

**Study the properties of the low-lying energy states  
for  $^{184,186}\text{Os}$  isotopes using IBM-1**

**دراسة خصائص مستويات الطاقة الواطنة لنظائر الاوزيميوم  $^{184,186}\text{Os}$  باستخدام  
IBM-1**

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**بحث مستل**

**Abstract:-**

The energy levels,  $B(E2)$  values, the square of rotational energy and the moment of inertia, staggering in  $\gamma$ - band energies and the potential energy surface for  $^{184,186}\text{Os}$  isotopes have been investigated using IBM-1. The calculated results good agreement with experimental data.

*Keywords:* IBM; Energy levels;  $B(E2)$  values; Staggering in  $\gamma$ - band energies; Potential energy surface

**الخلاصة**

تم حساب مستويات الطاقة، قيم  $B(E2)$ ، طاقة الدوران للحزمة الارضية لنظائر الاوزيميوم  $^{184,186}\text{Os}$  باستخدام نموذج تفاعل البوزونات الاول. وكانت النتائج متوافقة مع القيم العملية. كذلك تم حساب تمايل في حزمة  $\gamma$  وجهد طاقة السطح حيث اظهرت النتائج ان هذه النظائر تقع ضمن المنطقة الانتقالية  $SU(3)-O(6)$  وقريبة من التحديد  $SU(3)$ .

**1.Introduction**

The interacting boson model (IBM-1) was proposed by Arima and Iachello in (1974)[1, 2], it has become widely accepted to describe and predicting low-energy collective properties of complex nuclei. This model adopted in terms of the group  $U(6)$ [2], it has three dynamical symmetries corresponding to different nuclear shapes:  $U(5)$  a spherical nucleus that can vibrate,  $SU(3)$  an ellipsoidal deformed axially symmetric rotor, and  $O(6)$  an axially asymmetric rotor. Even-even osmium isotopes are interesting because it's lies in complex region and hard to populated experimentally. Many studies have been conducted on the structure of Osmium nucleus in recent years, P. Sarriguren et. al. were studied the evolution of shapes with the number of nucleons in various chains of Yb, Hf, W, Os, and Pt isotopes from neutron number  $N = 110$  up to  $N = 122$ , in (2008)[3]. K. Nomura. et al in (2011)[4], used interacting boson model (IBM) to calculate the transitions from prolate to oblate ground-state shapes. The energy levels (positive parity), the reduced probability of  $E2$  transitions, the intrinsic quadrupole moment  $Q_0$ , and the potential energy surface for  $^{184}\text{W}$  and  $^{184}\text{Os}$  nuclei were calculated using IBM-1 by F. I. Sharrad et al. in (2013)[5]. In (2015) [6],

the energy levels for the ground-state band, the properties of the ground and excited-state bands, negative parity band, the  $\gamma$ -band and  $\beta$ -band states for  $^{178-188}\text{Os}$  isotopes, have been calculated using Interacting Boson model by I. Mamdouh and M, Al-Jubbori.

In this present work were investigated the energy levels, probability of electromagnetic transitions  $B(E2)$ , and potential energy surface for  $^{184-194}\text{Os}$  isotopes using IBM-1

**2. Theory**

**2.1. The Interacting Boson Model (IBM-1) :-**

The Hamiltonian operator according to IBM- 1 describes the system of s (L= 0) and d (L= 2) boson can be written as follows[1, 7]

$$\hat{H} = \varepsilon \hat{n}_d + a_0 \hat{p}^\dagger \cdot \hat{p} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{T}_3 \cdot \hat{T}_3 + a_4 \hat{T}_4 \cdot \hat{T}_4 \quad (1)$$

where:-

( $\varepsilon, a_0, a_1, a_2, a_3,$  and  $a_4$ ) are parameter used in IBM-1 ,

$\varepsilon = (\varepsilon_d - \varepsilon_s)$  is the boson energy

$\hat{n}_d = (d^\dagger \cdot \tilde{d})$  Boson number type of (d-boson)

$\hat{p} = 1/2 [(d \cdot \tilde{d}) - (\tilde{s} \cdot s)]$  Pairing bosons operator

$\hat{L} = \sqrt{10} [d^\dagger \times \tilde{d}]^1$  Angular momentum operator

$\hat{Q} = [d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]^{(2)} + CHI [d^\dagger \times \tilde{d}]^{(2)}$  Quadrupole operator

$\hat{T}_3 = [d^\dagger \times \tilde{d}]^{(3)}$  Octupole operator

$\hat{T}_4 = [d^\dagger \times \tilde{d}]^{(4)}$  Hexadecapole operator

A form of the IBM Hamiltonian suitable for the study of shape phase transitions from SU(3) to O(6) as follows[7, 8]:-

$$\hat{H} = a_0 \hat{p}^\dagger \cdot \hat{p} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} \quad (2)$$

The ratio  $a_0/4a_2$  is closed to (-1), the equation above is used, but if it's larger the appropriate Hamiltonian is [2]:-

$$\hat{H} = a_0 \hat{p}^\dagger \cdot \hat{p} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{T}_3 \cdot \hat{T}_3 \quad (3)$$

The parameters  $a_2$  and  $a_1$  determine only features of the eigenvalue spectrum. Furthermore,  $\chi$  (CHI) can be used as a single parameter describing the O(6)- SU(3) transition, since  $\chi=0$  corresponds to O(6) eigenfunctions and  $\chi = -\frac{\sqrt{7}}{2}$  corresponds to SU(3) eigenfunctions[9].

The  $n_d \varepsilon_d$  term were added to the equation (2) so the Hamiltonian Writes as follows [10]:-

$$\hat{H} = \varepsilon \hat{n}_d + a_0 \hat{p}^\dagger \cdot \hat{p} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} \quad (4)$$

The terms in  $n_d \varepsilon_d$  or  $\hat{T}_3$  will to reduce the ratio  $E4_1^+ / E2_1^+$  and thus must be kept small for well deformed nuclei [8].

The general form of B(E2) operator in IBM-1 is given by reduce electric quadrupole transition probability B(E2) operator as follows [11,12, 13]:-

$$\hat{T}_\mu^{E2} = \alpha_2 [d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]_\mu^{(2)} + \beta_2 [d^\dagger \times \tilde{d}]_\mu^{(2)} \quad (5)$$

The reduced  $E2$  transition probability ( $E2$ ) is given by [11,12] :-

$$B(E2; I_i \rightarrow I_f) = \frac{1}{(2I_i+1)} |\langle I_f || T^{(E2)} || I_i \rangle|^2 \quad (6)$$

## 2.2. Staggering in $\gamma$ - band Energies

The staggering in  $\gamma$  - band energies  $S(J)$  and the transition between different structural symmetries in nucleican be written as follows [7, 14] :-

$$S(J) = \frac{\{E(J_\gamma^+) - E[(J-1)_\gamma^+]\} - \{E[(J-1)_\gamma^+] - [(J-2)_\gamma^+]\}}{E(2_1^+)} \quad (7)$$

The odd–even staggering is quite strongly pronounced in nuclear regions characterized by SU(5) and O(6) and relatively weaker in nuclei near the SU(3) region[15].

## 2.3. The square of rotational energy and the moment of inertia

The moment of inertia and the square of rotational energy are [7,16,17]

$$\frac{2\mathcal{I}}{\hbar^2} = \frac{4L-2}{E(L)-E(L-2)} = \frac{4L-2}{E_\gamma} (\text{MeV})^{-1} \quad (8)$$

where:

$\mathcal{I}$  is moment of inertia,  $E_\gamma$  is transition energy and  $L$  is angular momentum. Also,

$$(\hbar\omega)^2 = \left[ \frac{E(L)-E(L-2)}{\sqrt{L(L+1)} - \sqrt{(L-2)(L-1)}} \right]^2 (\text{MeV})^2 \quad (9)$$

## 2.4. Potential energy surface

The calculation the potential energy surface is one of methods to knowledge the deformation of nuclear structure. The general formula for the potential energy surface as a function of geometrical variables  $\beta$  and  $\gamma$  is given by[1, 8]:-

$$E(N, \beta, \gamma) = \frac{N\varepsilon_d\beta^2}{(1+\beta^2)} + \frac{N(N+1)}{(1+\beta^2)^2} (\alpha_1\beta^4 + \alpha_2\beta^3 \cos 3\gamma + \alpha_3\beta^2 + \alpha_4) \quad (10)$$

Where:

$N$  is the total boson number,  $\beta^2$  is the quadrupole deformation parameter and  $\gamma$  : is a symmetry angle.

## 3. Results and discussion

The energy levels, reduced electric transition probabilities  $B(E2)$ , relative  $B(E2)$  values, the square of rotational energy and the moment of inertia, the staggering in  $\gamma$ - band energies, and potential energy surface, for  $^{184,186}\text{Os}$  isotopes have been studied and compared with the experimental data using IBM-1 code PHINT [18].

### **3.1. Energy levels**

The  $^{184,186}\text{Os}$  isotopes, with  $Z=76$  and  $N=108, 110$ , have ratio  $R4/2$  equals 3.21, 3.16, and the  $\beta$ -band above the  $\gamma$ -band which is incompatible with the  $SU(3)$  limit. So the equations 1, 2, and 3 are suitable used to get better results and comparing with experimental data. The parameters of  $^{184,186}\text{Os}$  isotopes are shown in Tables (1, 2, 3). The calculated of the energy levels are shown in Fig. (1), can see when we added EPS and OCT terms better than without them, generally the calculations are in agreement compare with experimental data.

### **3.2. Reduced probability of electric quadrupole transition $B(E2)$**

Electromagnetic properties were described by IBM-1[2, 8], to calculated the absolute  $B(E2)$  values of  $^{184,186}\text{Os}$  isotopes the E2SD and E2DD were estimated correspond to selection rules[2, 8], and it's shown in Table 4. The calculated and experimental values of absolute  $B(E2)$  values given in Table 4, shows the values of  $BE(2;2_1^+ \rightarrow 0_1^+)$ ,  $BE(2;4_1^+ \rightarrow 2_1^+)$  decreases with increasing atomic mass number, from this Table can see the results are in agreement with experimental values. The relative  $B(E2)$  values of the  $^{184,186}\text{Os}$  isotopes were calculated, the intraband and interband transitions from  $2_\gamma^+$ ,  $4_\gamma^+$ ,  $5_\gamma^+$ , and  $6_\gamma^+$  states were compare the calculated values with experimental data [19], as see in Fig. (3, 4), shows the results are in agreement with experimental data[19].

### **3.3. Staggering in $\gamma$ -band energies**

Can observed the effect odd-even staggering in the  $\gamma$ -bands it is among the most sensitive phenomena carrying information about the symmetry changes[15], as see in Fig.(5, 6 and 7), in Fig. (5) the  $^{184,186}\text{Os}$  isotopes  $S(J)$  not appear but in Fig.(6 and 7) the  $S(J)$  is weak because it's close to  $SU(3)$  limit. This results agreement with previous study in Ref.[14].

### **3.4. The square of rotational energy and the moment of inertia**

Fig.(8, and 9) shows the square of rotational energy and the moment of inertia for  $^{184,186}\text{Os}$  isotopes. As see the  $^{184}\text{Os}$  appears a gradual increase in moment of inertia between the lower angular momentum states, then change in behavior and then again extends gradually, this effect is known as back bending[2], that is occurs in some heavy nuclei because the rotational energy increases the energy required to break a pair of coupled nucleons. When this effect occurs, the unpaired nucleons go into different orbits and change the nuclear moment of inertia [16]. But in  $^{186}\text{Os}$  does not appear any back bending, namely the properties for  $^{186}\text{Os}$  isotope no change.

### **3.5. Potential energy surface**

The potential energy surface were calculated using equation (10) to determined the last shape for  $^{184,186}\text{Os}$  isotopes. Fig. (10 and 11), shows the isotopes under study have prolate deformed deeper than oblate shape. where the  $\beta_{min}=1.4$  for  $^{184,186}\text{Os}$  isotopes as for  $SU(3)$  limit.

## **4. Conclusions**

In this work, the IBM-1 model was applied for  $^{184,186}\text{Os}$  isotopes, this isotopes lie in transition region  $SU(3)$ - $O(6)$  limits. The energy levels have been calculated in three different procedures. The results of added the OCT or EPS term to the eq. 2 are in agreement with experimental data and better than without them.  $B(E2)$  values, relative  $B(E2)$  values are in agreement with experimental data. The square of rotational energy and the moment of inertia were produced reasonably with experimental values, where the  $^{184}\text{Os}$  isotope has the backbending. odd-even staggering in the  $\gamma$ -bands has been studied, the  $S(J)$  is weak because it's close to  $SU(3)$  limit. The final shape for  $^{184,186}\text{Os}$  isotopes were found by calculating potential energy surfaces. They have prolate shape more than oblate.

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Evaluated Nuclear Structure Data File (ENSDF), online database,  
National Nuclear Data Center,  
(Site Visited: 3 January, 2015).

**Table .1. Adopted values for the parameters used for IBM-1 calculations. All parameters are given in MeV except CHI. using eq. 2**

Isotopes	N	EPS	PAIR	ELL	Q Q	OCT	HEXA	CHI
$^{184}\text{Os}$	12	0.0	0.0110	0.0312	-0.0238	0.0	0.0	-2.958
$^{186}\text{Os}$	11	0.0	0.300	0.0389	-0.0200	0.0	0.0	-2.898

**Table .2. Adopted values for the parameters used for IBM-1 calculations. All parameters are given in MeV except CHI. using eq. 2+OCT term.**

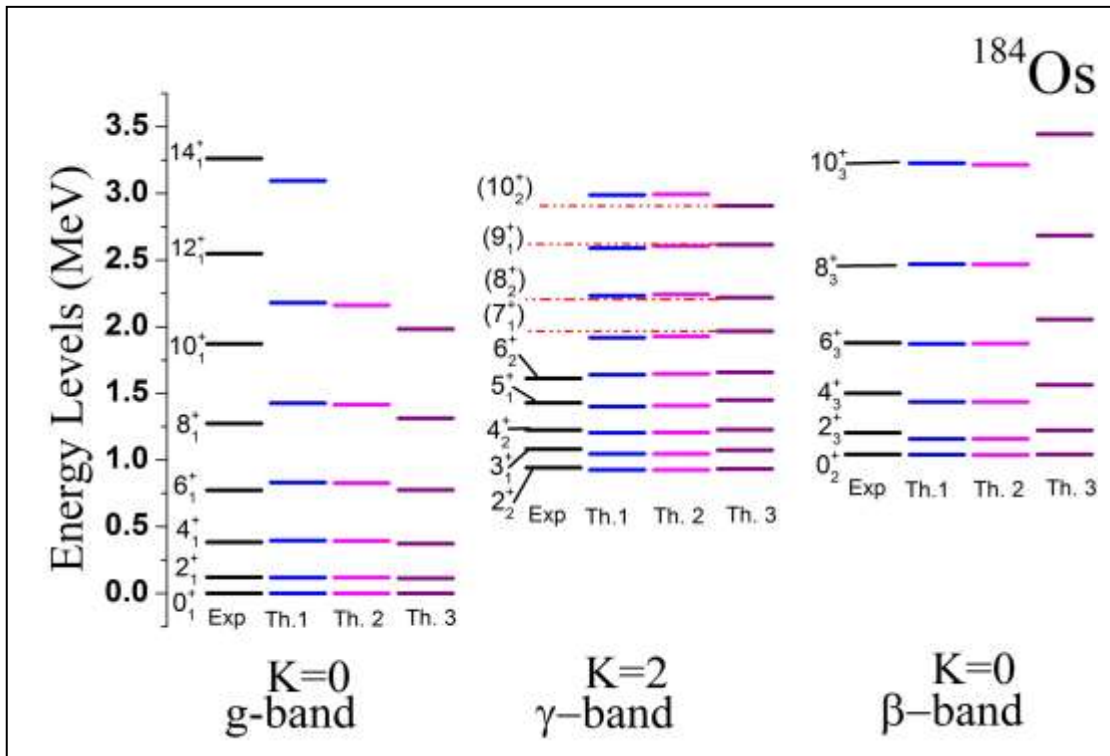
Isotopes	N	EPS	PAIR	ELL	Q Q	OCT	HEXA	CHI
$^{184}\text{Os}$	12	0.0	0.0139	0.0300	-0.0219	0.0020	0.0	-2.9580
$^{186}\text{Os}$	11	0.0	0.242	0.0328	-0.0178	0.0046	0.0	-2.7325

**Table .3. Adopted values for the parameters used for IBM-1 calculations. All parameters are given in MeV except CHI. using eq. 2 + EPS term.**

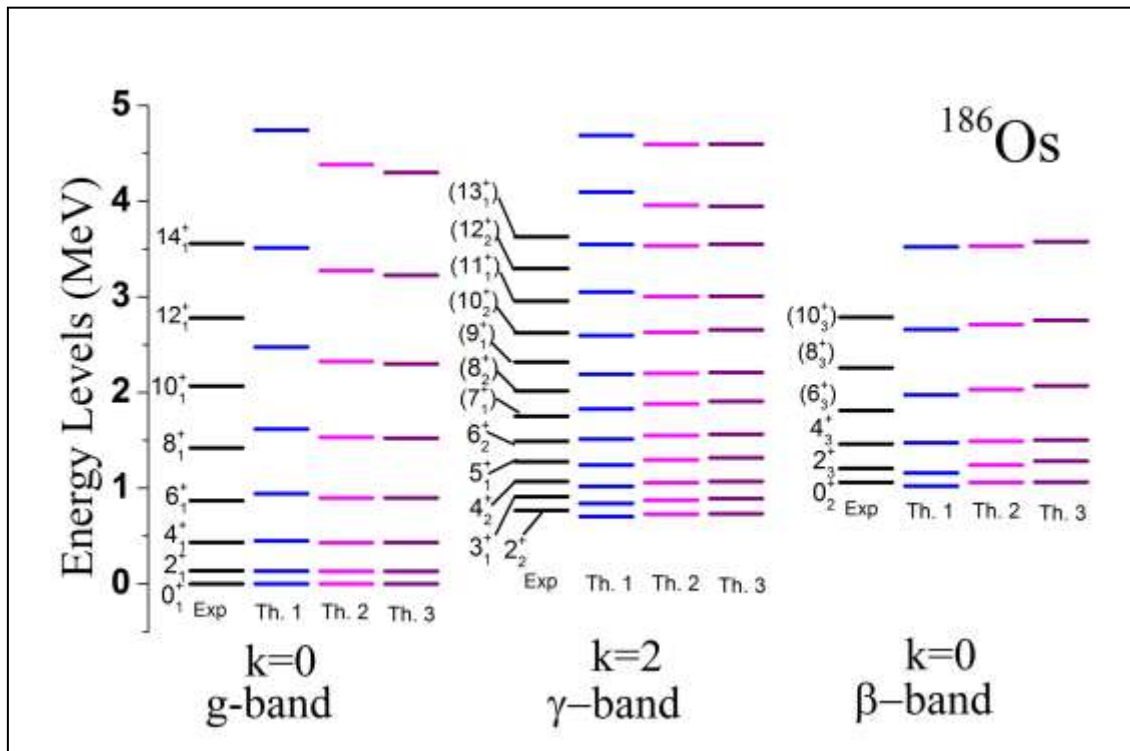
Isotopes	N	EPS	PAIR	ELL	Q Q	OCT	HEXA	CHI
$^{184}\text{Os}$	12	0.383	0.0223	0.0172	-0.0239	0.0	0.0	-2.7680
$^{186}\text{Os}$	11	0.383	0.300	0.0245	-0.0173	0.0	0.0	-2.7485

**Table .4. The values of parameters (E2SD, E2DD in ( $e^2 b^2$ )) of B(E2) for  $^{184-186}\text{Os}$  isotopes.**

Isotopes	E2SD	E2DD
$^{184}\text{Os}$	1	-1.088
$^{186}\text{Os}$	1	-0.686



**figure . 1.** (color online) The calculated low-lying energy levels (Th.1: using eq.2, Th. 2: using eq.  $2+a_3$  term, Th. 3: using eq.  $2+ \varepsilon\hat{n}_d$  term) and the experimental data of  $^{184}\text{Os}$ , taken from Ref. [19].

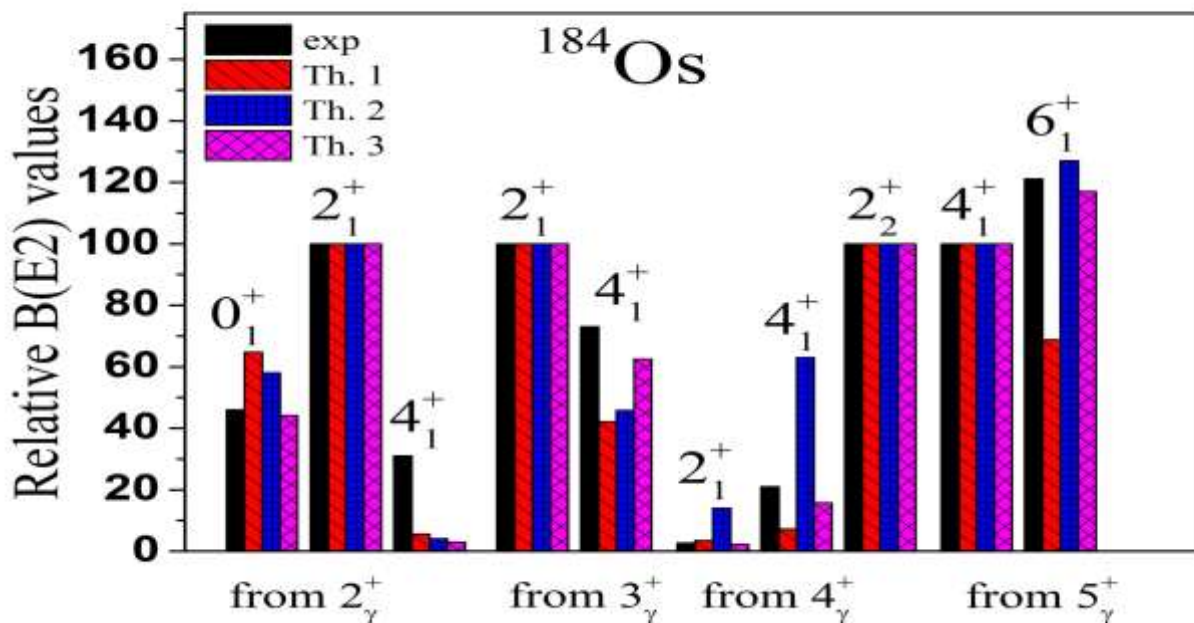


**Figure . 2.** (color online) The calculated low-lying energy levels (Th.1: using eq.2, Th. 2: using eq.  $2+a_3$  term, Th. 3: using eq.  $2+ \varepsilon\hat{n}_d$  term) and the experimental data of  $^{186}\text{Os}$ , taken from Ref. [19].

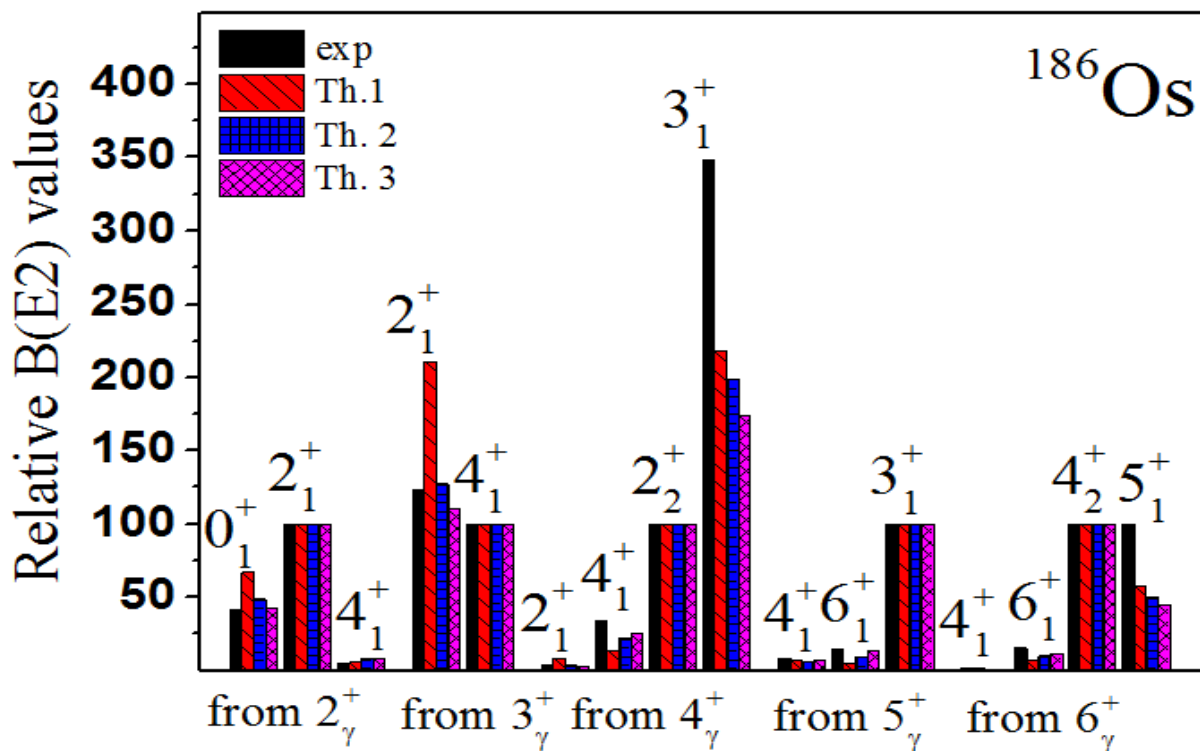
**Table .5.B(E2) values for  $^{184,186}\text{Os}$  isotopes in  $(e^2b^2)$**

Isotopes	$J_i \rightarrow J_f$	EXP B(E2) [19]	Th. 1 B(E2)	Th. 2 B(E2)	Th. 3 B(E2)
$^{184}\text{Os}$	$2_1^+ \rightarrow 0_1^+$	0.610	0.610	0.610	0.610
	$4_1^+ \rightarrow 2_1^+$	0.870	0.850	0.870	0.870
	$6_1^+ \rightarrow 4_1^+$	-----	0.910	0.930	0.940
	$2_2^+ \rightarrow 0_1^+$	-----	0.011	0.013	0.023
	$2_2^+ \rightarrow 2_1^+$	-----	0.018	0.020	0.052
	$2_2^+ \rightarrow 4_1^+$	-----	0.001	0.001	0.002
	$3_1^+ \rightarrow 2_1^+$	-----	0.019	0.020	0.040
	$3_1^+ \rightarrow 4_1^+$	-----	0.008	0.010	0.025
	$4_2^+ \rightarrow 2_1^+$	-----	0.010	0.005	0.007
	$4_2^+ \rightarrow 4_1^+$	-----	0.021	0.020	0.050
	$4_2^+ \rightarrow 2_2^+$	-----	0.290	0.040	0.320
$^{186}\text{Os}$	$2_1^+ \rightarrow 0_1^+$	0.580	0.580	0.580	0.580
	$4_1^+ \rightarrow 2_1^+$	0.840	0.820	0.820	0.820
	$6_1^+ \rightarrow 4_1^+$	1.160	0.860	0.880	0.880
	$2_2^+ \rightarrow 0_1^+$	0.063	0.024	0.028	0.031
	$2_2^+ \rightarrow 2_1^+$	0.140	0.035	0.057	0.073
	$2_2^+ \rightarrow 4_1^+$	0.0075	0.002	0.005	0.006
	$3_1^+ \rightarrow 2_1^+$	-----	0.040	0.050	0.057
	$3_1^+ \rightarrow 4_1^+$	-----	0.020	0.038	0.049
	$4_2^+ \rightarrow 2_1^+$	0.020	0.010	0.010	0.010
	$4_2^+ \rightarrow 4_1^+$	0.155	0.040	0.064	0.080
	$4_2^+ \rightarrow 2_2^+$	0.450	0.279	0.298	0.320
$4_2^+ \rightarrow 3_1^+$	-----	0.610	0.587	0.556	

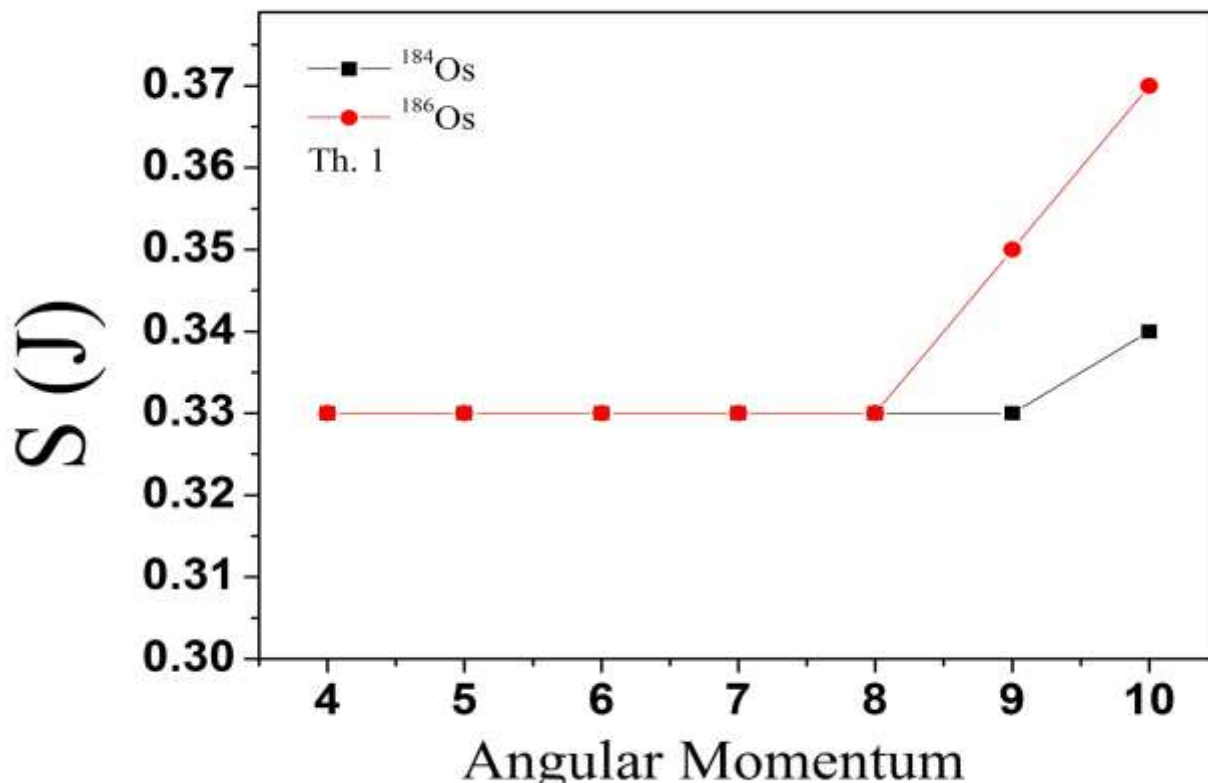




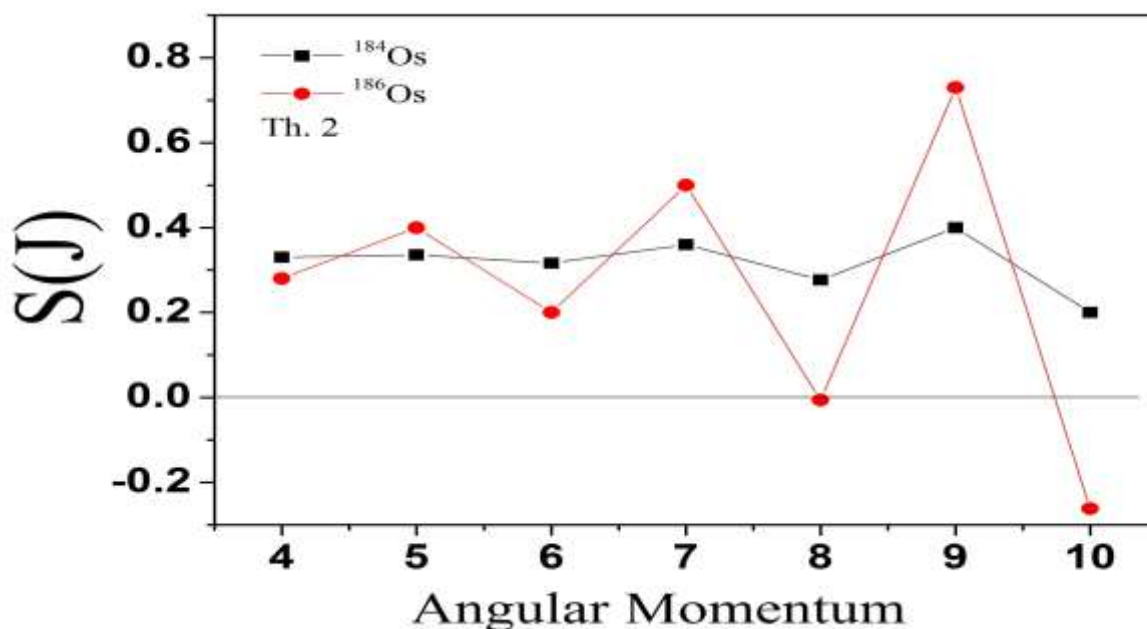
**Figure.3.** (color online) Calculated (Th.1 is using eq.2, Th. 2 is using eq. 2+ $a_3$  term, Th. 3 is using eq. 2+  $\varepsilon\hat{n}_d$  term) and experimental values of relative B(E2) for  $^{184}\text{Os}$ , experimental values taken from Ref. [19].



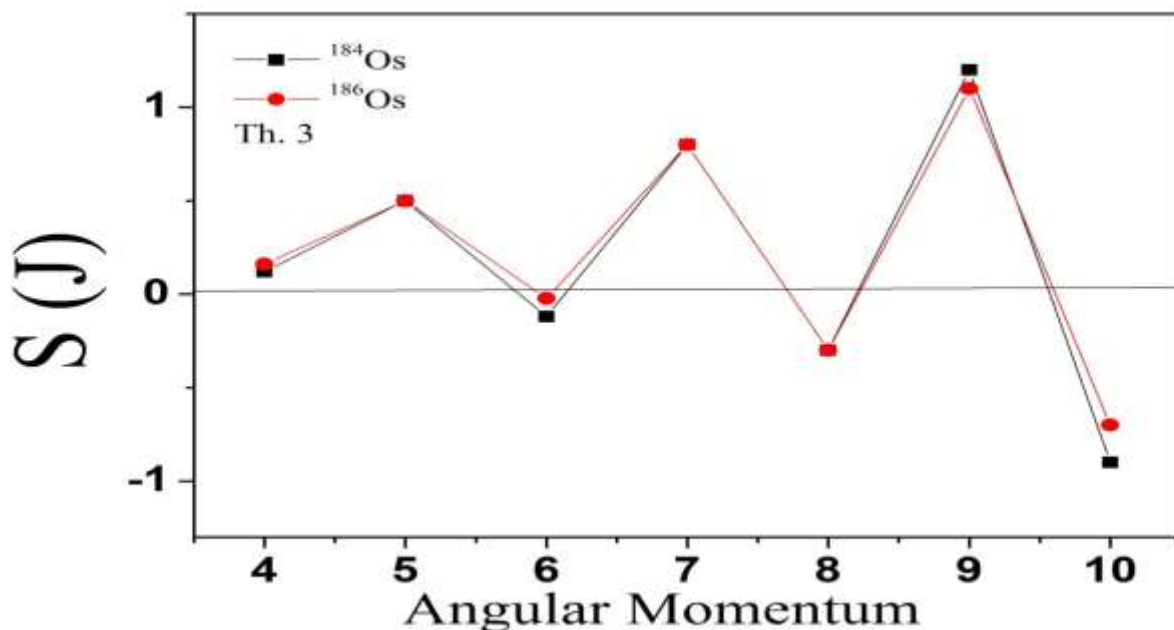
**Figure.4.** (color online) Calculated (Th.1:using eq.2. Th. 2: using eq. 2+ $a_3$  term. Th. 3: using eq. 2+  $\varepsilon\hat{n}_d$  term) and experimental values of relative B(E2) for  $^{186}\text{Os}$ , experimental values taken from



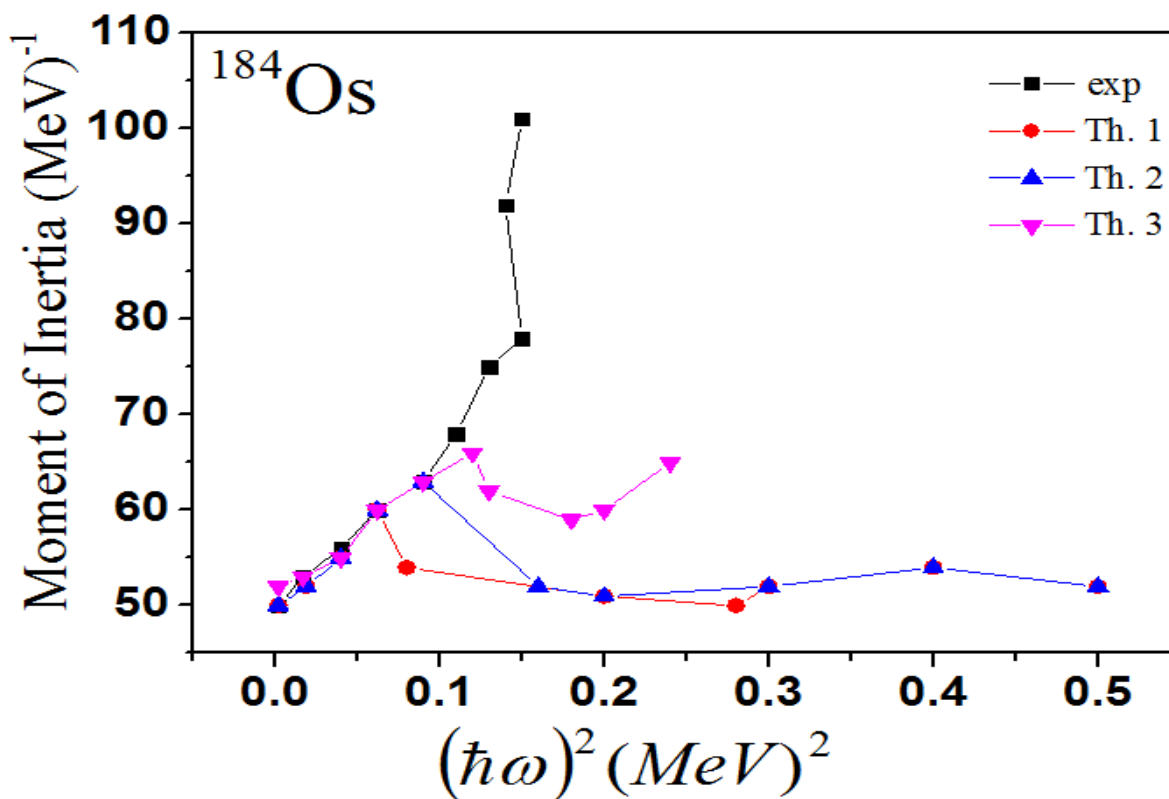
**Figure. 5.** Staggering S(J) in  $\gamma$ - band calculations for  $^{184-186}\text{Os}$  isotopes, using eq. 2



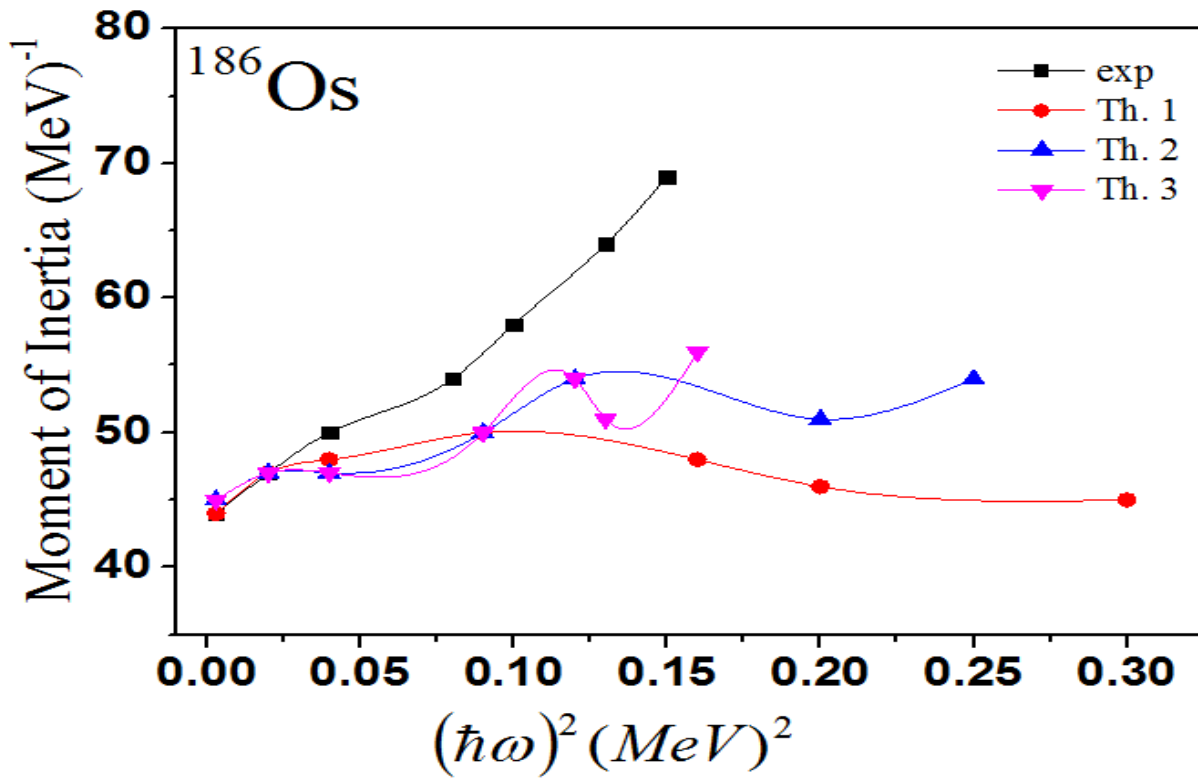
**Figure. 6.** Staggering S(J) in  $\gamma$ - band calculations for  $^{184-186}\text{Os}$  isotopes, using eq. 2 + OCT term.



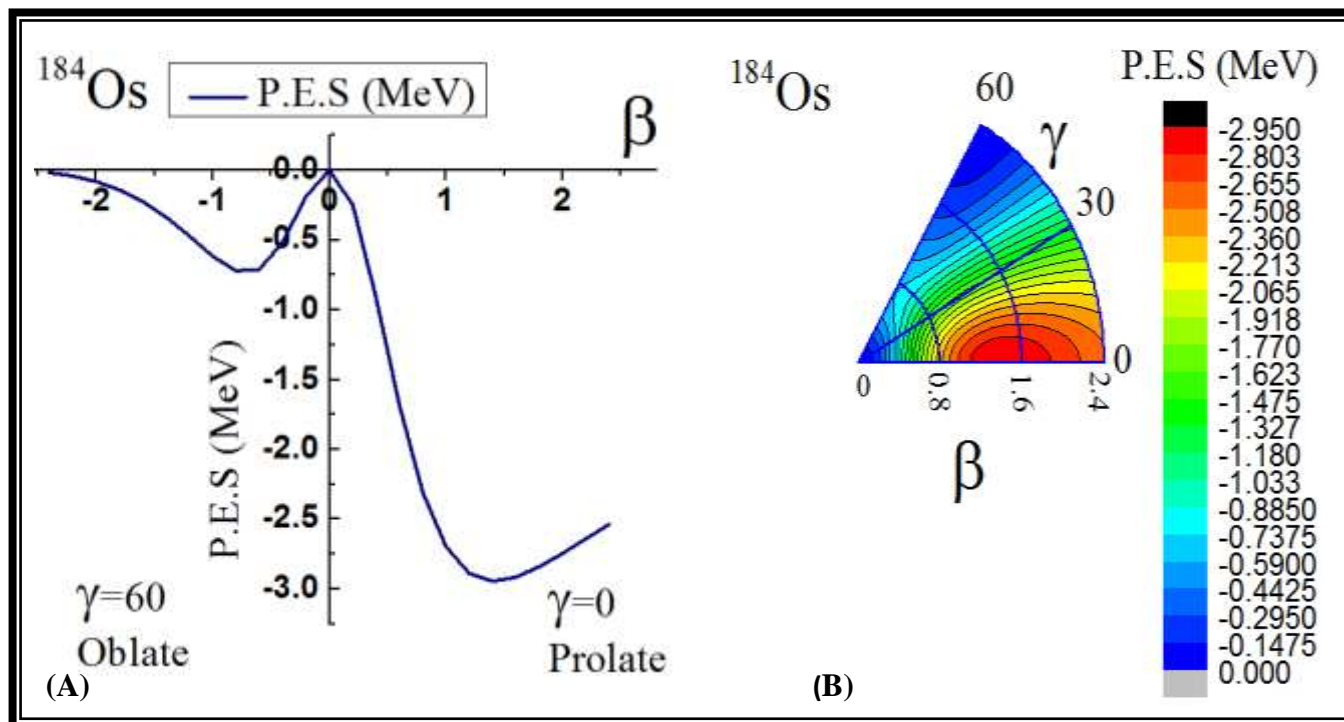
**Figure. 7.** Staggering  $S(J)$  in  $\gamma$ - band calculations for  $^{184-186}\text{Os}$  isotopes, using eq. 2 +EPS term.



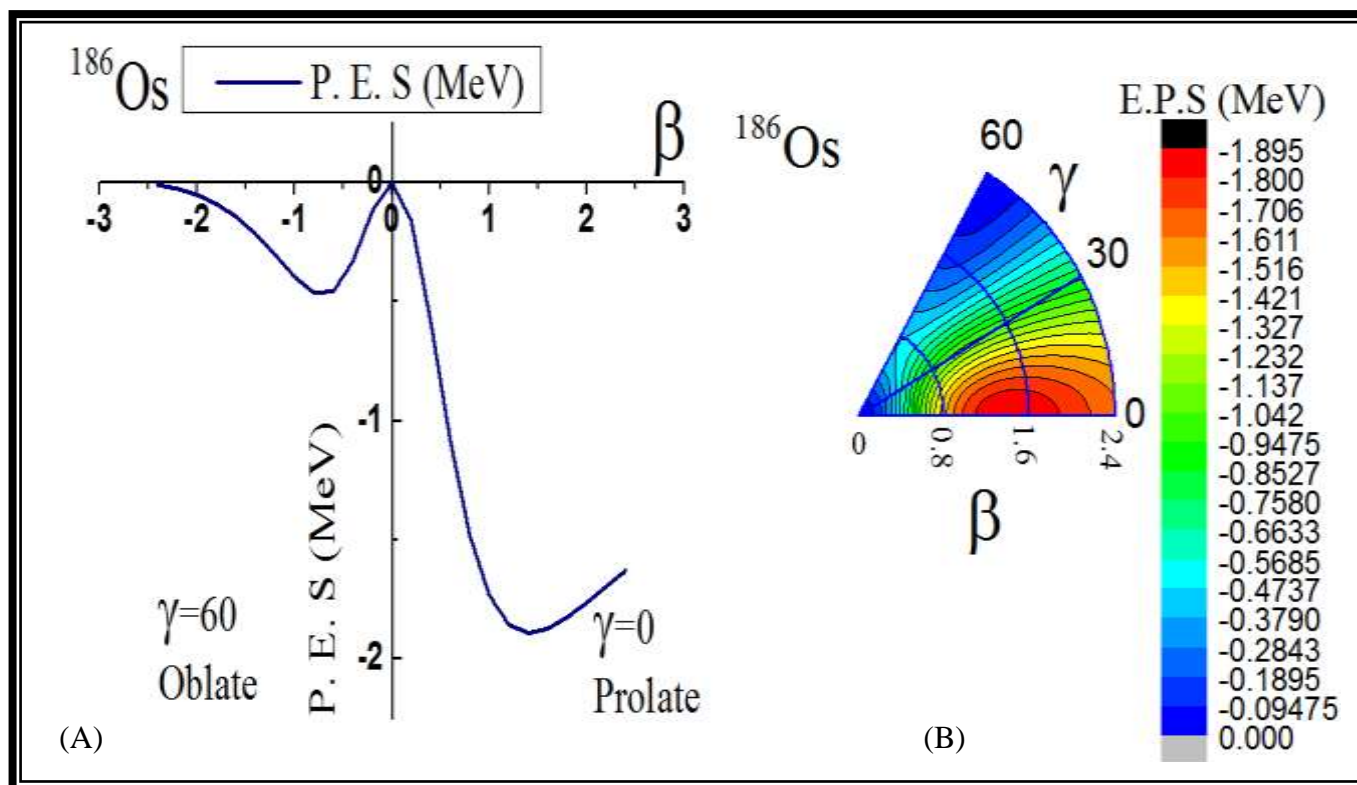
**Figure.8.** (color online)The calculated moment of inertia  $\frac{2g}{\hbar^2}$  versus the square of rotational energy  $(\hbar\omega)^2$  (Th.1: using eq.2, Th. 2: using eq. 2+ $a_3$  term, Th. 3: using eq. 2+  $\varepsilon\hat{n}_d$  term)for  $^{184}\text{Os}$  isotope, the experimental data taken from Ref.[19].



**Figure. 9.** (color online) The calculated moment of inertia  $\frac{2g}{\hbar^2}$  versus the square of rotational energy  $(\hbar\omega)^2$  (Th.1: using eq.2, Th. 2: using eq. 2+ $a_3$  term, Th. 3: using eq. 2+  $\varepsilon\hat{n}_d$  term) for  $^{186}\text{Os}$  isotope, the experimental data taken from Ref.[19].



**Figure. 10.** (A) The potential energy surface for  $^{184}\text{Os}$  as a function of  $\beta$  at  $\gamma=0^\circ$  and  $60^\circ$ . (B) The potential energy surface in  $\beta$ - $\gamma$  plane for  $^{184}\text{Os}$  Nucleus.



**Figure. 11. :** (A) The potential energy surface for  $^{186}\text{Os}$  as a function of  $\beta$  at  $\gamma=0^\circ$  and  $60^\circ$ . (B) The potential energy surface in  $\beta$ - $\gamma$  plane for  $^{186}\text{Os}$  Nucleus.