Electromagnetic scattering from dielectric bodies of revolution with attached wires*

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ABSTRACT

In this paper we investigate the electromagnetic scattering by dielectric bodies of revolution (DBOR) with attached wires in the present of common region (junction) between wire and Body of Revolution (BOR) surface, where the composite and irregular body is formed. A surface integral equation (SIE) technique is developed to analyze the scattering properties. Two vector integral equations (IEs) via Maxwell's equation, Greens theorem, and the boundary conditions are used. The formulations are generated by the standard electric field integral equation (EFIE) and PMCHWT, and numerically evaluated by the method of moment (MoM) technique with Galerkin's approach. The numerical results are validated for DBOR and compared with that of other researchers, while for DBOR-wire-junction compared with conductor bodies of revolution (CBOR)-wire-junction, which satisfied by comparing with experimental data. Furthermore, the addition of attached wires significantly alters the constant CBOR and DBOR cross-section, and the effect of changing dielectric constant (ε_r) will be disparate among different values on the radar cross section (RCS).

Keyword:- **D**ielectric **B**ody **O**f **R**evolution (DBOR) , **A**ttached **W**ires**, J**unction

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INTRODUCTION

Electromagnetic (EM) scattering from conducting, dielectric and composite bodies is an important and challenging problem in the field of computational electromagnetic (CEM). Furthermore, composite structures of metallic and homogeneous dielectric materials have many important applications such as radar technology, antenna design and microwave engineering...etc. Analytical solutions are available for only very limited geometries. For bodies having an arbitrary shape, one has to resort to some approximate numerical technique. A variety of approaches have been developed to study this problem, which includes the MoM, finite element method (FEM), and the finite difference time domain (FDTD) method.

When the bodies are homogeneous, MoM is preferred because the problem can be formulated in terms of surface integrals equation (SIE)over the conducting and dielectric surfaces (Rao *et al.*, 1991). In special for BORs the problem is formulated in terms of integrals over generatrixes (Mautz and Harrington,1979) (Resende *et al.*, 2007). For dielectric BOR (DBOR), many combinations of electric field integral equation (EFIE) and magnetic field integral equation (MFIE) have been investigated (Mautz and Harrington,1979) (Kishk and Shafai, 1986).

EFIE-PMCHWT is the usual formulation for general composite structures (Rao *et al.*, 1991) (Yla-Oijala *et al.*,2005). In this formulation EFIE is applied on metallic surfaces and the Poggio-Miller- Chang-Harrington-Wu-Tsai (PMCHWT) formulation (Mautz and Harrington,1979) is applied on the dielectric interfaces, but it is not sufficient for removing the interior resonances if the structure includes closed metallic surfaces. In that case, in addition, on those surfaces the combined field integral equation (CFIE) must be applied (Mautz and Harrington,1977).

Such solutions (or combinations of them) can be used in the analysis of scattering by composite body (DBOR-wire-junction) as they are shown in Figures.(1) and (2). However, the problem of DBOR-wire was formulated in a system of wire radiators coupled to DBOR without junction region (Junker *et al.*,1993), based on MoM and deals with this problem in the same way of conducting bodies (Shaeffer and Medgysi,1981).

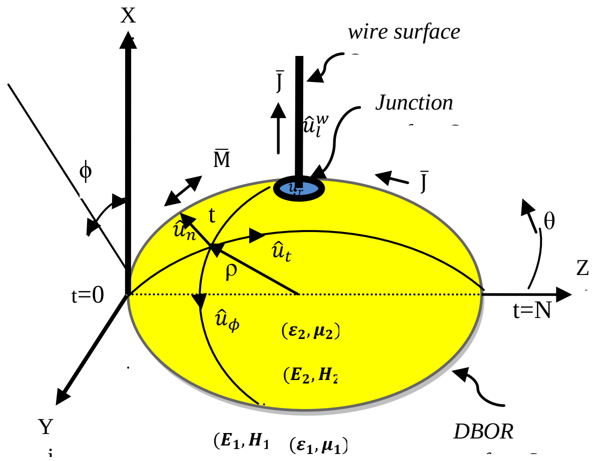


Fig.(1): Homogeneous dielectric object (ε_2 , μ_2) embedded in a homogenous medium (ε_1 , μ_1) with electric attached wire. $\bar{J}$ and $\bar{M}$ are the equivalent surface electric and magnetic currents for the exterior region.

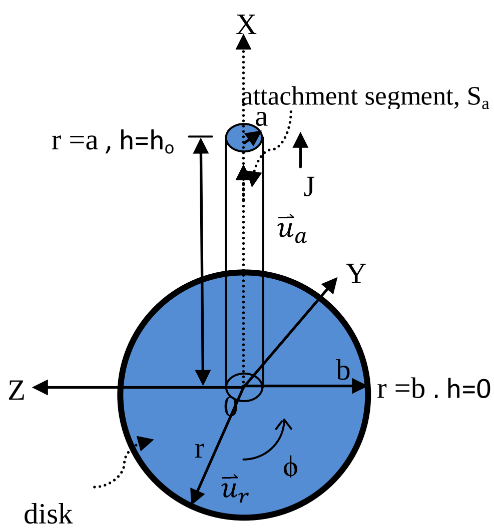


Fig.(2): Attachment region at the wire/BOR surface.

Formulation of the boundary value problem

In this section, wires attached to DBORs are considered. The geometry of Figure.(3a) is typical of the EM boundary value problem of a structure consist of DBOR-wire-junction. The original problem of Figure.(3a) can be divided into two problems: one of the interior region, and the other for the exterior region. For this purpose, the equivalence principle (Stutzman and Thiele,1981) is used and the two equivalent problems are illustrated in Figures.(3b) and (3c). The field components in each region of Figure.(3a) can be constructed readily from the equivalent currents. The boundary conditions to be satisfied are:

$$\hat{n} \times \bar{E}^{et} = 0 \tag{1a}$$

$$\hat{n} \times \bar{H}^{et} = 0 \tag{1b}$$

$$\hat{n} \times \bar{E}^{dt} = 0 \tag{1c}$$

$$\hat{n}\times\overline{H}^{dt}=0$$
(1d)

and the surface equivalent currents are:

$\overline{J}_{we}=\hat{n}\times\overline{H}^{e}$, on S_{we} (2a)

$\overline{J}_{wd}=\hat{n}\times\overline{H}^{d}$, on S_{wd} (2b)

$\overline{J}_{je}=\hat{n}\times\overline{H}^{e}$, on S_{je} (2c)

$\overline{J}_{jd}=\hat{n}\times\overline{H}^{d}$, on S_{jd} (2d)

$\overline{J}_{de}=\hat{n}\times\overline{H}^{e}$, on S_{de} (2e)

$\overline{M}_{de}=-\hat{n}\times\overline{E}^{e}$,on S_{de} (2f)

Eqs.(1) form a set of four equations in the two unknowns $\bar{J}$ and $\overline{M}$.

By enforcing the boundary conditions that the tangential component of the electric field must vanish at the conductor surface, and that the tangential components of both electric and magnetic fields must be continuous across the dielectric surface, a system of integro-differential equations is obtained from which the unknowns J_{wd}, J_{we} , J_{jd} , J_{je} , J_{de} and M_{de} can be determined. This system of equations can be written in operator form (Junker *et al.*,1995) as the following:

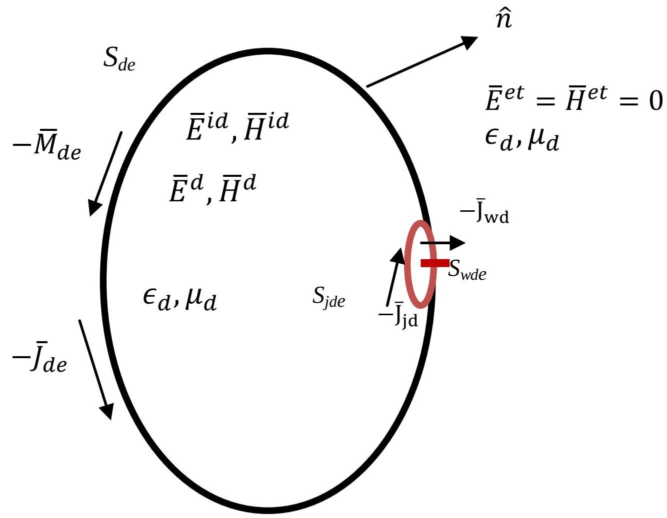
$$\overline{E}^{d}_{\tan}(\overline{J}_{de}+\overline{J}_{wd}+\overline{J}_{jd},\overline{M}_{de})=\overline{E}^{id}_{\tan}$$
, on S_{wd} and S_{jd} (3a)

$$\overline{E}^{e}_{\tan}(\overline{J}_{de}+\overline{J}_{we}+\overline{J}_{je},\overline{M}_{de})=-\overline{E}^{ie}_{\tan}$$
, on S_{we} and S_{je} (3b)

$$\overline{E}^{e}_{\tan}(\overline{J}_{de}+\overline{J}_{we}+\overline{J}_{je},\overline{M}_{de})+\overline{E}^{d}_{\tan}(\overline{J}_{de}+\overline{J}_{wd}+\overline{J}_{jd},\overline{M}_{de})=\overline{E}^{id}_{\tan}-\overline{E}^{ie}_{\tan}$$
,on S_{de} (3c)

$$\overline{H}^{e}_{\tan}(\overline{J}_{de}+\overline{J}_{we}+\overline{J}_{je},\overline{M}_{de})+\overline{H}^{d}_{\tan}(\overline{J}_{de}+\overline{J}_{wd}+\overline{J}_{jd},\overline{M}_{de})=\overline{H}^{id}_{\tan}-\overline{H}^{ie}_{\tan}$$
,on S_{de} (3d)

The above system of equations consists of the EFIE on the conductor surfaces, and both the EFIE and the MFIE on the DBOR surface. This formulation is referred to as the EFIE-PMCHWT (Kishk and Shafai, 1986).



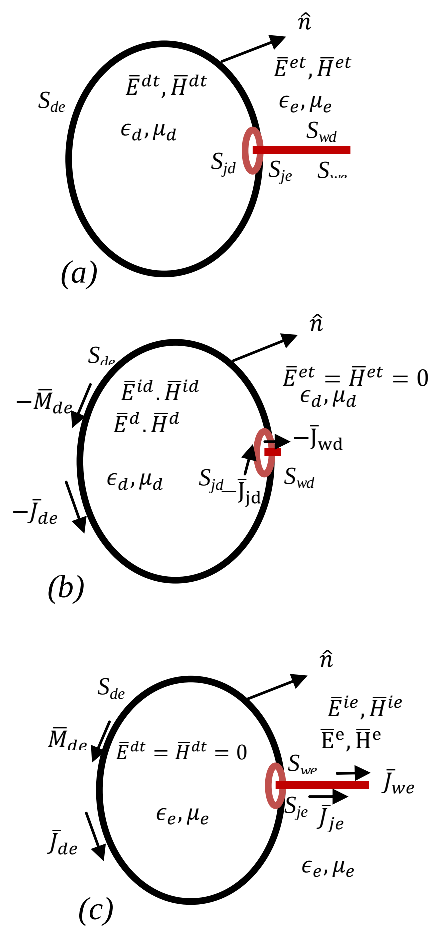


Fig.(3): (a) The original problem, (b) The interior problem and (c) The exterior problem.

The field vectors are defined in terms of magnetic and electric vector and scalar potential functions (Yeung, 2000).

PMCHWT integral equation for DBOR object.

Surface Integral Equation (SIE) methods are popular in solving EM scattering containing homogeneous dielectric materials. The scattering of EM waves from a homogeneous object having permittivity ϵ_2 and permeability μ_2 in a homogeneous background medium (ϵ_1, μ_1), as shown in Fig.(1), can be solved by PMCHWT integral equations (Yla-Oijala,2008), as follow:

$$\hat{n} \times \bar{E}^{inc} = \hat{n} \times [L_1(\bar{J}\,) + L_2(\bar{J}\,)] - \hat{n} \times [K_1(\bar{M}\,) + K_2(\bar{M}\,)] \tag{4}$$

$$\hat{n} \times \bar{H}^{inc} = \hat{n} \times [K_1(\bar{J}\,) + K_2(\bar{J}\,)] + \hat{n} \times \left[\frac{1}{\eta_1^2} L_1(\bar{M}\,) + \frac{1}{\eta_2^2} L_2(\bar{M}\,)\right] \tag{5}$$

Where J and M are the incident electric and magnetic current densities, $\eta_i = \sqrt{\mu_i/\epsilon_i}$ is the wave impedance for region i (i=1,2), $\bar{E}^{inc}$ and $\bar{H}^{inc}$ are the incident electric and magnetic field, respectively. The operators L_i and K_i are defined as (Medgyesi and Putnam,1985):

$$L_i(X) = j\omega\mu_i \int_s [XG_i + 1/(\omega^2 \mu_i \epsilon_i)\, \nabla\nabla'.XG_i\,]\, ds \tag{6}$$

$$K_i(X) = \int_s [X \times \nabla G_i\,]\, ds \tag{7}$$

Where S is the surface of the scatterer, and G_i is the scalar Green's function of background medium (i=1) or dielectric region (i=2) expressed as:

$$G_i(r, \acute{r}) = \frac{e^{-jk_i|r-\acute{r}|}}{4\pi|r-\acute{r}|} \tag{8}$$

with $k_i = \omega\sqrt{\mu_i\epsilon_i}$

Moment Method Solution

The SIEs.(3) are solved by MoM. The described problem can be solved in an efficient manner by a judicious choice of basis and testing functions. First, consider the DBOR of Fig.(1). Due to the axial symmetry, two components of electric and magnetic current can be identified: one directed along the generating arc $(\hat{u}_t)$, and the other in the circumferential direction $(\hat{u}_\phi)$. In order to solve PMCHWT integral equations (4) and (5), the unknown surface current densities are first expanded in terms of Fourier modes (Wu and Tsai,1977)

$$\bar{J}_s(t,\phi) = \bar{J}^t(t,\phi) + \bar{J}^\phi(t,\phi) = \sum_{n=-\infty}^{\infty}\left[\bar{J}_n^t(t) + \bar{J}_n^\phi(t)\right]e^{jn\phi} \tag{9a}$$

$$\bar{M}_s(t,\phi) = \eta_a\left(\bar{M}^t(t,\phi) + \bar{M}^\phi(t,\phi)\right) = \eta_a \sum_{n=-\infty}^{\infty}\left[\bar{M}_n^t(t) + \bar{M}_n^\phi(t)\right]e^{jn\phi} \tag{9b}$$

Where η_a is the wave impedance for interior or exterior regions.

These components can be further expanded on the generating arc in terms of sub-domain basis functions f_i as:

$$\left.\begin{aligned}\bar{J}_n^t(t) &= \hat{u}_t \sum_{i=1}^{N_d-1} I_{ni}^t f_i(t)\\ \bar{J}_n^\phi(t) &= \hat{u}_\phi \sum_{i=1}^{N_d-1} I_{ni}^\phi f_i(t)\end{aligned}\right\} \tag{10a}$$

$$\left.\begin{aligned}\bar{M}_n^t(t) &= \hat{u}_t \sum_{i=1}^{N_d-1} k_{ni}^t f_i(t)\\ \bar{M}_n^\phi(t) &= \hat{u}_\phi \sum_{i=1}^{N_d-1} k_{ni}^\phi f_i(t)\end{aligned}\right\} \tag{10b}$$

Where the $f_i(t) = \frac{1}{\rho} T_i(t)$ and T_i is the triangular function as shown in Fig.(4a). The testing function is

$$\bar{W}_{mi}^{\alpha} = \hat{u}_{\alpha} f_i(t) e^{-jm\phi} \text{ , } \alpha\text{=t or } \phi \qquad (11)$$

For wire, the basis $\bar{J}^{w}$ and testing functions $\bar{W}^{w}$ are introduced by

$$\bar{J}^{w} = \hat{u}_{\ell}^{w} I_{\ell}^{w} T_{\ell}(h) = \sum_{\ell=1}^{N_w-1} \hat{u}_{\ell}^{w} I_{\ell}^{w} T_{\ell}(h) \qquad (12)$$

$$\bar{J}^{w} = \bar{W}^{w} \qquad (13)$$

Where $\hat{u}_{\ell}^{w}$ is the unit vector along wire segment, and $T_{\ell}(h)$ is the triangular function as shown in Fig.(4b) .

The junction basis $\bar{J}^{j}$ and testing functions $\bar{W}^{j}$ have the form

$$\bar{J}^{j}(p) = I^{j} \begin{cases} J_a^j , & p \in S_a \\ J_d^j , & p \in S_d \end{cases} \qquad (14a)$$

Where

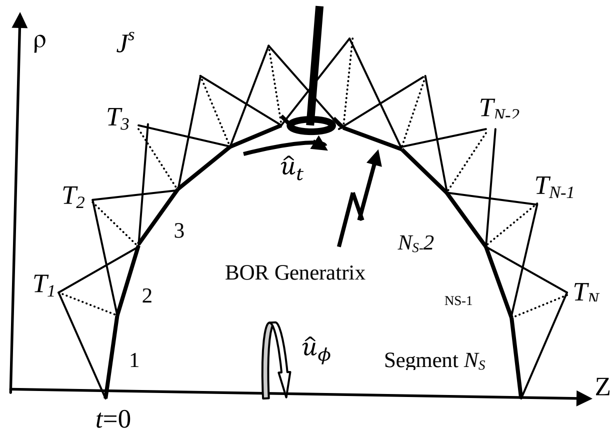
$$\bar{J}_a^j = \hat{u}_a \frac{T_a(h)}{2\pi a} \qquad (14b)$$

$$\bar{J}_d^j = -\hat{u}_r \frac{1}{2\pi r} \left(\frac{b-r}{b-a}\right) \qquad (14c)$$

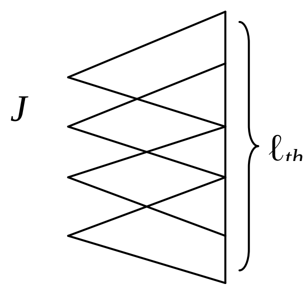
$$\bar{J}^{j} = \bar{W}^{j} \qquad (15)$$

Where $\bar{J}_a^j$ is the current density on the wire segment nearest the junction region, i.e., wire attachment segment S_a, $\bar{J}_d^j$ is the current density over the BOR surface near the junction regions (disk region) S_d, $\hat{u}_a$ and $T_a(h)$ are an outward–directed unit vector and half–triangle function as shown in Fig.(4c) on the attachment segment, respectively; $\hat{u}_r$ is a unit vector on the (annular) disk surface away from the wire, *r* is the radial distance on the disk, *b* is the outer disk radius, *a* is the wire radius as shown in Fig.(2). A similar formulation for the junction currents is given in (Shaeffer and Medgysi,1981) and (Newman and Pozar,1978).

(a)



(b)



(c)

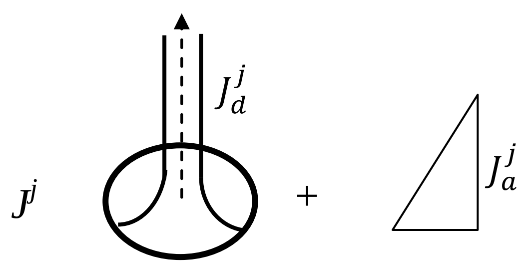


Fig.(4): Triangular basis functions (a) over the BOR generatrix.(b) on l_{th} segment of attachment wire. (c) junction region representation.

The number of data points on the DBOR, wire and junction region are N_d , N_w and N_a , respectively. The scattered is thus modeled by adding the three region data points (Newman and Pozar,1978).Then, The total number of data points are $N=N_d+N_w+N_a$.

$$\begin{bmatrix} Z_n^{EE} & Z_n^{EH} \\ \eta_1 Z_n^{HE} & \eta_1^2 Z_n^{HH} \end{bmatrix} \begin{bmatrix} I_n \\ \eta_1 M_n \end{bmatrix} = \begin{bmatrix} V_n^E \\ \eta_1 V_n^H \end{bmatrix} \quad (16)$$

The $I_{ni}^t, I_{ni}^{\phi}, k_{ni}^t, k_{ni}^{\phi}$, I^w, and I^j are the unknown current coefficients.

In order to balance the impedance elements(Ruiet.al,2010)(Godaymi2007)(Cao, and Gao,2008), the matrix equation can be written

Now the composite body (DBOR/wire/junction) have a system of equations by substituting Eqs.(10), (12), and (14), into Eqs.(3) and using the linearity of the operators, yield:

$$\sum_{n=-\infty}^{\infty}\left[\sum_{j=1}^{N_d-1} I_{nj}^d \bar{E}_{\tan}^d\left(\bar{J}_{nj}^d,0\right)+\sum_{l=1}^{N_w-1} I_l^{wd}\bar{E}_{\tan}^d\left(\bar{J}_w^d,0\right)+\sum_{j=1}^{Na-1} I_j^{jd}\bar{E}_{\tan}^d\left(\bar{J}_j^d,0\right)+\eta_d\sum_{j=1}^{N_d-1} K_{nj}\bar{E}_{\tan}^d\left(0,\bar{M}_{nj}\right)\right] = \bar{E}_{\tan}^{id}\left(\bar{J}^d\right) \quad \text{on } S_{wd} \text{ and } S_{jd} \quad (17a)$$

$$\sum_{n=-\infty}^{\infty}\left[\sum_{j=1}^{N_d-1} I_{nj}^e \bar{E}_{\tan}^e\left(\bar{J}_{nj}^e,0\right)+\sum_{l=1}^{N_w-1} I_l^{we}\bar{E}_{\tan}^e\left(\bar{J}_w^e,0\right)+\sum_{j=1}^{N_a-1} I_j^{je}\bar{E}_{\tan}^e\left(\bar{J}_j^e,0\right)+\eta_e\sum_{j=1}^{N_d-1} K_{nj}\bar{E}_{\tan}^e\left(0,\bar{M}_{nj}\right)\right] = \bar{E}_{\tan}^{ie}\left(\bar{J}^e\right)$$

$$\text{, on } S_{we} \text{ and } S_{je} \quad (17b)$$

$$\sum_{n=-\infty}^{\infty}\left[\sum_{j=1}^{N_d-1} I_{nj}\left\{\bar{E}_{\tan}^d\left(\bar{J}_{nj}^d,0\right)+\bar{E}_{\tan}^e\left(\bar{J}_{nj}^e,0\right)\right\}+\sum_{j=1}^{N_w-1}\left\{I_l^{wd}\bar{E}_{\tan}^d\left(\bar{J}_w^d,0\right)+I_l^{we}\bar{E}_{\tan}^e\left(\bar{J}_w^e,0\right)\right\}+\sum_{j=1}^{Na-1}\left\{I_j^{jd}\bar{E}_{\tan}^d\left(\bar{J}_j^d,0\right)+I_j^{je}\bar{E}_{\tan}^e\left(\bar{J}_j^e,0\right)\right\}+\sum_{j=1}^{N_d-1} K_{nj}\left\{\eta_d\bar{E}_{\tan}^d\left(0,\bar{M}_{nj}\right)+\eta_e\bar{E}_{\tan}^e\left(0,\bar{M}_{nj}\right)\right\}\right] = \bar{E}_{\tan}^{id}\left(\bar{J}^d\right)-\bar{E}_{\tan}^{ie}\left(\bar{J}^e\right) \quad \text{, on } S_{de} \quad (17c)$$

$$\sum_{n=-\infty}^{\infty}\left[\sum_{j=1}^{N_d-1} I_{nj}\left\{\bar{H}_{\tan}^d\left(\bar{J}_{nj}^d,0\right)+\bar{H}_{\tan}^e\left(\bar{J}_{nj}^e,0\right)\right\}+\sum_{j=1}^{N_w-1}\left\{I_l^{wd}\bar{H}_{\tan}^d\left(\bar{J}_w^d,0\right)+I_l^{we}\bar{H}_{\tan}^e\left(\bar{J}_w^e,0\right)\right\}+\sum_{j=1}^{Na-1}\left\{I_j^{jd}\bar{H}_{\tan}^d\left(\bar{J}_j^d,0\right)+I_j^{je}\bar{H}_{\tan}^e\left(\bar{J}_j^e,0\right)\right\}+\sum_{j=1}^{N_d-1} K_{nj}\left\{\eta_d\bar{H}_{\tan}^d\left(0,\bar{M}_{nj}\right)+\eta_e\bar{H}_{\tan}^e\left(0,\bar{M}_{nj}\right)\right\}\right] = \bar{H}_{\tan}^{id}\left(\bar{J}^d\right)-\bar{H}_{\tan}^{ie}\left(\bar{J}^e\right) \quad \text{, on } S_{de} \quad (17d)$$

An advantage of the Galerkin's procedure is the resulting symmetry in the inner products when the coordinates of the testing and basis functions are interchanged.

Therefore, the Eqs.(17) have a system of new equations which may be expressed in matrix form as follow :

$$
\begin{bmatrix}
([\mathbf{Z}^{e}_{de,de}]_{n} + \boldsymbol{\eta}_{r}[\mathbf{Z}^{d}_{de,de}]_{n}) & ([\mathbf{Y}^{e}_{de,de}]_{n} + [\mathbf{Y}^{d}_{de,de}]_{n}) & \boldsymbol{\eta}_{r}[\mathbf{Z}^{d}_{de,wd}]_{n} & [\mathbf{Z}^{e}_{de,we}]_{n} & \boldsymbol{\eta}_{r}[\mathbf{Z}^{d}_{de,jd}]_{n} & [\mathbf{Z}^{e}_{de,je}]_{n} \\
([\mathbf{Y}^{e}_{de,de}]_{n} + [\mathbf{Y}^{d}_{de,de}]_{n}) & (-[\mathbf{Z}^{e}_{de,de}]_{n} + \frac{\mathbf{1}}{\boldsymbol{\eta}_{r}}[\mathbf{Z}^{d}_{de,de}]_{n}) & [\mathbf{Y}^{d}_{de,wd}]_{n} & [\mathbf{Y}^{e}_{de,we}]_{n} & [\mathbf{Y}^{d}_{de,jd}]_{n} & [\mathbf{Y}^{e}_{de,je}]_{n} \\
\boldsymbol{\eta}_{r}[\mathbf{Z}^{d}_{wd,de}]_{n} & [\mathbf{Y}^{d}_{wd,de}]_{n} & [\mathbf{Z}^{d}_{wd,wd}] & [\mathbf{0}] & [\mathbf{Z}^{d}_{wd,jd}] & [\mathbf{0}] \\
[\mathbf{Z}^{e}_{we,de}]_{n} & [\mathbf{Y}^{e}_{we,de}]_{n} & [\mathbf{0}] & [\mathbf{Z}^{e}_{we,we}] & [\mathbf{0}] & [\mathbf{Z}^{e}_{we,je}] \\
\boldsymbol{\eta}_{r}[\mathbf{Z}^{d}_{jd,de}]_{n} & [\mathbf{Y}^{d}_{jd,de}]_{n} & [\mathbf{Z}^{d}_{jd,wd}] & [\mathbf{0}] & [\mathbf{Z}^{d}_{jd,jd}] & [\mathbf{0}] \\
[\mathbf{Z}^{e}_{je,de}]_{n} & [\mathbf{Y}^{e}_{je,de}]_{n} & [\mathbf{0}] & [\mathbf{Z}^{e}_{je,we}] & [\mathbf{0}] & [\mathbf{Z}^{e}_{je,je}]
\end{bmatrix}
\begin{bmatrix}
[\mathbf{I}^{\alpha}]_{n} \\ [\mathbf{k}^{\alpha}]_{n} \\ [\mathbf{I}^{wd}] \\ [\mathbf{I}^{we}] \\ [\mathbf{I}^{jd}] \\ [\mathbf{I}^{je}]
\end{bmatrix}
=
\begin{bmatrix}
[\mathbf{V}^{\alpha}_{de}]^{E}_{n} \\ [\mathbf{V}^{\alpha}_{de}]^{H}_{n} \\ [\mathbf{V}^{wd}] \\ [\mathbf{V}^{we}] \\ [\mathbf{V}^{jd}] \\ [\mathbf{V}^{je}]
\end{bmatrix}
\tag{18}
$$

Where $\eta_r = \sqrt{\mu_a/\epsilon_a}$, and $\alpha = t$ or ϕ

The full representation of the sub matrices of the impedance and admittance and excitation are presented in details in the thesis chapters and appendices.

The radar cross section evaluation

The far field is the far field scattered by structure of DBOR-wire-junction. For plane wave excitation, and through the reciprocity theorem (Wu and Tsai,1977) one may find the radiation field $\bar{E}^s$ at a distance r from the origin due to the surface currents J and M on S a

$$
\bar{\mathrm{E}}^{s} \cdot \hat{u}^{r} = \iint_{s} (\bar{J}(\bar{r}) \cdot \bar{E}^{r} - \bar{M}(\bar{r}) \cdot \bar{H}^{r})\, ds \tag{19}
$$

Where $\hat{u}^r$ is a unit vector specifying the polarization, $\overline{E}^r$ and $\overline{H}^r$ is the electric and magnetic field is due to $\hat{u}^r$, respectively, and given by:

$$\overline{\mathrm{E}}^{\mathrm{r}} = -\frac{jk\eta}{4\pi\, r} e^{-jk\, r_r}\, \hat{u}^r\, e^{-j\bar{k}_r \cdot \bar{r}} \qquad (20a)$$

$$\overline{\mathrm{H}}^{\mathrm{r}} = -\frac{j}{4\pi\, r} e^{-jk\, r_r} (\overline{\mathrm{k}}_{\mathrm{r}} \times \hat{u}^r)\, e^{-j\bar{k}_r \cdot \bar{r}} \qquad (20b)$$

Where k_r is the propagation vector of the plane wave, k is the propagation constant and η is the intrinsic impedance of the medium outside S.

$$\begin{aligned} (R_n^{t\theta})_i &= \langle \bar{E}_\theta^r, \bar{J}_{ni}^t \rangle \\ (R_n^{\phi\theta})_i &= \langle \bar{E}_\theta^r, \bar{J}_{ni}^\phi \rangle \\ (\mathcal{R}_n^{t\phi})_i &= \langle \bar{H}_\phi^r, \bar{M}_{ni}^t \rangle \\ (\mathcal{R}_n^{\phi\phi})_i &= \langle \bar{H}_\phi^r, \bar{M}_{ni}^\phi \rangle \end{aligned} \qquad (21)$$

And for the ϕ-polarized plan wave

$$\begin{aligned} (R_n^{t\phi})_i &= \langle \bar{E}_\phi^r, \bar{J}_{ni}^t \rangle \\ (R_n^{\phi\phi})_i &= \langle \bar{E}_\phi^r, \bar{J}_{ni}^\phi \rangle \\ (\mathcal{R}_n^{t\theta})_i &= -\langle \bar{H}_\theta^r, \bar{M}_{ni}^t \rangle \\ (\mathcal{R}_n^{\phi\theta})_i &= -\langle \bar{H}_\theta^r, \bar{M}_{ni}^\phi \rangle \end{aligned} \qquad (22)$$

In the case of BOR, and for θ-polarized plane wave the contribution of measurement matrix [R] is written by (Harrington and Mautz,1969)

In other hand side, the contribution to R^w from the wire in both θ-and ϕ-polarized can be written in the form:

$$\begin{aligned} R^{w,\theta} &= \langle \bar{E}_\theta^r, \bar{J}_l^w \rangle \\ R^{w,\phi} &= \langle \bar{E}_\phi^r, \bar{J}_l^w \rangle \end{aligned} \qquad (23)$$

Similarly, the contribution to R^j from the junction are

$$\begin{aligned} R^{j,\theta} &= \langle \bar{E}_\theta^r, \bar{J}_a^j \rangle + \langle \bar{E}_\theta^r, \bar{J}_d^j \rangle \\ R^{j,\phi} &= \langle \bar{E}_\phi^r, \bar{J}_a^j \rangle + \langle \bar{E}_\phi^r, \bar{J}_d^j \rangle \end{aligned} \qquad (24)$$

The scattering cross section σ^{pq} is defined by

$$\sigma^{pq} = 4\pi r^2 \frac{|\bar{E}_{pq}^s|^2}{|\bar{E}^i\ |^2} \qquad (25)$$

Where p is either θ or ϕ and q is either θ or ϕ, E^i and E^s is the incident and scattered field. For large r the relation between scattering matrix and Eq.(25) can be written as:

$$\sigma^{pq} = 4\pi |S^{pq}|^2 \qquad (26)$$

Where

$$\mathrm{S}^{\mathrm{pq}} = \frac{-\mathrm{j}\omega\mu}{4\pi} \{[[\mathrm{R\_n\char94 tp}\ ][\mathrm{R\_n\char94 \phi p}\ ][\mathcal{R}\mathrm{\_n\char94 tq}\ ][\mathcal{R}\mathrm{\_n\char94 \phi q}\ ]] \cdot [\mathrm{I\_n\char94}\ \ ]\mathrm{\char94 T} + [[\mathrm{R\_\char94 wp}\ ][\mathrm{I\_\char94 wq}\ ]] + [[\mathrm{R\_\char94 jp}\ ][\mathrm{I\_\char94 jq}\ ]]\} \qquad (27)$$

So, the radar cross section RCS normalized to wavelength is given by substituting (27) in (26), to obtain a formula to conducting and dielectric BOR-wire-junction, respectively.

$$\frac{\sigma^{pq}}{\lambda^2} = \frac{\eta^2 k^4}{16\pi^3}\left|\sum_{-n}^{n}\left(R_n^{tp} I_n^{tq} + R_n^{\phi p} I_n^{\phi q}\right) + R^{wp} I^{wq} + R^{jp} I^{jq}\right|^2 \tag{28a}$$

$$\frac{\sigma^{pq}}{\lambda^2} = \frac{\eta^2 k^4}{16\pi^3}\left|\sum_{-n}^{n}\left(R_n^{tp} I_n^{tq} + R_n^{\phi p} I_n^{\phi q} + \mathcal{R}_n^{tp} k_n^{tq} + \mathcal{R}_n^{\phi p} k_n^{\phi q}\right) + R^{wp} I^{wq} + R^{jp} I^{jq}\right|^2 \tag{28b}$$

Results

To demonstrate the validity of the formulation, numerical results are presented for a homogeneous DBOR illuminated by a plane wave, for which the PMCHWT is known. The first example is a lossy dielectric sphere, whose radius are set to be *ka*=1,and the value of dielectric constant ε_r=1.44. The bistatic RCS computed by the PMCHWT integral equation methods is shown in Fig.(5). It can be seen from figure that the horizontal polarization (HP) and vertical polarization (VP) results are in agreement with (Wu and Tsai,1977). Moreover, the matrix elements integrals are computed numerically by using 20 point Gaussian quadrature.

The second example is closed cylindrical rod with rounded ends of radius *a*=0.2 λ and length $l_c = 1.1\lambda$ is next examined by SIE. It is chosen as another test to illustrate the validity of formulation and MoM to complex bodies, as depicted in Fig.(6). Figure (6) shows that the solution accuracy of PMCHWT results in the two polarizations (HP and VP) in convergence with that of EFIE with ε_r=1. However, the convergence represented by the coincided with conductor part of (Andreasen, 1965).

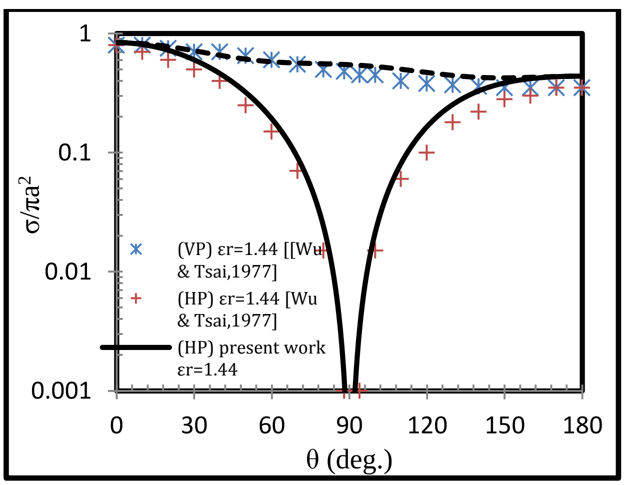


Fig.(5): Bistatic RCS of dielectric sphere (ε_r=1.44 and radius a=0.159λ).

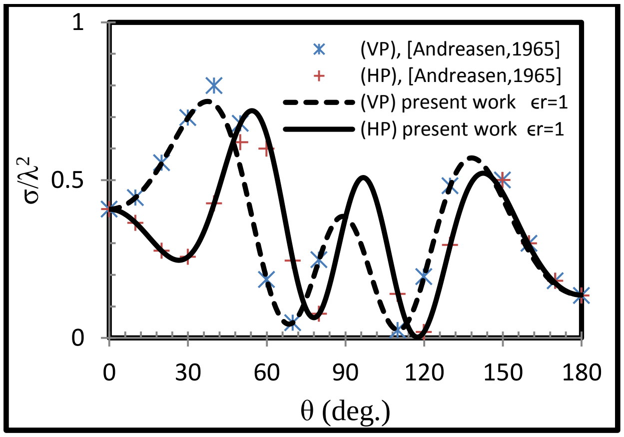


Fig.(6): Bistatic RCS for the dielectric cylindrical rod with rounded ends

The applications were in two parts. The first was CBOR-wire-junction by solving the EFIE with MoM technique as outlined. The computed results are compared with measured monostatic RCS obtained by (Shaeffer,1982) and normalized to the cross sections of a sphere, σ_o=0.00525m^2, which was computed using the Mie series solution for a perfect conducting sphere. Note, the junction dimensions for all applications are a=0.005λ and b=0.107λ.

The results are computed by applying Eq.(28a) for the conducting sphere of radius a=0.444λ (ka=2.79), with one attached wire of length ($l_w = 0.444\lambda$), as shown in Fig.(7). It can be seen that the computation results obtained using the MoM is in a good agreement with the measured, where the RCS of a sphere does not change for a body of radial symmetry alone. Also, The scattering cross section from the flat-faced cylinder, radius a=0.344λ and length $l_c = 1.98\lambda$ with attached wires of length $l_w = 0.880\lambda$, is computed for HP and comparison with experimental data. The results are presented in Fig.(8) with two attached wires and shows a good agreement with the measured. The effect of thin wires and junction to the total backscattered RCS is noticeable.

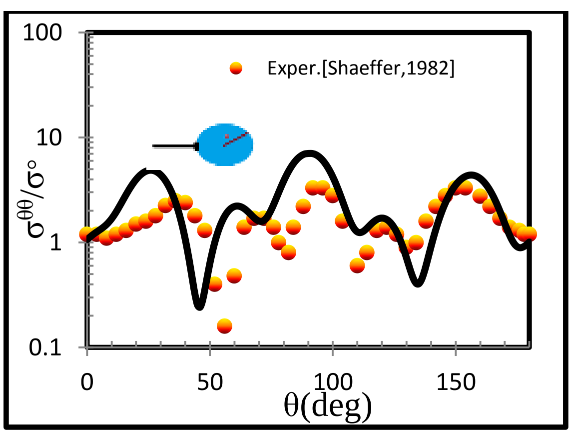


Fig.(7): Backscattering RCS in HP from conducting sphere with one attached wire.

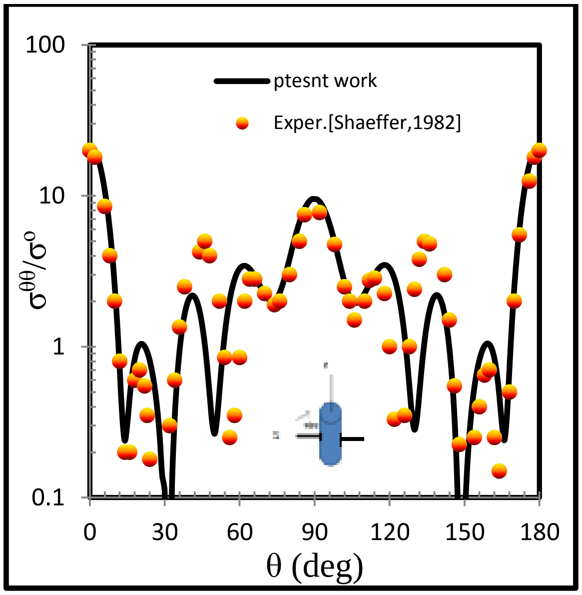


Fig.(8): Monostatic RCS in HP from conducting cylinder with two attached wire.

The numerical results are presented to generalize the EFIE-PMCHWT on a selected applications consist of DBOR-wire-junction, as second application. Furthermore, the method of numerical solution for a particular scattering problem consists of choosing a value for ε_r and the number of attached wires.

The first results are computed for the a dielectric sphere with ε_r=4 and radius such that *ka*=2.79, with one attached wire of length $l_w = 0.444\lambda$. Figure(9) shows the backscattered RCS in HP by applying Eq.(28b) in the case of dielectric sphere with single wire. In comparing the results with the conductor case, the addition of wire produces a result that has less than number of lobe and no lobe symmetry. This result may be explained by the fact that the effect of single wire and dielectric constant appears for backside incident $\theta>90^o$, where the wire is partially shielded by sphere. Moreover, the null is less deep than on the front illuminated side with sharp decreased in RCS between $\theta=30^o$ to $\theta=150^o$ as a common effect of wire and dielectric constant.

To explain the effect of two attached wires, another important results is taking by a dielectric flat-faced cylinder with ε_r=4 and radius *a*=0.344λ and length

$l_c = 1.98\lambda$, where anyone of the two wires has a length of $l_w = 0.880\lambda$. A similar phenomenon has been observed previously in the case of conducting bodies with two attached wires, but the result here shows that the effect of ε_r and thin wires on total RCS is noticeable. It can be seen that the results show the same effect for the two wires, located at cylinder center, on the RCS, whereas the common effect with ε_r is changed the total backscattered RCS between angles $\theta=10^o$

to $\theta=170^{o}$ from that in the conductor case as shown in Fig.(10).

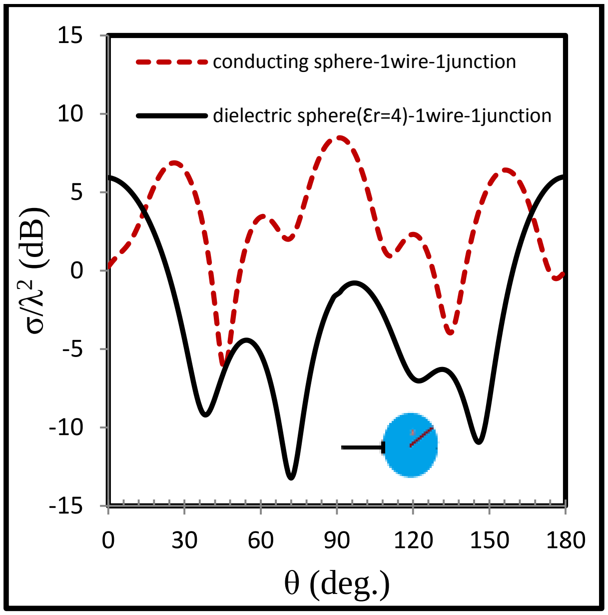


Fig.(9): Backscattered RCS in HP of EFIE-PMCHWT formulation for a homogeneous dielectric sphere with one attached wire (radius $a=0.159\lambda$, $\varepsilon_r=4$ and $l_w=0.444\lambda$.

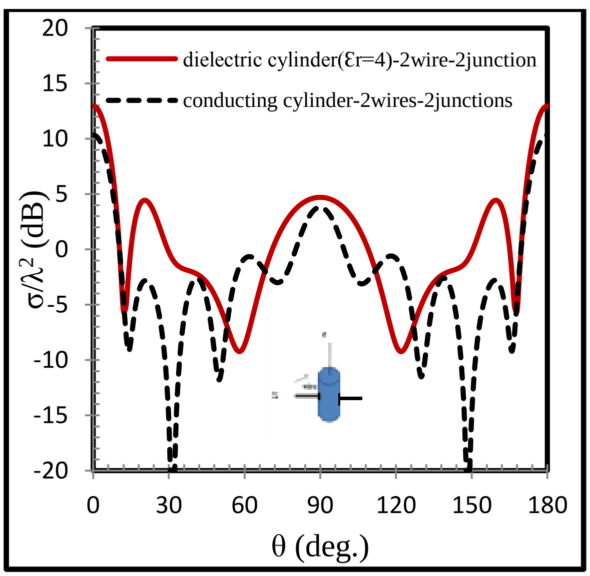


Fig.(10): Backscattered RCS in HP of EFIE-PMCHWT formulation for finite homogeneous dielectric cylinder with two attached wires (radius $a=0.344\lambda$, $\varepsilon_r=4$, $l_c=1.98\lambda$ and $l_w=0.880\lambda$.

The important part of the applications is to determine the efficacy of the composite body scattering to more complicated and practical geometries. The calculations of RCS for missile configuration with wings modeled as wire-loops, shown in Fig.(11), was obtained and compared with other measurements in (Shaeffer,1982).

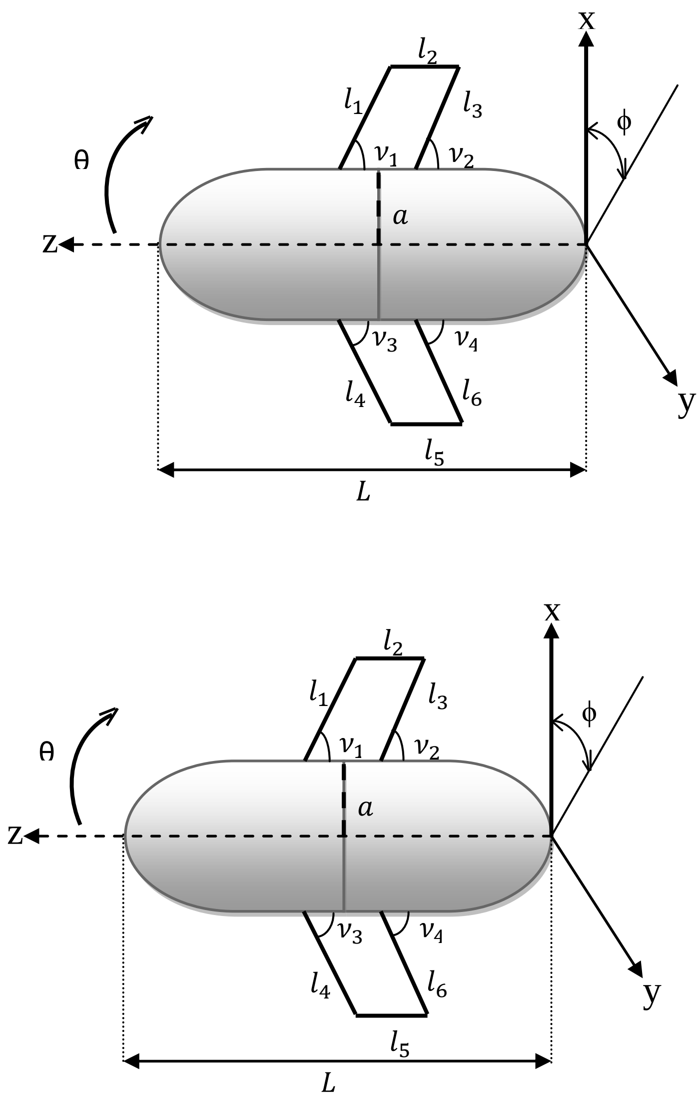


Fig.(11): Geometric shape for missile configuration with wings model,
($l_{w1}=l_1+l_2+l_3$, $l_{w2}=l_4+l_5+l_6$, $l_1=l_3=l_4=l_6$, $l_2=l_5$ and $\nu_1=\nu_2=\nu_3=\nu_4$).

This modeling approach is an extension of the stick representation used by (Lin and Richmond,1975), but the only difference here is that the body represented by a BOR. The body was a cylinder with hemisphere endcaps of diameter $d/\lambda=0.688$ (ka=2.16) and total length $=2.6\lambda$, the wire-loop wing of length $l_{w1}=l_{w2}=0.826\lambda$ and $l_2=0.76l_1$with slant angle of wings $\nu_i=45^o$ (i=1,...,4). The computed and measured results, in the case of CBOR-wire-junction, are in a good agreement for HP of wire-loops representing to the wings and shown in Fig.(12), whereas the wings do not contribute to the RCS for vertical polarization because the wings are normal to the incident electric vector.

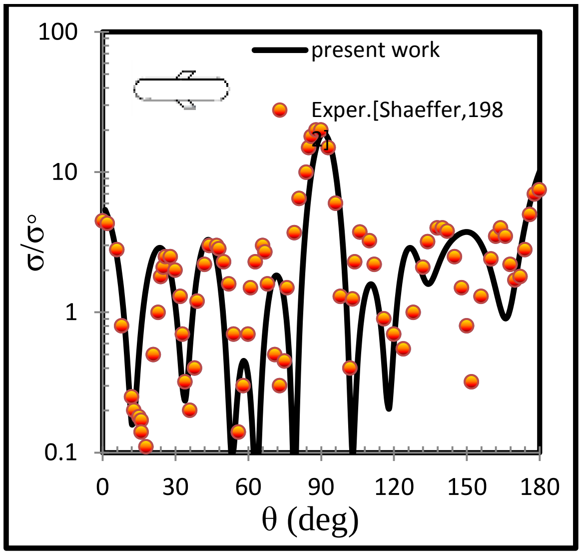


Fig.(12): Monostatic RCS in HP from conducting cylinder with hemisphere endcaps and wire-loop wings.

It is interesting to note that the effect of single value for dielectric constant were appearing in total RCS in the case of DBOR-wire-junction. Another important result finding were that for several possible value of ε_r. it is represented by a missile consisting of a dielectric cylindrical rod with rounded ends of radius a=0.32λ and total length $l_c = 2.6\lambda$, and ε_r = 2,4, and 6, with wire-loop wings of length $l_w = 0.826\lambda$. Figure(14) shows the monostatic cross section for the HP for different values of ε_r. However, this result has not previously been described, but compared with conducting case. It should be noted that the effect of changing ε_r will be disparate among these values as shown in Fig.(13).

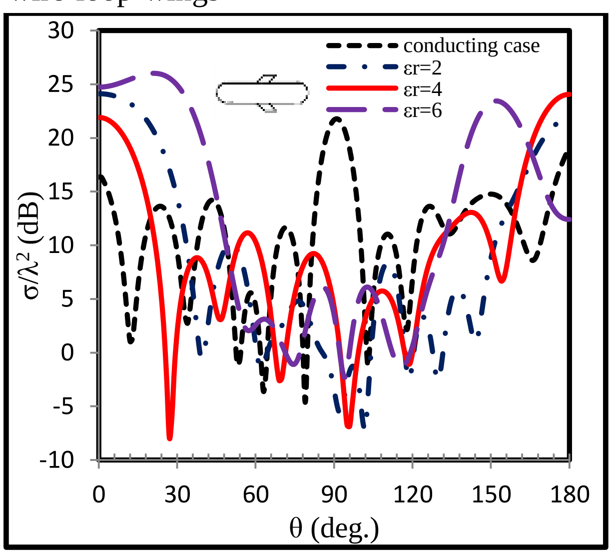


Fig.(13): Backscattered RCS in HP of EFIE-PMCHWT formulation for homogeneous dielectric cylindrical rod with rounded ends of different values of ε_r, with two wire-loop wings.

Conclusion

The choice of a formulation is important if the surface is irregular or material contrast is changed. The solution of EFIE-

PMCHWT formulations for EM scattering by composite body with irregular shape using the MoM with Galerkin's procedure has been presented. In the case of DBOR-wire-junction, the moment matrix for DBOR is twice as moment matrix of CBOR. The EFIE-PMCHWT formulations with MoM of Galerkin' s approach give largely reasonable results compared with that of CBOR-wire-junction, and the lossy DBOR-wire-junction, is treated efficiently with these formulations. Adoption of the comparative results of CBOR-wire-junction with case of DBOR-wire-junction was due to the lack of available results, the fact that the last one subject in the new application. Furthermore, the addition of attached wires significantly alters the constant CBOR and DBOR cross-section.

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الاستطارة الكهرومغناطيسية من الاجسام العازلة المتناظرة محوريا مع الاسلاك الملحقة

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الخلاصة

في هذا البحث ندرس الاستطارة الكهرومغناطيسية بواسطة الاجسام العازلة والمتناظرة محوريا بوجود الاسلاك الملحقة ومنطقة مشتركة (الوصلة) بين السلك وسطح الجسم المتناظر محوريا، مكونة جميعها جسم غير منتظم. ان تقنية المعادلة السطحية قد طورت لتحليل خصائص الاستطارة. وقد استخدمت المعادلتين التكامليتين الاتجاهيتين والتي تعتمد على قوانين ماكسويل ودوال كرين بشروط حدودية ملائمة. ان الصيغة المتولدة كانت بواسطة المعادلة التكاملية للمجال الكهربائي EFIE وصيغة الـ PMCHWT، اما الحسابات العددية فكانت بواسطة طريقة العزوم (MoM) مع صيغة كالركين. تم التحقق من صحة النتائج العددية للجسم العازل والمتناظر محوريا وقورنت ايضا مع باحثين اخرين، في حين قورنت نتائج الجسم العازل والمتناظر محوريا بوجود السلك الملحق والوصلة مع نتائج الجسم الموصل والمتناظر محوريا مع السلك والوصلة ، وتحققت صحتها بالمقارنة مع النتائج العملية. وجد ان اضافة الاسلاك تغير كثيرا من المقطع العرضي الثابت للاجسام الموصلة والعازلة بدون اسلاك، وان تاثيرتغيير قيم ثابت العزل يكون متباين على مساحة المقطع العرضي للاستطارة.