General Control Method for Complete Synchronization of Two-Arbitrary 2-D Chaotic Systems in Discrete-Time

طريقة سيطرة عامة من اجل مواقتة تامة لنظامين فوضويين ثنائي الأبعاد في الزمن المتقطع

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Abstract:
An advanced general method of synchronization is proposed in this research. Where a couple of arbitrary two dimensions (2-D) chaotic systems in discrete time were synchronized. Scheme of synchronization based on nonlinear controllers and Lyapunov stability theory. Master and slave generalized of Lorenz system and Fold system were used for (2-D) chaotic systems. The advantage of modified method is revealed by mathematical analytic and numerical simulation.
Key words: (2-D) discrete time systems, Chaos synchronization, Lyapunov stability theory, nonlinear controllers.

الخلاصة:
في هذا البحث تم اقتراح طريقة عامة ومطورة من اجل المواقتة. حيث تم مواقتة زوج من الأنظمة الفوضوية المتحكمة ثنائية الأبعاد (2-D) في الزمن المتقطع. بني مخطط المواقتة على أساس المتحكمات اللاخطية ونظرية لايابونوف للاستقرارية. تعميم (السيد- الخادم) لنظام لورينزو ولنظام فولد استخدمت للأنظمة الفوضوية ثنائية الأبعاد (2-D). استخدام التحليل الرياضي والمحاكاة العددية من اجل إظهار ميزات هذه الطريقة المطورة .
الكلمات المفتاحية: (2-D) أنظمة الزمن المتقطع، مواقتة الفوضى، نظرية لايابونوف للاستقرارية، المتحكمات اللاخطية.

1. INTRODUCTION

Recently, synchronization of chaotic systems in discrete-time attracts attention of researchers of many areas of science and technology. One of the main reasons for this interest is the possibility to apply to communications security [1,2]. Many mathematical models of different scientific processes were defined using discrete-time dynamical systems.

It has been presenting many types of chaotic synchronization as complete synchronization [3], anti-synchronization [4], generalized synchronization [5], projective synchronization [6] and generalized projective synchronization [7]. The representation of various methods and techniques to synchronize chaotic systems in order to achieve some types of synchronized chaos in continuous time systems, such as OGY method [8, 9], PC method [10], feedback approach [11], active and adaptive control [12, 13], sliding mode approach [14], back stepping design technique [15].

In this work, it has been using nonlinear controllers and Lyapunov stability theory. A general method in discrete-time is proposed to achieve complete synchronization between two arbitrary (2-D) chaotic systems. In order to verify the effectiveness of modified approach, the proposed scheme is applied in two (2-D) chaotic maps: Lorenz discrete-time system [16] and Fold discrete-time system [17].

2. MATHEMATICAL ANALYSIS

Consider the following master system:

$$\{x_i\,(k+1) = f_i\big(X(k)\big), \qquad 1 \le i \le 2 \tag{1}$$

Where $X(k)$= $(x_i\,(k))$, is the state vector of master system, and (f_i) as the slave system.

Taking the following system:

$$\begin{cases} y_i\,(k+1) = \sum_{j=1}^{2} b_{ij} y_j(k) + g_i\big(Y(k)\big) + u_{ij} \qquad 1 \le i \le 2 \end{cases} \tag{2}$$

Where $Y(k)$= $(y_i(k))$, is the state vector of slave system, (b_{ij}) and (g_i) are nonlinear functions, (u_{ij}) are controllers to be determined.

Many 2-D chaotic maps can be written under the form of master system, Equ. (1), such as Hénon map, Lozi map, Fold map and Lorenz discrete-time system.
The synchronization problem is to find the controllers(u_i) $where\ 1 \le i \le 2$, which stabilize the synchronization error.

$$e_i(k) = y_i(k) - x_i(k) \qquad where\ 1 \le i \le 2 \tag{3}$$

The aim of synchronization is to make $\lim_{k\to\infty} e_i(k) = 0, where\ (i = 1,2)$. The synchronization errors between the master system, Equ.(1) and the slave system, Equ.(2) can be derived as follow:

$$\begin{cases} e_i\,(k+1) = \sum_{j=1}^{2} b_{ij} e_j(k) + u_i + L_i + N_i \qquad 1 \le i \le 2 \end{cases} \tag{4}$$

Where

$$\begin{cases} L_i = \sum_{j=1}^{2} b_{ij} x_j(k) \qquad 1 \le i \le 2 \end{cases} \tag{5}$$

And

$$N_i = g_i - f_i \qquad where\ 1 \le i \le 2 \tag{6}$$

The master system, Equ. (1) and the slave system, Equ. (2)are globally synchronized under the following controllers:

$$\begin{cases} u_1 = \left(\frac{l}{\sqrt{2}} - b_{11}\right) e_1(k) + \left(\frac{l}{\sqrt{2}} - b_{12}\right) e_2(k) - L_1 - N_1 \\ u_2 = \left(\frac{l}{\sqrt{2}} - b_{21}\right) e_1(k) + \left(\frac{l}{\sqrt{2}} - b_{22}\right) e_2(k) - L_2 - N_2 \end{cases} \tag{7}$$

Where the control constant is chosen as: $|l| < 1$ (8)

By substituting Equ. (7) in to Equ. (4), the synchronization errors can be written as:

$$e_1(k+1) = \left(\frac{l}{\sqrt{2}}\right) e_1(k) + \left(\frac{l}{\sqrt{2}}\right) e_2(k)$$
$$e_2(k+1) = \left(\frac{l}{\sqrt{2}}\right) e_1(k) - \left(\frac{l}{\sqrt{2}}\right) e_2(k) \quad (9)$$

For stability analysis, let us consider the following quadratic Lyapunov function [18]:

$$V(e(k)) = \sum_{i=1}^{2} e_i^2(k), \quad (10)$$

By using Equ. (9), we obtain:

$$\begin{cases} \Delta V(e(k)) = V(e(k+1)) - V(e(k)) \\ = \sum_{i=1}^{2} e_i^2(k+1) - \sum_{i=1}^{2} e_i^2(k) \\ = \frac{l^2}{2} e_1^2(k) + l^2 e_1(k) e_2(k) + \frac{l^2}{2} e_2^2(k) + \frac{l^2}{2} e_1^2(k) - l^2 e_1(k) e_2(k) + \frac{l^2}{2} e_2^2(k) - e_1^2(k) - e_2^2(k) \\ = (l^2 - 1)(e_1^2(k) - e_2^2(k)) \end{cases}$$

Since the value of control constant (l) is chosen less than one, according to Equ. (8), then the value of $(l^2 - 1)(e_1^2(k) - e_2^2(k))$ will necessarily be negative.

Then we get: $\Delta V(e(k)) < 0$, thus by Lyapunov stability It has demonstrated on goal of Synchronization which is [18]:

$$\lim_{k\to\infty} e_i(k) = 0, \qquad i = 1,2 \quad (11)$$

So, the drive system, Equ. (1) and the response system, Equ. (2) are completely synchronized.

3. NUMERICAL SIMULATION

To demonstrate using of chaos synchronization criterion which proposed herein, an example of chaotic systems is considered in this section. The master system is described by the 2-D Lorenz discrete-time system.

$$\begin{cases} x_1(k+1) = (1+\alpha\beta) x_1(k) - \beta x_2(k) x_1(k) \\ x_2(k+1) = (1-\beta) x_2(k) + \beta x_1^2(k) \end{cases} \quad (12)$$

Where $x_1(k), x_2(k)$are the states and α, β are bifurcation parameters of the system. The 2-D Lorenz discrete time system has a chaotic attractor when $\alpha = 1.25$ and $\beta = 0.75$ [18]. The chaotic attractor of the system depicts in Fig. (1).

The slave system is described by the controlled Fold discrete-time system, as below:

$$\begin{cases} y_1(k+1) = y_2(k) + a y_1(k) + u_1 \\ y_2(k+1) = b^2 + y_1(k) + u_2 \end{cases} \quad (13)$$

Where $y_1(k), y_2(k)$are the states, a, b are bifurcation parameters of the *controlled* system and (u_1, u_2) are synchronization controllers. The Fold discrete-time system is chaotic when $a = -0.1$ and $b = -1.7$ [19]. The chaotic attractor of slave system depicts in Fig. (2).

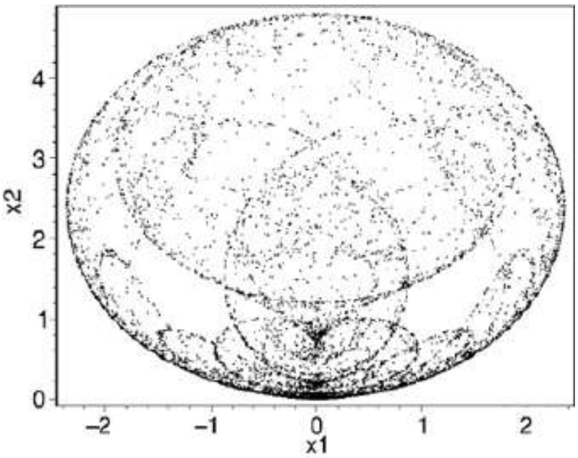

Fig.1: chaotic attractor of Lorenz discrete-time system.

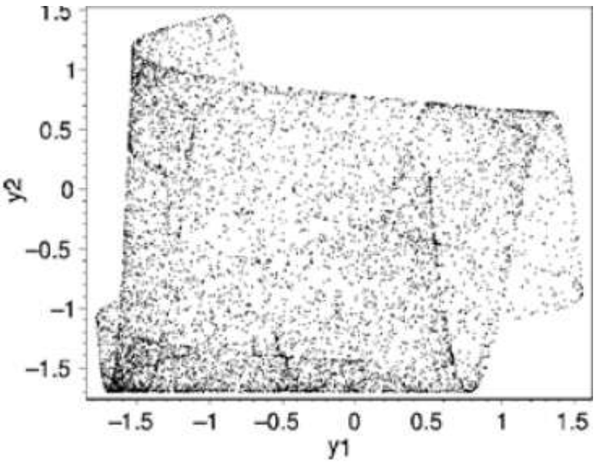

Fig.2: chaotic attractor of Fold discrete-time attractor.

According to Equ. (7), the synchronization controllers (u_1, u_2) are designed as follow:

$$\begin{cases} u_1 = \left(\frac{1}{3\sqrt{2}} - \alpha\right) e_1(k) + \left(\frac{1}{3\sqrt{2}} - 1\right) e_2(k) - y_2(k) - \alpha y_1(k) + (1 + \alpha\beta)x_1(k) - \beta x_2(x) x_1(k) \\ u_2 = \left(\frac{1}{3\sqrt{2}} - 1\right) e_1(k) - \left(\frac{1}{3\sqrt{2}}\right) e_2(k) - y_1(k) + (1 - \beta)x_2(k) + \beta x_1^2(k) - b^2 \end{cases} \quad (14)$$

And according to Equ. (9), the synchronization errors are derived as follow:

$$\begin{aligned} e_1(k+1) &= \left(\frac{1}{3\sqrt{2}}\right) e_1(k) + \left(\frac{1}{3\sqrt{2}}\right) e_2(k) \\ e_2(k+1) &= \left(\frac{1}{3\sqrt{2}}\right) e_1(k) - \left(\frac{1}{3\sqrt{2}}\right) e_2(k) \end{aligned} \quad (15)$$

After the result has been proved mathematically, algorithm was implemented using MATLAB to prove it simultaneously.

The following algorithm explains Synchronization errors between Lorenz discrete-time system and Fold discrete-time attractor according of Equ.(15).

1. e1(k)=-.5;e2(k)=-.5:initial values for e1(k+1) 2. e1(k+1)=(1/3*sqrt(2))*e1(k)+(1/3*sqrt(2))*e2(k) 3. for k=1:22 e1(:,k+1)=e1(:,1).*e1(:,k); end 4. e1(k)=-.4;e2(k)=1.2:initial values for e2(k+1)	5. e2(k+1)=(1/3*sqrt(2))*e1(k)-(1/3*sqrt(2))*e2(k) 6. for k=1:22 e2(:,k+1)=e2(:,1).*e2(:,k); end 7. x=[e1(k+1);e2(k+1)]; plot (x)

The result of implemented algorithm shown in Fig. (3).

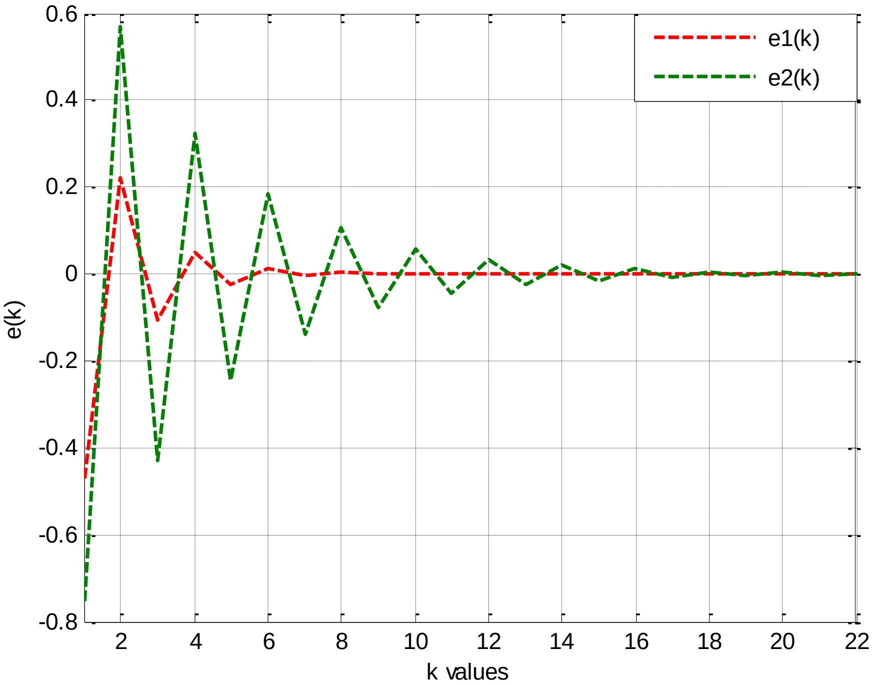

Fig.3: Synchronization errors between Lorenz discrete-time system and Fold

4. CONCLUSION

In this research, It has been represented modified nonlinear control method for coupled of arbitrary 2D chaotic systems in discrete-time. This method has been proven that the proposed controller was based on effective results. Fig. (3) is represented the result of numerical example which was utilized to illustrate the effectiveness of the proposed method.

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