Artin's Characters Table of the Group $Q_{2m} \times C_3$ when m is an Even Number

جدول شواخص ارتن للزمرة $Q_{2m} \times C_3$ عندما m عدد زوجي

Assistant Lecturer Rajaa Hassan Abass
University of Kufa, College of Education for Girls, Department of Mathematics
rajaah.alabidy @uokufa.edu.iq

Abstract

The main purpose of this paper is to find the general form of Artin's characters table of the group
$Q_{2m} \times C_3$ when m is an even number where Q_{2m} is the quaternion group of order 4m and C_3 is the cyclic group of order3 we prove that this table depends on Artin's characters table of a quaternion group Q_{2m} of order 4m when m is an even number. which is denoted by $Ar(Q_{2m} \times C_3)$.

المستخلص

الهدف الرئيسي لهذا البحث هو ايجاد الصيغة العامة لجدول شواخص ارتن للزمرة $Q_{2m} \times C_3$ عندما m عدد زوجي حيث ان Q_{2m} هي الزمرة الرباعية العمومية ذات الرتبة 4m و C_3 هي الزمرة الدائرية ذات الرتبة 3 ،وقد برهنا أن هذا الجدول يعتمد على جدول شواخص آرتن للزمرة الرباعية العمومية Q_{2m} ذات الرتبة 4m عندما m عدد زوجي. الذي يرمز له بـ $Ar(Q_{2m} \times C_3)$

Introduction

Representation theory is a branch of the Mathematics that studies abstract algebraic structures by representing their elements as linear transformations of vector spaces and studies modules over these abstract algebraic structures .So that representation theory is a power full tool because it reduces problems in abstract algebra to problems in a linear algebra which is a very well understood theory .

For a finite group G, let $\overline{R}$ (G) denotes the abelian group generated by Z - valued characters of G under operation of pointwise addition. Inside this group there is a subgroup generated by Artin characters (The characters induced form the principal characters of cyclic subgroups of G), which will be denoted by T(G).The factor group $\overline{R}$ (G) /T (G) is called the Artin Cokernel of G denoted by AC(G).

A well known theorem dues to Artin asserted that T(G) has a finite index in $\overline{R}$ (G) i.e, [$\overline{R}$ (G):T(G)] is finite so AC(G) is a finite abelian group.
The exponent of AC(G) is called Artin exponent of G and denoted by A(G), In 1967 T.Y. lam [9] gave the definition of AC(G). In 1996 K.K Nwabuez [5] studied A(G) of p-groups. In 2009 S.J.Mahmood [8] found the general from of Artin's characters table $Ar(Q_{2m})$ when m is an even number.
The aim of this paper is to find the general from of the Artin's characters table of the group
$Q_{2m} \times C_3$ when m is an even number .

1.Preliminaries

In this section we introduce some important definitions and basic concepts about the induced character,
the Artin's characters tables, the Artin's characters table of C_{p^s} , the Artin's characters table of the Quaternion group Q_{2m} when m is an even number and the group $Q_{2m}\times C_3$.

Definition (1.1): [7]

Two elements of G are said to be **Γ- conjugate** if the cyclic subgroups they generate are conjugate in G , this defines an equivalence relation on G. It is classes are called Γ- classes.

Example (1.2):

Consider a cyclic group C_3=< x > such that:

1 is Γ- conjugate 1

Then the Γ- class [1] = {1}

$< x > = < x^2 >$

Then x and x^2 are Γ- conjugate , and $[x] = \{ x , x^2\}$

So that there are two Γ- classes of C_3 :[1] and [x]

In general for C_{p^s} *where p is any prime number,there are s+1 distinct*

Γ- classes which are [1], [x], $[x^p]$, … ,$[x^{p^{s-1}}]$.

Definition (1.3):[3]

Let H be a subgroup of G and let φ be a class function on H, ***the induced class function on G***, is given by :

$$\varphi'(g) = \frac{1}{|H|}\sum_{h\in G}\varphi^{\circ}(hgh^{-1})$$

where φ° is defined by:

$$\varphi^{\circ}(x) = \begin{cases} \varphi(x) & if \quad x \in H \\ 0 & if \quad x \notin H \end{cases}$$

Proposition (1.4) :[6]

Let H be a subgroup of G and φ be a character of H, then φ' is a character of G.

Definition (1.5):[4]

The character φ' of G is called ***induced character*** on G.

Theorem (1.6):[2]

Let H be a cyclic subgroup of G and $h_1, h_2, \dots, h_m$ are chosen representative for m-conjugate classes, then :

1- $\varphi'(g) = \frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i)$ if $h_i \in H \cap CL(g)$

2- $\varphi'(g) = 0$ if $H \cap CL(g) = \phi$.

Definition (1.7):[3]

Let G be a finite group, all characters of G induced from a principal character of cyclic subgroups of G are called ***Artin's characters of G.***

Example (1.8):

To find the Artin's character of C_3,

there are two cyclic subgroups of C_3, which are {1} and C_3 = <x > and let φ be principal character, then :

by using theorem (1.6)

$$\varphi'(g) = \begin{cases} \dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if \quad h_i \in H \cap CL(g) \\ 0 & if \quad H \cap CL(g) = \phi \end{cases}$$

if H = {1} and G = C_3

since $H \cap CL(1) = \{1\}$, then

$\varphi'_1(1) = \frac{3}{1}.\ \varphi(1) = 3.1=3$

since $H \cap CL(x) = \phi$, then

$\varphi'_1(x) = 0$

since $H \cap CL(x^2) = \phi$, then

$\varphi'_1(x^2) = 0$

if H = C_3

since $H \cap CL(1) = \{1\}$, then

$\varphi'_2(1) = \frac{3}{3}.\ \varphi(1) = 1$

since $H \cap CL(x) = \{x\}$, then

$\varphi'_2(x) = \frac{3}{3}.\ \varphi(x) = \frac{3}{3}.1=1$

since $H \cap CL(x^2) = \{x^2\}$, then

$\varphi'_2(x^2) = \frac{3}{3}.\ \varphi(x^2) = \frac{3}{3}.1=1$

then we get the two Artin's characters φ'_1 and φ'_2.

Proposition (1.9):[2]

The number of all distinct Artin's characters on a group G is equal to the number of Γ-classes on G. furthermore, Artin's characters are constant on each Γ-classes.

Definition (1.10):[1]

Artin's characters of finite group G can be displayed in a table ***called Artin's characters table of G*** which is denoted by Ar (G).

The first row is the Γ- conjugate classes, the second row is the number of elements in each conjugate classes, the third row is the size of the centralize $|C_G(CL_\alpha)|$ and the rest row contain the values of Artin's characters.

Example (1.11):

In the Artin's character table of C_3 there are two Γ- classes, [1], [x] then, from proposition (1.10) they obtain two distinct Artin's characters

And From example (1.8) we obtain the values of Artin's characters, then the table of it as follows:

Ar(C_3) =

Γ- classes	[1]	[x]
$\|CL_\alpha\|$	1	1
$\|C_{C_3}(CL_\alpha)\|$	3	3
φ'_1	3	0
φ'_2	1	1

Table (1)

Theorem (1.12):[1]

The general form of Artin's character table of C_{p^s} when p is a prime number and n is an integer number is given by:

Ar(C_{p^s})=

Γ-classes	[1]	$[x^{p^{s-1}}]$	$[x^{p^{s-2}}]$	$[x^{p^{s-3}}]$	…	$[x^p]$	$[x]$
$\lvert CL_\alpha \rvert$	1	1	1	1	…	1	1
$\lvert C_{p^s}(CL_\alpha)\rvert$	P^s	p^s	P^s	p^s	…	p^s	P^s
φ'_1	p^s	0	0	0	…	0	0
φ'_2	P^{s-1}	P^{s-1}	0	0	…	0	0
φ'_3	P^{s-2}	P^{s-2}	P^{s-2}	0	…	0	0
⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮
φ'_s	P	P	P	P	…	P	0
φ'_{s+1}	1	1	1	1	…	1	1

Table (2)

Example (1.13):

Consider the cyclic group C_8 ,

To find the Artin's character table we use theorem (1.12) as follows:

The group $C_8 = C_{2^3}$ then:

Ar (C_{2^3}) =

Γ- classes	[1]	$[x^{2^2}]$	$[x^2]$	[x]
$\lvert CL_\alpha \rvert$	1	1	1	1
$\lvert C_{2^3}(CL_\alpha)\rvert$	2^3	2^3	2^3	2^3
φ'_1	2^3	0	0	0
φ'_2	2^2	2^2	0	0
φ'_3	2	2	2	0
φ'_4	1	1	1	1

Table (3)

Theorem (1.14): [8]

The Artin's characters table of the quaternion group Q_{2m} when m is an even number is given as follows :

Ar (Q_{2m}) =

Γ- classes	Γ- classes of C_{2m}							
	[1]	$[x^m]$					[y]	[xy]
$\lvert CL_\alpha \rvert$	1	1	2	2	…	2	m	m
$\lvert C_{Q_{2m}}(CL_\alpha) \rvert$	4m	4m	2m	2m	…	2m	4	4
Φ_1	**$2Ar(C_{2m})$**						0	0
Φ_2							0	0
⋮							⋮	⋮
Φ_l							0	0
Φ_{l+1}	m	m	0	0	…	0	2	0
Φ_{l+2}	m	m	0	0	…	0	0	2

Table(4)

where l is the number of Γ- classes of C_{2m} and Φ_j ; $1 \leq j \leq l+2$ are the Artin characters of the Quaternion group Q_{2m}.

Example (1.15):

To construct $Ar(Q_8)$ by using theorem (1.14) we get the following table :

$Ar(Q_8)=Ar(Q_{2^3})=$

Γ- classes	[1]	$[x^4]$	$[x^2]$	[x]	[y]	[xy]
$\lvert CL_\alpha \rvert$	1	1	2	2	4	4
$\lvert C_{Q_{2^3}}(CL_\alpha) \rvert$	16	16	8	8	4	4
Φ_1	2^4	0	0	0	0	0
Φ_2	2^3	2^3	0	0	0	0
Φ_3	2^2	2^2	2^2	0	0	0
Φ_4	2	2	2	2	0	0
Φ_5	2^2	2^2	0	0	2	0
Φ_6	2^2	2^2	0	0	0	2

Table (5)

The Group $Q_{2m}\times C_3$ (1.17)

The direct product group ($Q_{2m}\times C_3$) where Q_{2m} is Quaternion group of order 4m with two generators x and y is denoted by

$$Q_{2m}=\{x^k y^j : x^{2m}=y^4=1, yx^m y^{-1}=x^{-m}, 0\leq k\leq 2m-1, j=0,1\}$$

and C_3 is a cyclic group of order 3 consisting of elements $\{I,z,z^2\}$ .The direct product group $Q_{2m}\times C_3$ is denoted by

$$Q_{2m}\times C_3=\{(q,c):q\in Q_{2m}, c\in C_3\} \text{ and } |Q_{2m}\times C_3|=|Q_{2m}|.|C_3|=4m.3=12$$

2. The main results

In this section we find the general form of Artin's characters table of the group $Q_{2m}\times C_3$ when m is an even number

Proposition (2.1):

The general form of the Artin's characters table of the group $Q_{2m}\times C_3$ when m is an even number is given as follows:

$Ar(Q_{2m}\times C_3)$=

Γ- classes of $(Q_{2m})\times\{I\}$							**Γ- classes of $(Q_{2m})\times\{z\}$**					
Γ- classes	$[1,I]$	$[x^m,I]$	…	$[x\ ,I]$	$[y,I]$	$[xy,I]$	$[1,z]$	$[x^m,z]$	…	$[x,z]$	$[y,z]$	$[xy,z]$
$\lvert CL_\alpha\rvert$	**1**	**1**	**…**	**2**	**m**	**m**	**1**	**1**	**…**	**2**	**m**	**M**
$\lvert C_{Q_{2m}\times C_3}(CL_\alpha)\rvert$	**12m**	**12m**	**…**	**6m**	**12**	**12**	**12m**	**12m**	**...**	**6m**	**12**	**12**
$\Phi_{(1,1)}$	**3Ar(Q_{2m})**						**0**					
$\Phi_{(2,1)}$												
⋮												
$\Phi_{(l,1)}$												
$\Phi_{(l+1,1)}$												
$\Phi_{(l+2,1)}$												
$\Phi_{(1,2)}$	**Ar(Q_{2m})**						**Ar(Q_{2m})**					
$\Phi_{(2,2)}$												
⋮												
$\Phi_{(l,2)}$												
$\Phi_{(l+1,2)}$												
$\Phi_{(l+2,2)}$												

Table (6)

Where $Ar(Q_{2m})$ is Artin's characters table of the group Q_{2m}.

Proof :

Let $g\in(Q_{2m}\times C_3)$;$g=(q,I)$ or $g=(q,z)$or $g=(q,z^2)$, $q\in Q_{2m}$,$I,z,z^2\in C_3$

Case (I):

If H is a cyclic subgroup of $Q_{2m}\times\{I\}$,then:

1.H=$\langle(x,I)\rangle$ 2. H=$\langle(y,I)\rangle$ 3. H=$\langle(xy,I)\rangle$

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1\le j\le l+2$ then by using Theorem (1.6)

$$\Phi_j(g)=\begin{cases}\dfrac{|C_G(g)|}{|C_H(g)|}\displaystyle\sum_{i=1}^{m}\varphi(h_i) & if \quad h_i\in H\cap CL(g)\\ 0 & if \quad H\cap CL(g)=\phi\end{cases}$$

1. H=$\langle(x,I)\rangle$

(i) If g=(1,I) and g∈H

$$\Phi_{(j,1)}((1,I))=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{|C_H(I,1)|}.1=\frac{3.4m}{|C_H(I,1)|}.1=\frac{3|C_{Q_{2m}}(1)|}{|C_{\langle x\rangle}(1)|}.\varphi(1)=3.\Phi_j(1) \text{ since}$$

H ∩CL(1,I)={(1,I)}

(ii) if g=(x^m,I) and g∈H

$$\Phi_{(j,1)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{|C_H(g)|}.1=\frac{3.4m}{|C_H(g)|}.1=\frac{3|C_{Q_{2m}}(x^m)|}{|C_{\langle x\rangle}(x^m)|}.\varphi(g)=3.\Phi_j(x^m)$$

since H ∩CL(g)={g},φ(g)=1

(iii) if g=$(x^i,I), i\neq m$ and $i\neq 2m$ and g∈H

$$\Phi_{(j,1)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{6m}{|C_H(g)|}(1+1)=$$

$$\frac{3.2m}{|C_H(g)|}.(1+1)=\frac{3|C_{Q_{2m}}(q)|}{|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1}))=3.\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,I),q∈ Q_{2m} and q≠ x^m ,q≠1

(iv) if g∉H

$\Phi_{(j,1)}(g)=3.0=3.\Phi_j(q)$ Since H∩CL(g)= ϕ

2. H=$\langle(y,I)\rangle$ ={(1,I),(y,I),(y^2,I),(y^3,I)}

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{4}.1=3m=3.\Phi_{l+1}(1)$$

(ii) If g= (x^m,I)=(y^2,I) and g∈ H

$$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{4}.1=3m=3.\Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I) or g=(y^3,I) and g∈H

$$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{12}{4}.(1+1)=3.2=3.\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+1,1)}(g)=0$ since H∩CL(g)=∅

3- H=⟨(xy, I)⟩ ={(1,I),(xy,I),($(xy)^2$,I),($(xy)^3$,I)}

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+2,1)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{4}.1=3m=3.\Phi_{l+2}(1)$$

(ii) If g= (x^m, I)=($(xy)^2$,I)=(y^2,I) and g∈ H

$$\Phi_{(l+2,1)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{4}.1=3m=3.\Phi_{l+2}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) If g=(xy,I) or g=($(xy)^3$,I) =(xy^3,I) and g∈H

$$\Phi_{(l+2,1)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{12}{4}.(1+1)=3.2=3.\Phi_{l+2}(xy)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+2,1)}(g)=0$ since H∩CL(g)=∅

Case (II):

If H is a cyclic subgroup of (Q_{2m}× {z}),then:

1.H=⟨(x,z)⟩=$\langle(x,z^2)\rangle$ *2. H*=⟨(y,z)⟩ = $\langle(y,z^2)\rangle$ *3. H*=⟨(xy,z)⟩ = $\langle(xy,z^2)\rangle$

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1\le j\le l+2$ then by using Theorem (1.6)

$$\Phi_j(\mathrm{g})=\begin{cases}\dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if\quad h_i\in H\cap CL(g)\\ 0 & if\quad H\cap CL(g)=\phi\end{cases}$$

1.*H*=⟨(x,z)⟩=$\langle(x,z^2)\rangle$

(i) If g=(1,I) or g=(1,z)or g=(1,z^2) and g∈H

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(1,I)|}\cdot\varphi(g)=\frac{12m}{|C_H(1,I)|}.1=\frac{3.4m}{|C_{\langle(x,z)\rangle}(1,I)|}.1=\frac{3|C_{Q_{2m}}(1)|}{3|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$

since H ∩CL(g)={(1,I),(1,z),(1,z^2)}

(ii) If g=(1,I) or g=(x^m,I) or g=(x^m,z) or g=(1,z)or g=(x^m,z^2) or g=(1,z^2) and g∈H

(a) if g=(1,I) or g=(1,z) or g=(1,z^2) and g∈H.

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$=\frac{12m}{|C_H(g)|}.1=\frac{3.4m}{|C_{\langle(x,z)\rangle}(g)|}.1=\frac{3|C_{Q_{2m}}(1)|}{3|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$ since H ∩CL(g)={g},φ(g)=1

(b)If g=(x^m,I) or g=(x^m,z) or g=(x^m,z^2) and g∈H

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{|C_H(g)|}.1=\frac{3.4m}{|C_H(g)|}.1=\frac{3|C_{Q_{2m}}(x^m)|}{3|C_{\langle x\rangle}(x^m)|}.\varphi(x^m)=\Phi_j(x^m)$$

since H ∩CL(g)={g},φ(g)=1

(iii) If $g=\{(x^i,I),(x^i,z),(x^i,z^2\}$,$i\neq m, i\neq 2m$ and g∈H

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{6m}{|C_H(g)|}(1+1)$$

$$\frac{3.2m}{|C_H(g)|}.(1+1)=\frac{3|C_{Q_{2m}}(q)|}{3|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1}))=\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,z)=(q,z^2),q∈ Q_{2m} and q≠ x^m ,q≠1

(iv) if g∉H

$\Phi_{(j,2)}(g)=0$ Since H∩CL(g)= ϕ

2. H=$\langle (y,z)\rangle$ ={(1,I),(y,I),(y^2,I),(y^3,I),(1,z),(y,z),(y^2,z),(y^3,z), (1,z^2),(y,z^2),(y^2,z^2),(y^3,z^2)}

(i) If g=(1,I) or g=(1,z)or g=(1,z^2)and g∈H H∩CL(g)={(1,I),(1,z),(1,z^2)}

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{12}.1=m=.\Phi_{l+1}(1)$$

(ii) If g= (x^m,I)=(y^2,I) or g= (y^2,z) or g=(y^2,z^2) and g∈ H

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{12}.1=m=\Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I)or g= (y,z)or g=(y,z^2) or g=(y^3,I_2)or g= (y^3,z) or g= (y^3,z^2) and g∈H

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{12}{12}.(1+1)=2=\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+1,2)}(g)=0$ since H∩CL(g)=∅

3.H=$\langle (xy,z)\rangle$ ={(1,I),(xy,I),($(xy)^2$,I)=(y^2,I),($(xy)^3$,I)=(xy^3,I),(1,z),(xy,z), ($(xy)^2$,z)),($(xy)^3$,z), (1,z^2),(xy,z^2),($(xy)^2$,z^2)),($(xy)^3$,z^2)}

(i) If g=(1,I) or g=(1,z) or g=(1,z^2) H∩CL(g)={g}

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{12}.1=m=\Phi_{l+2}(1)$$

(ii) If g= (x^m,I)=($(xy)^2$,I)=(y^2,I)or g=($(xy)^2$,z)=(y^2,z)or g=($(xy)^2$,z^2)=(y^2,z^2) and g∈ H

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{12m}{12}.1=m=\Phi_{l+2}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) If g=(xy,I) or g= ($(xy)^3$,I) or g= (xy,z) or g= ($(xy)^3$,z) or g= (xy,z^2) or g= ($(xy)^3$,z^2)and g∈H

$$\Phi_{(l+2,2)}(g)=\frac{|C_{Q_{2m}\times C_3}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{12}{12}.(1+1)=2=\Phi_{l+2}(xy)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+2,2)}(g) = 0$ since H∩CL(g)=∅

Example (2.2):

To construct Ar($Q_8 \times C_3$) by using the theorem (2.1) we get the following table:
Ar($Q_2{}^3 \times C_3$)=

Γ- classes	[1,I]	[x^4,I]	[x^2,I]	[x,I]	[y,I]	[xy,I]	[1,z]	[x^4,z]	[x^2,z]	[x,z]	[y,z]	[xy,z]
$\lvert CL_\alpha \rvert$	1	1	2	2	4	4	1	1	2	2	4	4
$\lvert C_{Q_{2^3}\times C_3}(CL_\alpha)\rvert$	48	48	24	24	12	12	48	48	24	24	12	12
$\Phi_{(1,1)}$	48	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	24	24	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	12	12	12	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	6	6	6	6	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	12	12	0	0	6	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	12	12	0	0	0	6	0	0	0	0	0	0
$\Phi_{(1,2)}$	16	0	0	0	0	0	16	0	0	0	0	0
$\Phi_{(2,2)}$	8	8	0	0	0	0	8	8	0	0	0	0
$\Phi_{(3,2)}$	4	4	4	0	0	0	4	4	4	0	0	0
$\Phi_{(4,2)}$	2	2	2	2	0	0	2	2	2	2	0	0
$\Phi_{(5,2)}$	4	4	0	0	2	0	4	4	0	0	2	0
$\Phi_{(6,2)}$	4	4	0	0	0	2	4	4	0	0	0	2

Table (7)

References

[1] A.S.Abid, "Artin's Characters Table of Dihedral Group for Odd Number ", M.Sc. thesis, University of kufa,2006.
[2] C.Curits and I. Reiner, " Methods of Representation Theory with Application Finite Groups and Order ", John wily & sons, New York, 1981.
[3] I. M.Isaacs, "on Character Theory of Finite Groups ",Academic press, New York, 1976.
[4] J. P. Serre, "Linear Representation of Finite Groups", Springer- Verlage, 1977.
[5] K.Knwabusz, " Some Definitions of Artin's Exponent of Finit Group ", USA.National foundation Math.GR.1996.
[6] K. Sekigvchi, " Extensions and the Irreducibilities of The Induced Characters of Cyclic p-Group ", Hiroshima math Journal, p165-178 ,2002.
[7] L. E. Sigler, " Algebra ", Springer- verlage, 1976.
[8] S.J. Mahmood "On Artin cokernel of the Quaternion Group Q_{2m} when m is an Even Number" M.Sc. thesis, University of Kufa,2009.
[9] T.Y. Lam," Artin Exponent of Finite Groups ", Columbia University, New York, 1967.