Dynamic Crack Growth Analysis of Aluminum Thin Plates Using Analytical and Finite Element Analysis

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Received on:1/12/2014 & Accepted on:2/4/2015

ABSTRACT

In this work, the stresses and dynamic crack growth were studied and analyzed in thin flat plate with a surface crack at the center, subjected to cycling low velocity impact loading for two types of aluminum plates (2024, 6061). Analytical solution using classical plate theory by using Levy solution employed to determine stresses induced in plate due to the impact load. The impact load applied at the center of surface plate, was calculated by Hertzian contact law. Stresses with respect to time at the crack tip and the velocity of crack growth were calculated through elastic plastic fracture mechanics equations. The size of plastic zone around the crack tip which caused by impact loading was found by Dugdal's model theory. Numerical analysis using program (ANSYS11-APDL) based on finite element method used to analysis the stresses with respect to time at crack tip and then find the velocity of the crack growth under cycling impact loading. The results show that the stresses increase with increasing in the crack length (i.e.160Mpa at 7mm and 175Mpa at 10mm for Al-6061 also 140Mpa at 7mm and 165Mpa at 10mm for Al-2024).. It was found that the cumulative number of cycles leads to increase in the stress values with increasing in crack growth velocity.

Keywords: stress, velocity, crack tip, analysis, plate.

تحليل نمو الشق الداينميكي لصفائح الألمنيوم الرقيقة بأستخدام التحليل الرياضي والتحليل بالعناصر المحددة

الخلاصة

في هذا العمل تم دراسة وتحليل ألاجهادات وحركة نمو الشق لصفائح رقيقه ومستويه تحتوي على شق سطحي في وسطها ومعرضه لحمل صدمه متكرر ذات سرعة منخفضة ولنوعين من سبانك ألالمنيوم (2024,6061) . الحل التحليلي وذلك بأستخدام النظرية الكلاسيكية للصفائح (Levy solution) لحساب الاجهادات المتولده في الصفائح

https://doi.org/10.30684/etj.33.5A.6

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بالأعتماد على حمل الصدمه, حيث ان حمل الصدمة ناتج من أصطدام الذراع بمركز سطح الصفيحة ويحسب من قانون (Hertzian) للتلامس . ألاجهادات بالنسبه للزمن عند قمة الشق وسرعة نمو الشق تم حسابهما بمعادلات الية الكسر الميكانيكي للمرونة واللدونة, وبما أن حمل الصدمة يسبب وجود المنطقه اللدنة حول قمة الشق تم استخدام نظرية (Dugdal) لأيجاد حجم هذه المنطقه. التحليل العددي بأستخدام برنامج (ANSYS11-APDL) بألاعتماد على طريقة العناصر المحددة وذلك لتحليل وحساب الأجهادات بالنسبه للزمن عند قمة الشق ومن ثم حساب سرعة نمو الشق تحت تأثير حمل الصدمه المتكرر. وقد أظهرت النتائج بأن ألاجهادات المتولده تزداد بأزدياد طول الشق ور166ميكاباسكال عند 7ملم و175 ميكاباسكال عند 10ملم ل6061 وكذلكه10مليكاباسكال عند 7ملم وبالتالي زيادة سرعة نمو الشق.

INTRODUCTION

T tress analysis and study the cracks propagation within structures is very important to improve the design against fracture. In real life cracks may occur in some parts ▶ Of structure that may lead to failure like accidental cracking of welded connection, explosion of pressure vessel, buildings and sudden failure of jet aircraft. Therefore, stress analysis and study the cracks propagation within structures is very important to improve the design against fracture, In this work, stress of surface cracked thin plate under cyclic loading was analyzed. Cycling load involved in many structures such as automobiles (piston inside cylinder), wing of aircraft, bridges, and machines structures. There are many researchers were studied the stress at crack tip with different fluctuating stress-time modes.[1], dealt with the solution for stress and deformation field induced by a central crack, they analyzed the effects of plasticity on the stress and deformation field near the crack tip while[2], adopted a computationally efficient adaptive F.E.M strategy for accurate reliable evaluation of contact force under low velocity impact on isotropic plates is solved using a Hertzian contact law.[3], studied the effect of the impact modification on slow crack growth in poly, using the analytical solution (Paris relation and Dug dale model) and comparing it with the experimental solution.[4], presented the plane-strain crack subjected to mode I cyclic loading under small scale yielding was analyses, stress and strain fields was studied.[5], studied the fatigue short and long cracks behaviour in 2024 T4 aluminum alloy under rotating bending loading. In the short cracks region, cracks grow initially at a fast rate but deceleration occurs quickly and, depending on the stress level, they either arrest or are temporarily halted at a critical crack length.[6]. In this work, the buckling behavior for edge cracked plates under compression loading is studied considering the influence of the crack parameters (i.e. size, location and orientation), plate aspect ratio and plate boundary conditions.

The aim of this work to build up a model described the stress behavior and crack growth velocity at crack tip in thin plate under cyclic impact loads using **FORTRAN** program. **ANSYS11-APDL** package was employed to build up the model and analysis the stresses also calculated the velocity of crack propagation.

Analytical Solution

This study will concentrate on analyzing deflection and stresses using the analytical method. It was used a flat thin plate with different crack lengths at the center of the plate. The force acting on the plate is cycling impact loading which is function of two times, impact loading time and the cyclic loading time. Main equations uses from analytical solution:

Impact Load Description [7]:

$$F(t_i) = n^{2/5} \left[\frac{5V^2}{4M} \right]^{3/5} \left[\sin \frac{\pi t_i V}{2.94\delta_1} \right]^{3/2} \dots 1$$

$$n = \frac{1}{3\pi(K_1 + K_2)} \dots 2$$

$$M = \frac{1}{m_1} + \frac{1}{m_2}$$
...3

Total Deflection of Plate [8]:

$$W = \sum_{m}^{\infty} \left[\frac{F(t_i)A^4}{\pi^4 m^4 D} + C_2 \cosh \alpha_m y + \alpha_m y (C_3 \sinh \alpha_m y) \right] * \sin \alpha_m x \qquad \dots 4$$

Stress Intensity Factor [9]:

$$K_{\rm I}^{total} = K_{\rm I}^{impact} + K_{\rm I}^{cyclic} + K_{\rm I}^{plastic} \qquad \dots 5$$

$$K_{I}^{impact} = \frac{C_{f}\sigma_{impact}\sqrt{\pi.c}}{\Phi} \qquad \dots 6$$

$$K_{\rm I}^{cyclic} = \frac{c_f \sigma_a \sin w t_c \sqrt{\pi.c}}{\Phi} \qquad \dots 7$$

$$k_{I}^{\text{plastic}} = 1.12 \frac{\sigma}{\phi} \sqrt{\pi (a + r_p)} * \left(\sin^2 \varphi + \frac{a^2}{c^2} \cos^2 \varphi \right)^{1/4} \dots 8$$

And
$$K_{\rm II}^{impact} = \frac{C_f \tau_{impact} \sqrt{\pi.c}}{\Phi}$$
9

Where

$$\sigma_{\text{impact}} = \sigma_1 \cos^2 \theta_p + \sigma_2 \sin \theta_p \cos \theta_p \qquad \dots 10$$

$$\tau_{\text{impact}} = \sigma_2 \cos^2 \theta_p - \sigma_1 \sin \theta_p \cos \theta_p \qquad \dots 11$$

Crack Tip Stresses [9]:

$$\sigma_{\chi}(t) = \frac{\kappa_{\rm I}^{total}}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) - \frac{\kappa_{\rm II}^{impact}}{\sqrt{2\pi r}} \sin\frac{\theta}{2} \left(2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right) \qquad \dots 12$$

$$\sigma_{y}(t) = \frac{\kappa_{I}^{total}}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) + \frac{\kappa_{II}^{impact}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \qquad \dots 13$$

Crack Growth Velocity [9]:

$$\dot{a} = 0.38 \sqrt{\frac{E}{\rho}} \left(1 - \frac{a}{a_c} \right) \left(\frac{\sigma_{normal}}{\dot{\sigma}_{normal}} \right)$$

Numerical Analysis

Numerical analysis of structures subjected to various kinds of actions is an important issue for structural safety. A numerical method can be used to obtain an approximate solution; approximate numerical procedures have become very accurate and reliable for the solution with the advent of high speed digital computers [10]. Solving fracture mechanics problems involves performing a linear elastic or elastic-plastic state analysis and then using specialized post processing commands or macros to calculate desired fracture parameters. The following topics describe the two main aspects of procedure:

- 1. Selecting of element and meshing.
- 2. Modeling the crack region.
- 3. Calculating fracture parameters.

In this research the ANSYS software APDL is used for solving fracture problem. Selecting of element and meshing: The element Solid185 is used for 3D modeling of solid structures. The boundary conditions are clamped-clamped with simply supported at the other edges plate. The element solid185 is used for 3D modeling of solid structures. Solid element 185 **Fig.1** with Mesh200 used in this work **Fig.2**.

Modeling the crack region

Stress and deformation fields around the crack tip generally have high gradients. The precise nature of these fields depends on the material, geometry and other factors. To capture the rapidly varying stress and deformation fields, use a refined mesh in the region around the crack tip. For linear elastic problem the displacement near the crack tip (or crack front) vary as \sqrt{r} , where (r) is the distance from the crack tip. The stresses and strains are singular at the crack tip, varying as $1/\sqrt{r}$. To produce this singularity in stresses and strains, the crack tip mesh should have certain characteristics, the crack faces should be coincident and the elements around the crack tip (or crack front) should be quadratic, with the mid side nodes placed at the quarter points. Such elements are called singular elements. Fig.3

Loads and Boundary Conditions

Cycling impact loading is applied at the cracked thin flat plate; the boundary conditions are clamped-clamped with simply supported at the other edges plate. Three different plate dimensions were used in this work, takes (150×150) mm. Plate's thickness (6mm). The crack dimensions (length: 7mm, 10mm, width: 1.5mm and depth: 2mm) **Fig.4**

Results And Discussion

Analytical Solution

The effect of the crack lengths on the stresses

The analytical solution of the stress depends on the value of deflection, **Fig. (5 to 8)** were illustrated the effect of number of cycles on the stresses with crack lengths (7, 10) mm, for aspect ratios (1). The extension in length (deflection) is direct proportional with

... 14

applied stresses that means the increasing in crack length which leads to increase the values in stress. The instantaneous length of crack for aluminum 6061 greater than aluminum 2024. Because the aluminum 6061 is more ductile than the aluminum 2024 (young modulus for aluminum 2024 greater than aluminum 6061). Increasing in the cumulative number of cycles leads to increase in the stresses with nonlinear behavior so that the increasing in crack length also will be nonlinear .The yield region will be not appoint, so that there is some limiting values that strain hardening will be effect on the results therefor the rate of increasing curve slope will be low till 600 cycles then after that, the rate of curve slope will be high increasing. Materials especially metals tend to exhibit a yield stress, above which they deform plastically. This means that there is always a region around the tip of a crack in a metal, where plastic deformation occurs, and this plastic region is known as the crack tip plastic zone. The plastic zone size vary with the number of cycles and it increases with increase the number of cycles, because the increase in the number of cycles means increasing in the applied stresses, that is leading to increase in the plastic zone size. Maximum percentage error of stress (20.8%) between analytical solution and numerical analysis.

The effect of the crack growth velocity on the stresses

The crack growth velocity depends on the applied stress according to equation (5) for aluminum 6061 and aluminum 2024. While the increase in the number of cycles means increasing in the applied stresses, so that crack growth velocity increases with increase the cumulative number of cycles because the crack growth velocity depends on the applied stress. Also the crack growth velocity is directed proportional with crack length, that means increasing in the crack length is leading to increase the crack growth velocity, because the extension in crack length (deflection) causes increasing in the stresses will increase in the crack growth velocity as shown **Fig. (9 &10)**. From these figures noticed the rate of crack propagation will be increased to 400 cycles then after 400 cycles the rate of crack growth velocity will be decreased.

The crack growth velocity different from material to another where the crack growth velocity in the ductile materials like aluminum 6061 is greater than that for the hard material like aluminum 2024, because the crack growth velocity depends on the modulus of elasticity and the density of materials. Plastic zone size inversely proportional with yield stress therefor the plastic zone size for ductile materials like (aluminum 6061) is greater than that for the hard materials like (aluminum 2024).

ANSYS Program Analysis

The effect of the crack lengths on the stresses

The extension in length (deflection) is direct proportional with applied stresses that means the increasing in crack length which leads to increase the values in stress as shown in **Fig. (11 to 14)**. The instantaneous length of crack for aluminum 6061 greater than aluminum 2024. Because the aluminum 6061 is more ductile than the aluminum 2024 (young modulus for aluminum 2024 greater than aluminum 6061). Increasing in the cumulative number of cycles leads to increase in the stresses with nonlinear behavior so that the increasing in crack length also will be nonlinear. The yield region will be not

appoint, so that there is some limiting values that strain hardening will be effect on the results therefor the rate of increasing curve slope will be low till nearly 600 cycles then after that, the rate of curve slope will be high increasing. Materials especially metals tend to exhibit a yield stress, above which they deform plastically. This means that there is always a region around the tip of a crack in a metal, where plastic deformation occurs, and this plastic region is known as the crack tip plastic zone. The plastic zone size vary with the number of cycles and it increases with increase the number of cycles, because the increase in the number of cycles means increasing in the applied stresses that is leading to increase in the plastic zone size.

The effect of the crack growth velocity on the stresses

The crack growth velocity depends on the applied stress which increases with increase the cumulative number of cycles that means the crack growth velocity increases with increasing in the number of cycles. Maximum error percentage of crack growth velocity was (9.1%) for aluminum 6061 and (17.9%) for aluminum 2024 between analytical solution and numerical analysis. As shown in the **Fig. (15&16)** noticed after 500 cycles nearly for aluminum 2024, the crack propagation values were started more increasing and faster propagate before 500 cycles, while for aluminum 6061 after 525 cycles nearly the crack propagation values were started more increasing before 525 cycles, So that the plastic zone consists at aluminum 6061 before aluminum 2024 and the crack propagation at aluminum 6061 faster from aluminum 2024.

The main reason of this case, the effect of ductility of material on the stress values under cycling impact loading will become more clearly appeared in aluminum 6061rather than in aluminum 2024 and noticed from ANSYS program results the crack growth velocity values were increased with increasing the crack length. The crack propagation don't occur under impact load only because the plastic zone due to low velocity impact with applied cyclic loading which cause crack propagation.

Conclusions

The purpose of this paper was to study and analysis the stresses at crack tip in flat plate under cycling impact loading and calculate the velocity of crack propagation. The conclusions were: Plastic zone has significant effect on crack growth velocity under cycling impact loading for two materials used in this work. There are some specific values of stresses at which strain hardening effect on the results. Therefor the rate of curve slope will be lower until 600 cycles. After that, the rate of curve slope will back to increase. The number of cycles has significant effect on crack growth velocity specially at 400 cycles were the rate of increasing of crack growth velocity will be very high which is reflect the effect of plastic zone. The effect of ductility of material on the stress under cycling impact loading with cumulative number of cycles will become more pronounced in aluminum 6061rather than in aluminum 2024.

NOMENCLUTURE

А	Plate width
à	Velocity of crack propagation
a	Half crack length
a _c	Half-length of crack after the propagation
С	Crack depth
C_f	Correction factor
D	Flexural rigidity of plate
r _p	Radius of plastic zone
W	The total deflection in the plate
F(t _i)	Impact load
Lc	Crack length
n	parameter depended on material properties for impactor and plate
V	Linear velocity of impactor
Ar	Aspect ratio
K _I ^{mpact}	Stress intensity factor due to impact load mode I
K _I ^{cyche}	Stress intensity factor due to cyclic load
K _I plastic	Plastic stress intensity factor
K _{II} impact	Stress intensity factor due to impact load mode Π
σ_x , σ_y	Stresses at crack- tip region
$\sigma_{1,\sigma_{2}}$	Maximum and minimum stresses
$ heta_p$	Principal angle

 φ Angle of crack front from surface of the structure

Table(1). The specifications of aluminum 2024.[11]

Young modulus (E)Gpa	YieldTensile Strength (σ_y) Mpa	Ultimate tensile strength ($\sigma_{ult.}$) Mpa	Poissons ratio(ν)	Density (p)Kg/m ³
73	325	470	0.33	2780

Table(2	The specifications of aluminum (6061.[11]
	The specifications of analimitation of	

Young modulus	Yield Tensile	Ultimate tensile strength ($\sigma_{ult.}$) Mpa	Poisson's	Density
(E)Gpa	Strengt (σ_y) Mpa		ratio(ν)	(p)Kg/m ³
69	275	310	0.33	2700



ANSYS Program Flowchart



Figure (3) 3D model with solid185.



Figure (4) Load and B.C.



Figure (5) σ_x Analytical with number of cycles (Ar=1&Lc=7mm).



Figure (6) σ_y Analytical with number of cycles (Ar=1&Lc=7mm).



Figure (7) σ_x Analytical with number of cycles (Ar=1&Lc=10mm).



Figure (8) σ_y Analytical with number of cycles (Ar=1&Lc=10mm).



Figure(9) Crack velocity analytical with number of cycles (Ar=1&Lc=7mm).



Figure (10) Crack velocity analytical with number of cycles (Ar=1&Lc10mm).



Figure (11) σ_x Numerical with number of cycles (Ar=1&Lc=7mm).



Figure($12)\sigma_y$ Numerical with number of cycles (Ar=1&Lc=7mm).



Figure (13) σ_x Numerical with number of cycles (Ar=1&Lc=10mm).



Figure $(14)\sigma_y$ Numerical with number of cycles (Ar=1&Lc=10mm).



Figure (15) Crack velocity numerical with number of cycles (Ar=1&Lc=7mm).



Figure(16)Crack velocity numerical with number of cycles (Ar=1&Lc=10mm).

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