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Numerical Method of Interpolation By Using a Parametric Curve

طريقة عددية للأندراج بأستخدام منحني باراميتري

Mustafa Abbas Fadhel
Department of General Sciences/College of Basic Education
University of Al-Muthanna
E-mail: mustafa@mu.edu.iq

Abstract

In this paper we find new method to interpolation by using parameter polynomials. The problem studied here is the way of determine the consider element that is we can find it from solution equations system of every parameters curves that is we get from interpolation points. There are many studies in this field, the researchers are state the leading methods and formulate programmed examples to show the details of these studies. Compare this method with known methods by test examples and find the merits and drawbacks of each study.

الخـلاصـــــة

في هذا البحث وجدنا طريقة جديدة للاندراج بأستخدام متعددة الحدود البارمترية. مشكلة الدراسة هنا تكمن في أمكانية تحديد الثوابت الناتجة من حل انظمة معادلات لمنحنيات باراميترية نحصل عليها من نقاط الاندراج. هناك العديد من الدراسات في هذا المجال حيث أقترح الباحثون طرائق رئيسية وأمثلة مبرمجة لتوضيح تفاصيل هذه الدراسات. حيث قُورنت هذه الطريقة مع الطرائق المشهورة عن طريق أمثلة أختبارية ووجدنا أيجابيات و سلبيات كل دراسة.

1- Introduction

Interpolation methods such as Lagrange, least square, and spline are given a numerical function with a different certain ratio of error. This default error becomes a problem in many branches of science. In this paper, the calculation of interpolation points from parametric curves. We will examine yet another way of defining curves - the parametric description. We will see that this is, in some ways, far more useful than either the Cartesian description or the polar form. Although we shall only study planar curves the parametric description can be generalised to the description of spatial curve to find interpolation points.

2- Basic Definitions and Theorems :

In this section, the basic definitions and theorems with properties are presented as a function $y = f(x)$ is given only at discrete points such as $(x_0, y_0), (x_1, y_1), \dots, (x_{n-1}, y_{n-1}), (x_n, y_n)$. How does one find the value of 'y' at any other value of 'x' ?

Definition (2-1)[1] :

A continuous function $\boldsymbol{f(x)}$ may be used to represent the $\boldsymbol{n+1}$ data values with $\boldsymbol{f(x)}$ passing through the $\boldsymbol{n+1}$ nodes. Then one can find the value of $\boldsymbol{y}$ at any given value of $\boldsymbol{x}$. This is called interpolation. Of course, if $'\boldsymbol{x}'$ falls outside the range of $\boldsymbol{f(x)}$ for which the data is given, it is no longer interpolation but instead is called extrapolation. There are various methods for performing interpolation. In this paper, we will discuss various types of commonly used interpolation schemes (Lagrange, Least square and splines interpolations). Often we may use a cubic spline algorithm, which is normally the preferred type of interpolation[2].

Definition (2.2) [3,4].

A spline is a curve S(u) of degree n with knots $a_0, \dots, a_m$ and $a_i \leq a_{i+1}$ and $a_i \leq a_{i+n+1}$ for all i if:

1) S(u) is $(n-r)$ times differentiable at any r- knot

2) S(u) is a polynomial of degree $\leq$ n over each knot interval $[a_i, a_{i+1}]$ for $i = 0, \dots, m-1$.

Definition (2.3) [4].

Let $u = (u_j)_{j=1}^{n+p+1}$ be a non decreasing sequence of real numbers, and let an integer $q \geq 2$. The space of all spline curves in R^q of degree p and with knots u is defined as: $S_{p,u}^q = \{\sum_{j=1}^{n} c_j N_{j,p} \,|c_j \in R^q \text{ for } 1 \leq j \leq n\}$

More precisely, an element $f = \sum_{j=1}^{n} c_j N_{j,p}$ of $S_{p,u}^q$ is called a spline vector function or a parametric spline curve of degree p with knots u, and $(c_j)_{j=1}^{n}$ are called B-spline coefficients or control points of f.

Definition (2.4) [5].

We can describe the x, y coordinates of points on a curve in the plane with two function of t, one for x and one for y, these functions are called parametric equations, the input variable t is called parameter, as t changes and coordinates changes, the resulting curve is called the parametric curve. Suppose that x and y are both given as functions of a third variable t by the equations $x = x(t)$ and $y = y(t)$, each value of t determines a point (x, y), which we can plot in a coordinate plane. As t varies, the point

$(x, y) = P(t) = (x(t), y(t))$ varies and traces out a curve C, which it a **parametric curve**.

3-New Proposed Method

When we have $n+1$ interpolation points [let: $W_0, W_1, \dots, W_n$; $W_n = (x_n, y_n)$], then to find a parameter polynomial that passes through these points, we have $n+1$ values of the parameter t are chosen for the curve, we can choose this points as follow:

Let $t_0 = W_0 = (x_0, y_0) = x_0 + iy_0 \quad then\ t_0 = x_0 + iy_0$

$$then\ iy_0 = t_0 - x_0 \quad by\ square\ we\ get \quad -{y_0}^2 = {t_0}^2 - 2t_0x_0 + {x_0}^2$$

$$then\ {t_0}^2 - 2t_0x_0 + {x_0}^2 + {y_0}^2 = 0 \quad \dots.(3.1)$$

From equation (3.1) we can find the value of t_0, when got more values then take the real of it, and by the same way we can find $t_1, \dots, t_n$.

$Since \quad t_0 = W_0 = (x_0, y_0)$

$$then\ x(t_0) = x(W_0) = x_0 \ \ and\ \ y(t_0) = y(W_0) = y_0$$

$$Since\ x(t) = a_nt^n + \dots + a_1t + a_0$$

$$then\ x(t_0) = a_n{t_0}^n + \dots + a_1t_0 + a_0 \ = x_0$$

By the same way we can find other equations to get the following equations system:

$$x_i = a_n{t_i}^n + \dots + a_1t_i + a_0$$

$$when\ i = 1, \dots, n$$

then a system equations of $n+1$ determine a constant points $(a_0, a_1, \cdots, a_n)$ is solved to find $x(t)$, and by same a way we can find $y(t)$, therefor the parameter polynomial will be as $P(t) = (x(t), y(t))$.

4- Comparing the Results by Test Example:

In this section, we will take a test example and we find solution of it by two interpolation methods (cubic spline method and proposed method), later we will find ratio error of for this example and compare between the two results of these two methods to know which is given less error.

Notes:

- To find the ratio error in the test examples we will use $L_2 metric$.
- To find the distance by integration parameter functions we take the following formula

$$\int p(t) = \int \dot{x}(t) y(t) dt \quad .$$

where $x(t)$ *and* $y(t)$ are x and $y-axis$, respectively, for the parameter function $p(t)$. [6].

- To find the ratio error in the test examples we will use $L_2 metric$ for the parameter function one can find that, by using the following formula:

$$\left[\int |p(t) - q(t)|^2 dt\right]^{\frac{1}{2}} = \left[\int \left|\left(yp(t) - yq(t)\right)^2 \dot{xp}(t) dt\right|\right]^{\frac{1}{2}}$$

where $xp(t)$ *and* $yp(t)$ are the a x and $y-axis$, respectively, for the parameter function $p(t)$, also $xq(t)$ *and* $yq(t)$ are the a x and $y-axis$, respectively, for the parameter function $q(t)$.

Example: To construct the polynomial interpolating the data

x	-2	-1	0	1	2
$f(x) = x^3$	-8	-1	0	1	8

by using:

- **Cubic Spline Method**

The solution for this example by this method is given by:

$$f(x) = \begin{cases} -3/2\,x^3 - 9x^2 - 19/2\,x - 3 & , \quad x \in [-2,-1]\,, \\ 3/2\,x^3 - 1/2\,x & , \quad x \in [-1,0]\,, \\ 3/2\,x^3 - 1/2\,x & , \quad x \in [0,1]\,, \\ -3/2\,x^3 + 9x^2 - 19/2\,x + 3 & , \quad x \in [1,2]\,. \end{cases}$$

- **Proposed Method**

The solution for this example by this method is given by:

$$p(t) = (t \;\; , \;\; t^3)$$

Fig. 4.1 shows solutions of test example in both methods, also the table 4.1 shows the ratio error of both methods.

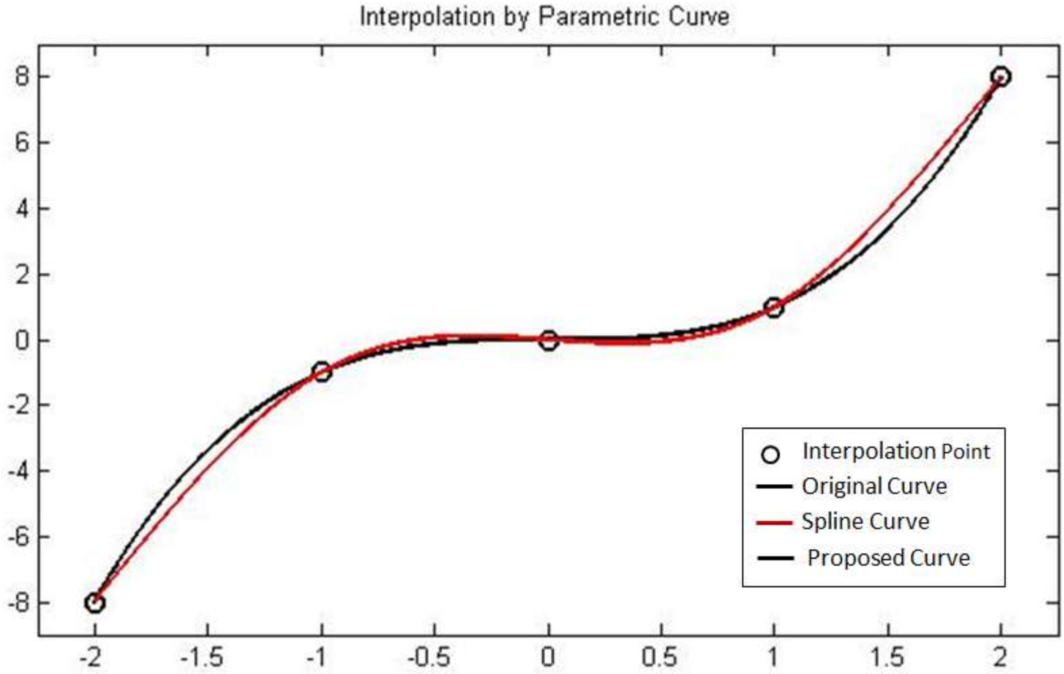

Fig. 4.1 All Solution Methods for Test Example

Table 4.1 shows the ratio error of both methods, also .Fig. 4.1 shows solutions of the test example in both methods.

Method	Cubic Spline	Proposed
Error	1.1156	0

Tab. 4.1 Ratio Error for both Solution Methods to Test Example

Conclusion:

We used parametric curve in interpolation via finding the constant points in a well arranged shape with interpolation points, where new proposed method passed through the interpolation points. Through investigating the pervious studies in this field, it became clearer that the proposed method of finding the constant points is more efficient than the analytic one. Finally, we testing proposed method by example where comparing with another methods and we find the ratio error for these interpolation methods.

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