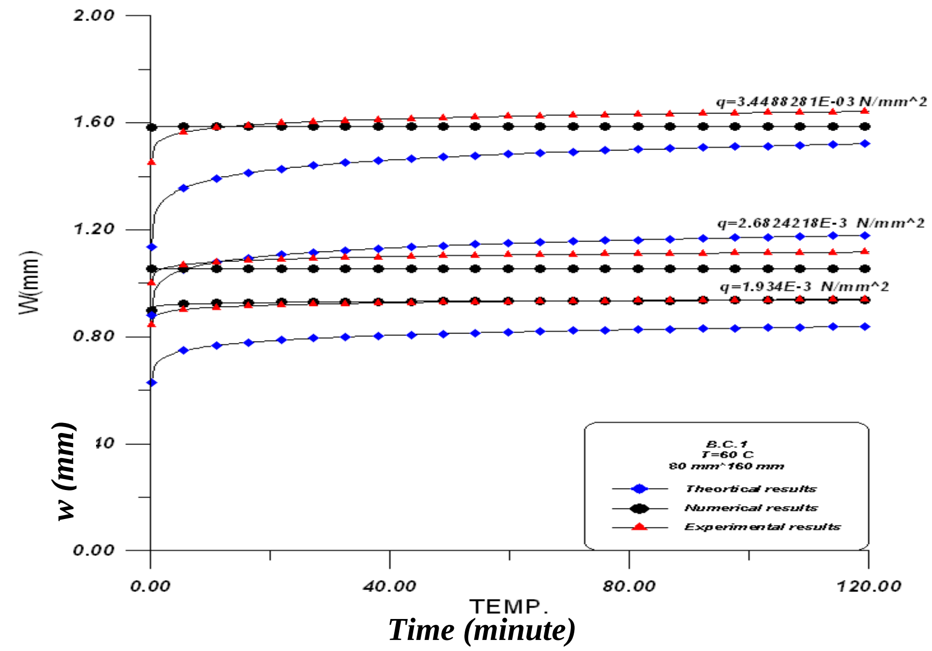


Figure.(10) Variation of central point deflection with distribution load at T=50 C°



Figure(11) Variation of central point deflection with distribution load at T=40 C°

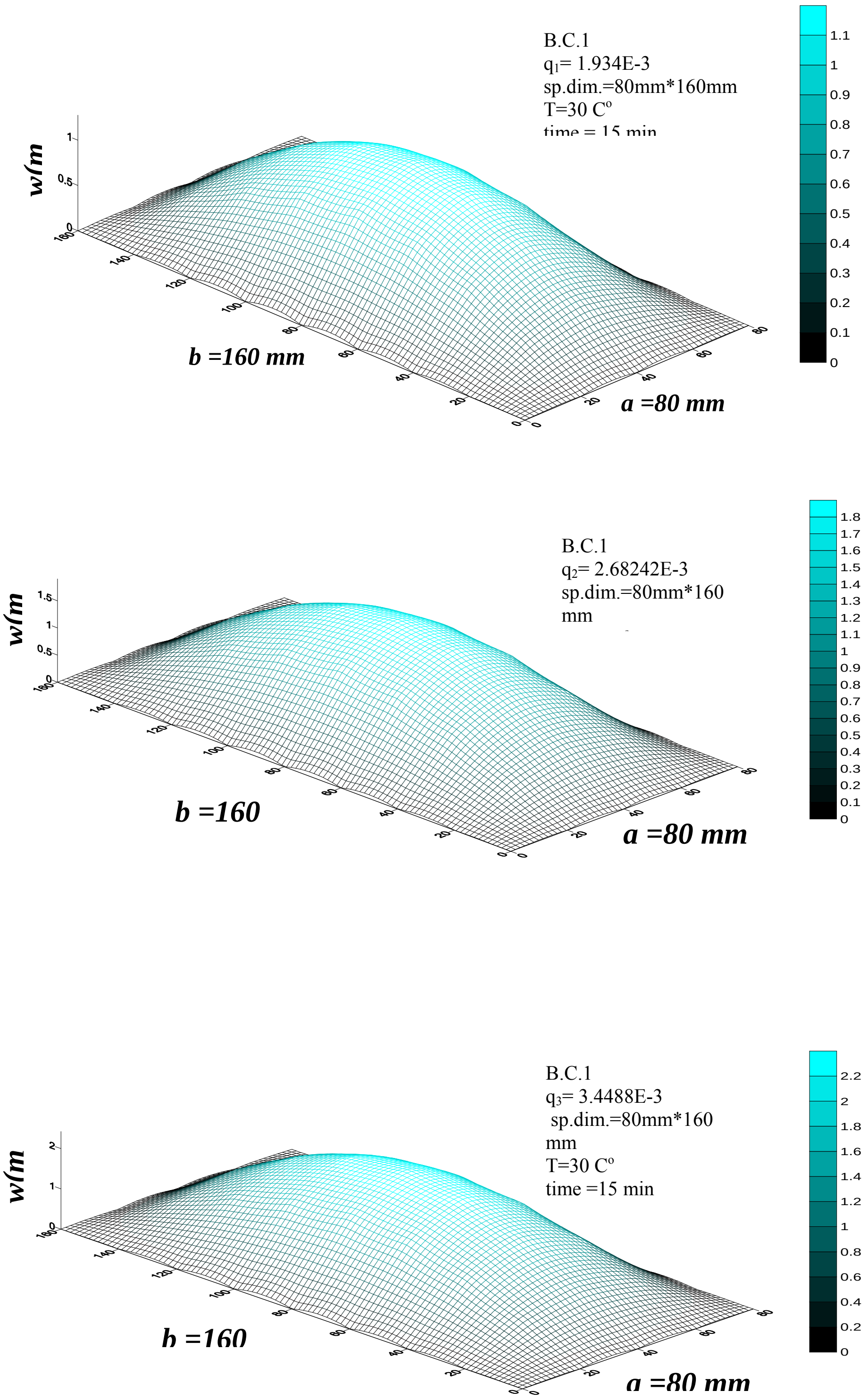


Figure.(12) Comparison of the 3D deflection among three distributed loads

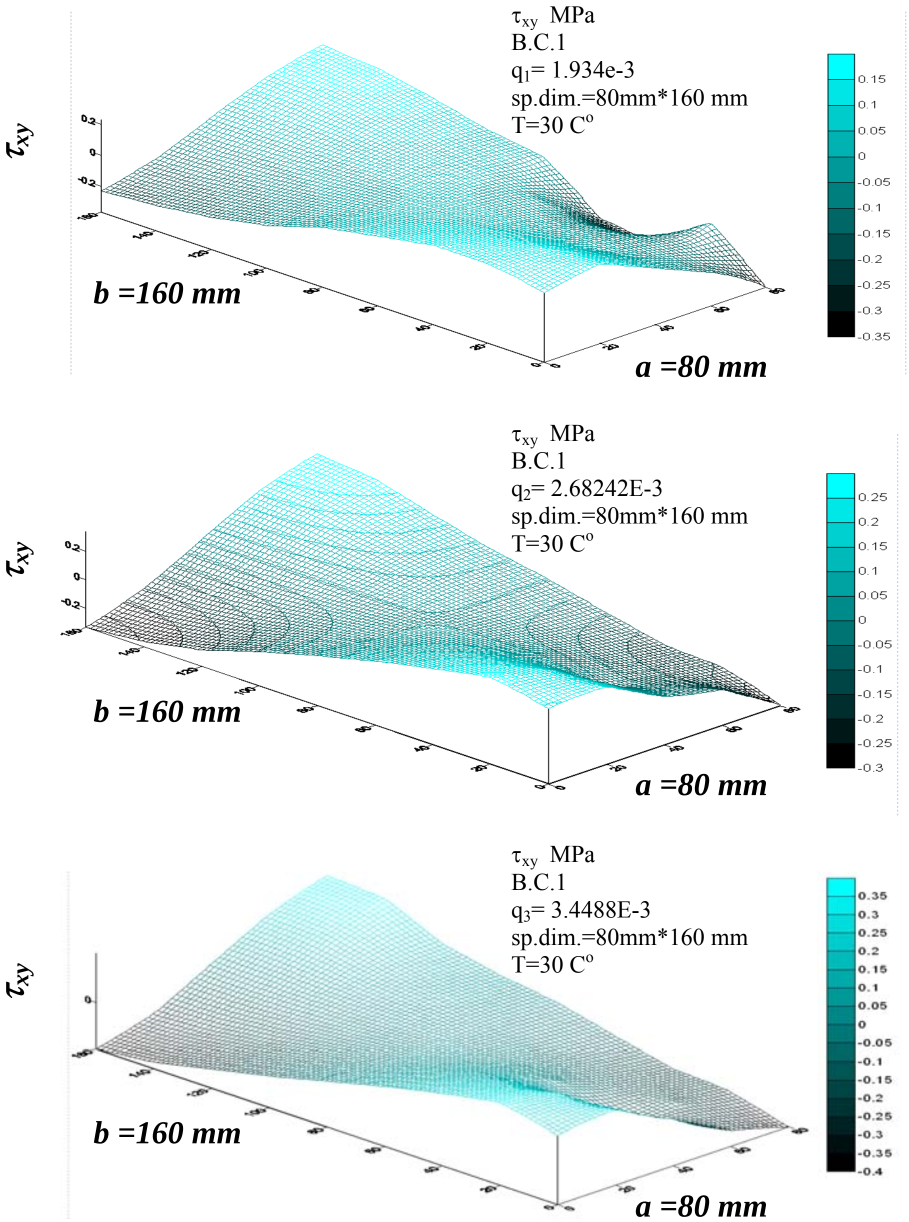
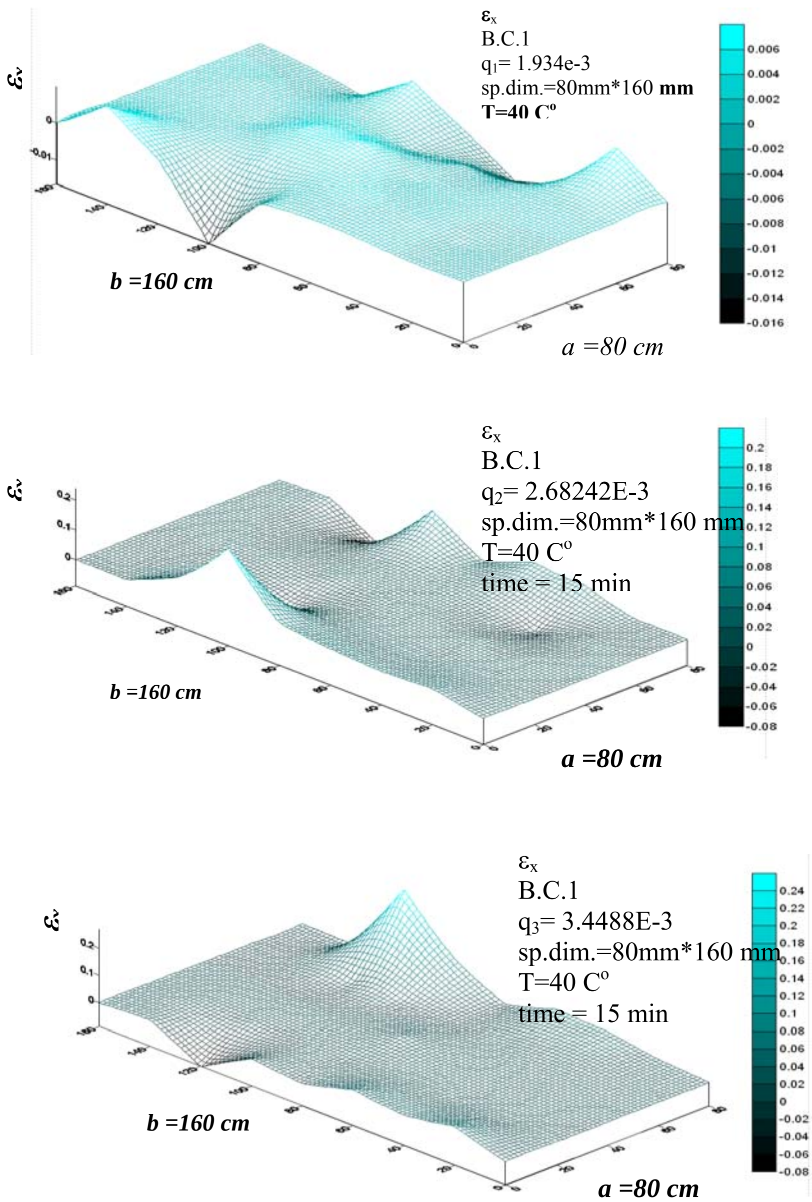


Figure.(13) 3D-shear stress at three different distributed loads.



Figure(14) 3D strain in x-axis direction

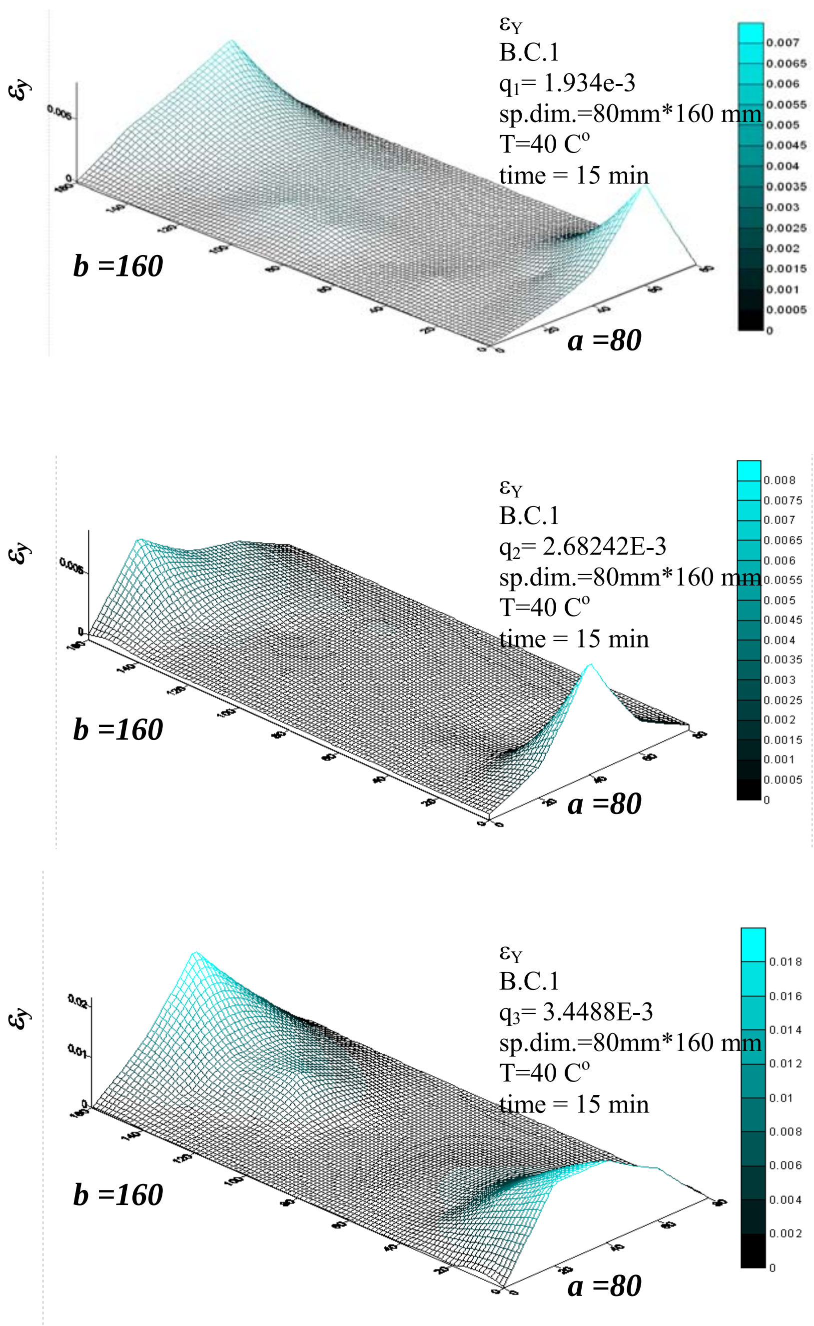


Figure.(15) 3D strain in y-axis direction

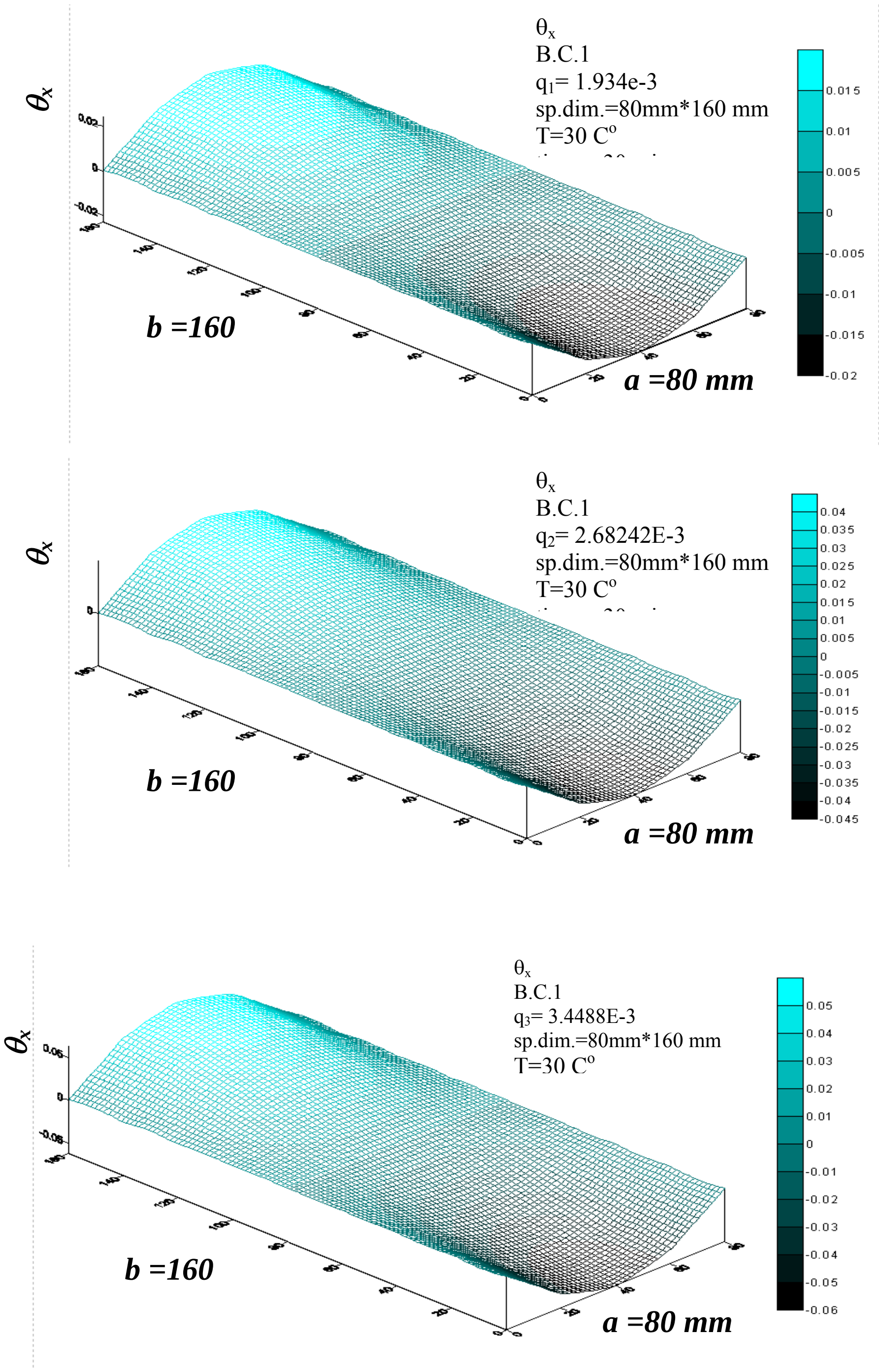
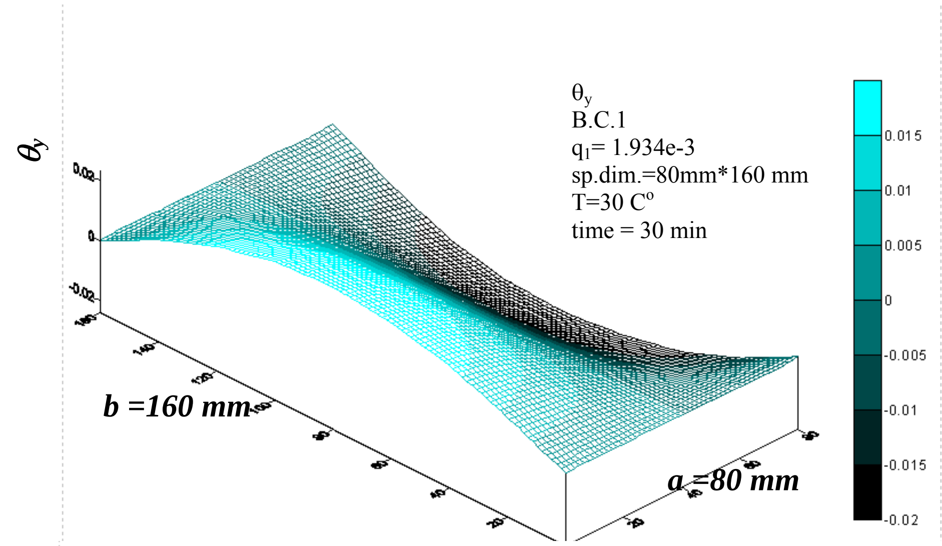
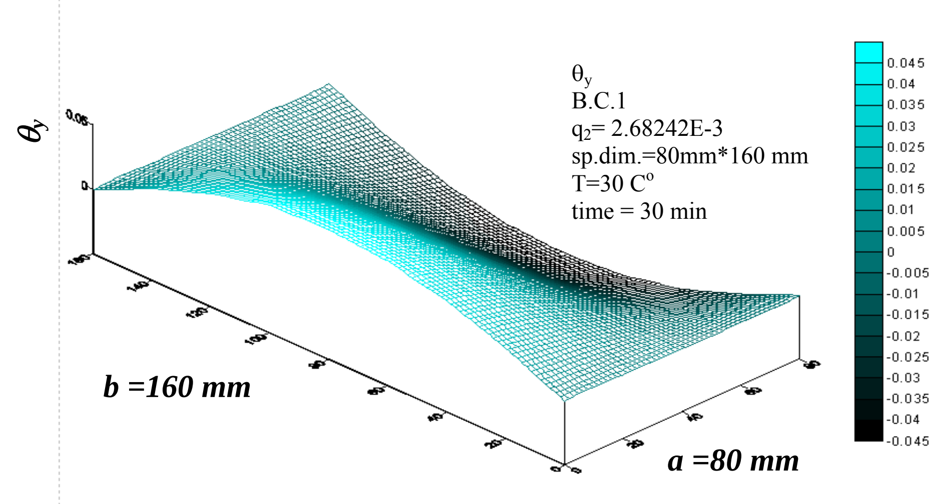


Figure.(16) 3D-Rotation about x-axis





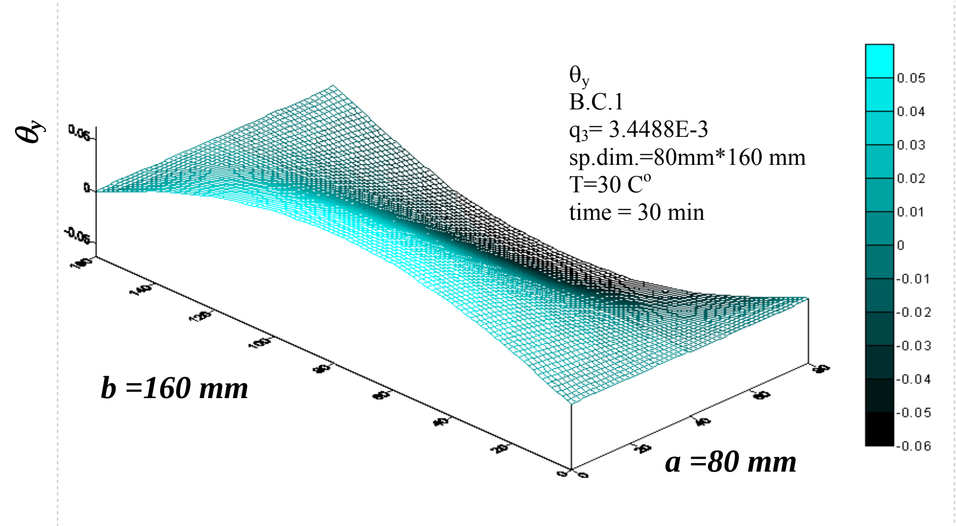


Figure.(17) 3D-Rotation about y-axis

Appendix (A)

$$[N] = \frac{w}{\{a^e\}} \quad \text{... (A-1)}$$

where :

$$\{a\}^e = \begin{Bmatrix} ai \\ aj \\ al \\ ak \end{Bmatrix} \quad \text{...(A-2)}$$

and ;

$$ai = \begin{Bmatrix} wi \\ \theta xi \\ \theta yi \end{Bmatrix} = \begin{Bmatrix} wi \\ -\left(\frac{\partial w}{\partial y}\right)i \\ -\left(\frac{\partial w}{\partial x}\right)i \end{Bmatrix} \quad \text{... (A-3)}$$

A polynomial expression is used to define the shape function of the twelve parameters.

$$w=\alpha_1+\alpha_2x+\alpha_3y+\alpha_4x^2+\alpha_5xy+\alpha_6y^2+\alpha_7x^3+\alpha_8x^2y+\alpha_9xy^2+\alpha_{10}y^3+\alpha_{11}x^3y+\alpha_{12}xy^3 \quad \text{... (A-4)}$$

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