Three-Dimensional Dynamic Analysis of Pile Foundations by Using Finite Element Method

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Abstract
In recent years, there is a dramatic progress in the development of theories for dynamic analysis of piles. The finite element method is a numerical approximation procedure to analyze physical problems by transforming them into mathematical models. This needs to be done numerically because it is either too complicated or impossible to solve problems by classical analytical methods.

In this study, a three- dimensional analysis by using finite element method will be adopted to predict the displacement response of pile foundation due to dynamic loads. A case study is carried out to investigate the effect of particles size of soil, length of pile, spacing between piles and pile cap size on the dynamic response of pile foundations.

The results indicate that, the maximum amplitude of displacement of pile foundation under dynamic loads can be occurred in loose sand in comparison with the other types of sand. In addition, the increase in pile length will lead to decrease in the amplitude of displacement due to increases in the mass of the foundation.

Keywords: Dynamic analysis, finite element method, pile foundations.

التحليل الحركي الثلاثي الأبعاد لأسس الركائز بإستخدام طريقةِ العناصرِ المحدودةِ

الخلاصة

في السَنَوات الأخيرة، هناك تقدّم كبير في تطوير النظرياتِ للتحليل الحركي للركائز. إنّ طريقةَ العناصر المحدودةِ إجراء تقريبٍ عددي لتَحليل المشاكلِ الفيزيائية بتَحويلها إلى النماذجِ الرياضية. هذا يعني مَنْ الضّرُوري أَنْ يكون الحَل بشكلٍ عددي لأنه أمّا أن يكون معقّد أو مستحيل جداً لحَلّ المشاكلِ بالطرقِ التحليليةِ الكلاسيكيةِ.

في هذه الدراسةِ، التحليلِ الثلاثي الابعاد باستخدامُ طريقةَ العناصرِ المحدودةِ سَيتبنّى لتَوُقُّع رَدِّ الإزاحةَ لأسس الركائز بسبب أَلأحمالِ الديناميكيةِ. حيث سيتم دراسة حالة للتَحرّي عن تأثيرِ حجم حبيباتِ التربةِ، طول الركيزة، المسافة بين الركائز، حجم قبعة الركائزعلى الرَدِّ الديناميكي لأسس الركائز.

تُشيرُ النَتائِجُ بأنّ، السعة القصوى لإزاحةِ أسس الركائز تحت الأحمالِ الديناميةِ يُمْكِنُ أَنْ تُحدث في الرملِ الرخو بالمقارنة بالأنواع الأخرى للرملِ. بالإضافة، الزيادة في طولِ الركيزة سَتُؤدّي إلى النقصانِ في سعة الإزاحةِ بسبب الزيادة في كتلةِ الركيزة.

INTRODUCTION

In many cases when the soil is loose or soft under the foundation which is subjected to dynamic loads, or when it is necessary to alter the foundation frequency, the pile foundation is used. Bowles (1996) mentioned that, the piles provide a greater apparent of soil stiffness.

One of the methods to represent a pile embedded in soil assumes that the soil profile is a continuous elastic medium. The continuity of the soil is modeled in this approach, and any force at a point will be transferred to the surrounding area, its effect decreasing with distance. This concept was proposed by Mindlin method (Mindlin, 1936), where the soil is represented as an elastic solid with the soil stiffness based on the integrated form of the Mindlin equations for the situation where a beam is loaded on the soil surface. The approach is still used today even though the soil does not behave as a perfectly elastic medium.

Another method is Winkler beam model which is a simplest idealization of a pile embedded in soil. The soil surrounding the pile shaft is modeled as a bed of independent springs. This model simplifies the situation with the assumption that the displacement of one spring has no effect on the displacement of any other springs. The results of a semi-infinite beam on a Winkler subgrade loaded at its tip can be determined using closed form solutions (Scott, 1981).

The mentioned approaches have limitations, where the elastic continuum method does not accurately portray the non-linearity of the system. In addition, the Winkler model ignores the continuity of the soil. These limitations can be overcome with the utilization of the finite element method to represent a pile element embedded in a soil medium.

Formulations of the Finite Element Method

In the finite element method, a continuum is divided into a number of elements. Each element consists of a number of nodes, and each node has a number of degrees of freedom that correspond to discrete values of the unknowns in the boundary value problem to be solved. In the present case, the degrees of freedom correspond to the displacement components.

The basic of the finite element equations for elastic analyses can be written as (Zienkiewicz and Taylor, 2005):

The displacement field in three dimensions includes three components as follows:

$$u = \sum_{i=1}^{n} N_i u_i, \quad v = \sum_{i=1}^{n} N_i v_i \quad \text{and} \quad w = \sum_{i=1}^{n} N_i w_i \qquad \ldots (1)$$

where

N_i is shape function at a given node and

u_i, v_i and w_i are the nodal displacement.

In matrix form:

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_{(3*1)} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & \cdots \\ 0 & N_1 & 0 & 0 & N_2 & 0 & \cdots \\ 0 & 0 & N_1 & 0 & 0 & N_2 & \cdots \end{bmatrix}_{(3*3N)} \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \\ \vdots \\ \vdots \\ \vdots \end{Bmatrix} \qquad \ldots (2)$$

or$\{u\}= [N]\{d_i\}$
The strain vector can be derived as:
$$\{\varepsilon\} = [B]\{d_i\} \qquad\qquad \dots (3)$$
Where
[B] is nodal strain-displacement matrix.
The stiffness matrix [K] can be written as (Zienkiewicz and Tylor, 2005):

$$K = \int_v [B]^T D [B]\, dv \qquad\qquad \dots (4)$$
where:
 D= elastic coefficient matrix.

Equation of Motion of the Dynamic Analysis
    The general equation of motion for a system is given by Zienkiewicz and Taylor (2005):
$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F^a\}$$
where:
[M] = mass matrix,
[C] = damping matrix, $\dots (5)$
[K] = stiffness matrix,
$\{\ddot{u}\}$ = nodal acceleration vector,
$\{\dot{u}\}$ = nodal velocity vector,
$\{u\}$ = nodal displacement vector, and
$\{F^a\}$= applied load vector.

Description of the Problem
    The objective is to investigate the effect of frequency, length of pile, spacing between piles, and pile cap size on the dynamic response of pile foundation.
    The soil profile consists of a sand stratum at a depth of 12.0 m from the ground surface. The problem represents a homogeneous layer of a natural deposit over bedrock. The properties of the soil are considered to be uniform throughout the depth of the layer which is a common assumption in soil dynamics. The soil is modeled as a linear elastic behavior, and the material properties of the soil are shown in Table (1).

Table (1): Properties of the soil (after Das, 2009)

Material Properties	Loose sand	Medium sand	Dense sand
Modulus of elasticity, E (kN/m^2)	20000	25000	35000
Poisson's ratio, υ	0.3	0.27	0.25
Dry density, ρ_s (kN/m^3)	14	16	18

    A group of piles supporting rigid foundation is subjected to harmonic load which represents dynamic force that may be applied from a machine. The pile group consists of four concrete piles of length 5.725 m with cross-section of (0.25$\times$ 0.25) m.
    The piles are poured at a depth of 5.0 m below the ground surface with a free length of 0.725 m. A concrete cap of 0.3 m thickness is poured on top of the piles with its bottom face 0.5 m above the ground surface. The behavior of concrete is

considered linearly elastic (Liu and Novak, 1991).The material properties of the concrete are shown in Table (2).The layout of the deep foundation is shown in Figure (1).

The pile group interaction factor which has been observed to have significant effect on the dynamic response on the system especially when the pile spacing is between 2.5D to 3D, where D is the overall diameter of the pile. To ignore the effect of group interaction factor, the centre to center distance between the piles is at least more than 5D.

Table (2): Material properties of the concrete foundation

Parameters	Value
Modulus of elasticity, E (kN/m^2)	31261×10^3
Poisson's ratio, υ	0.20
Bulk density, ρ_c (kN/m^3)	24
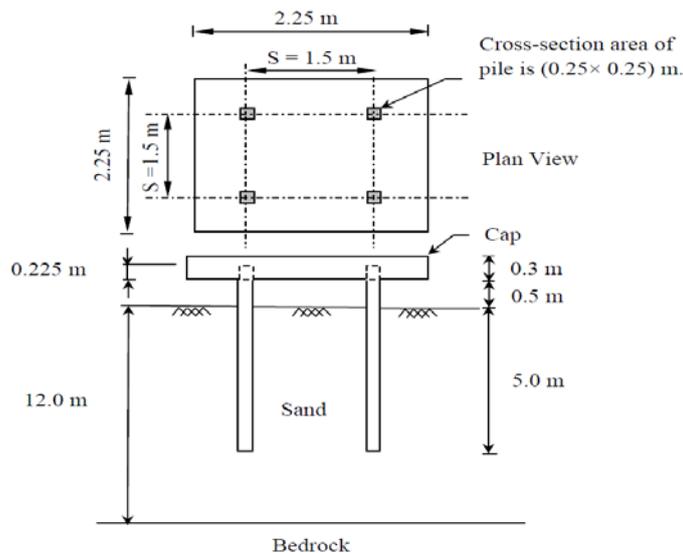
Figure (1): Layout of the deep foundation

Three-Dimensional Dynamic Analysis of Pile Foundations

For some problems, it is necessary to perform three-dimensional analysis, the finite element method is found to be an effective numerical technique to perform this type of analysis (Phan et al., 1979).

Most pile foundations consist of a group of piles rather than a single pile. Thus, an important component of the analysis of pile supported structures is the ability to perform an accurate and efficient analysis of pile groups. Just as for single pile, various numerical and analytical methods have been developed for the analysis of pile groups. Currently, the most accurate method of pile group analysis may be the 3D finite element analysis method.

To perform a numerical model, a brick element of eight nodes hexahedral linear isoparametric is used for the finite element idealization. The element has the eight nodes at the corners where each node has three degrees of freedom for the displacement.

The elementary boundaries with conditions of zero-displacement are used at the boundary. The boundary conditions are applied so that the bottom of the soil is fixed in displacement while the top surface of the soil is set to be free in displacement. The constraint on displacement in X and Z directions is applied on the nodes at the boundary in Y-Z and X-Y planes, respectively.

The three- dimensional finite element model and a cross-section in the model are shown in Figures (2) and (3), respectively.
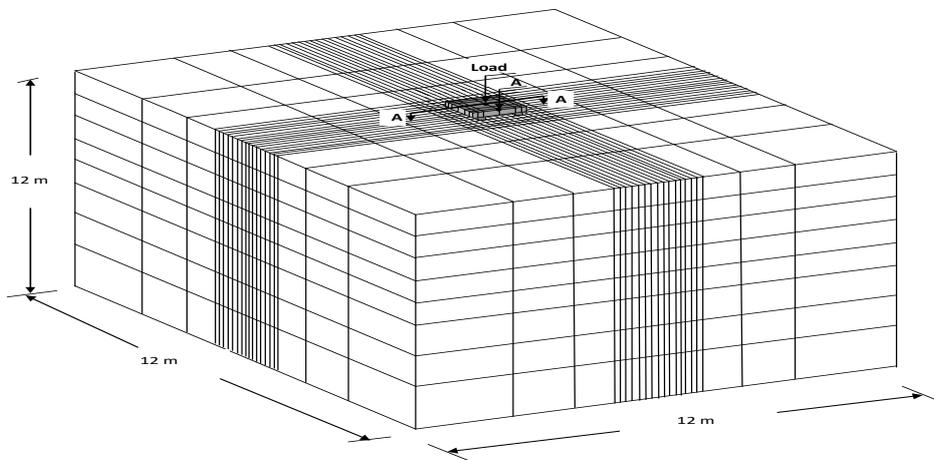
The dynamic load is applied at the surface of the foundation for a specific   node at

The center of the cap. The foundation is subjected to a steady state load of sinusoidal function of the form $F = F_o \sin(\omega t)$ with amplitude of force 9.941 kN and circular frequency, ω of 20.94 rad/sec.

To cure the artificial oscillation, the numerical damping is introduced into the analysis which is achieved by using $\gamma = 0.6$ and $\beta = 0.3025$ in the Newmark algorithm. The duration of the applied dynamic load is 10 sec, and the time step, $\Delta t = 0.19882$ sec. with a total of 50 steps are performed.

In this study, the finite element method is used to analyze problems in time domain employing the computer program (OpenSees). This program is originally produced by University of California. The program is not a code in the traditional sense, but rather it is a set of modules for creating a wide range of models and simulation procedures for structural and geotechnical systems. The program is a finite element application for which users specify a model and conduct an analysis in OpenSees using a Tool command language (Tcl).

Effect of particles size of soil on the dynamic response

The dynamic load is applied at the surface of the foundation at the center. Three different types of particle size of sands (loose, medium and dense) are considered. The parameters of each type of soil as mentioned in Table (1). The dynamic response of the foundation is shown in Figure (4).From this figure it can be seen that the maximum amplitude of displacement of pile foundation under dynamic loads occurred in the case of loose sand. This behavior can be attribute to the reduction in the stiffness of loose sand represented by the reduction in modulus of elasticity.

Max displacement in loose sand = 0.0771 mm
Max displacement in medium sand = 0.0544 mm
Max displacement in dense sand = 0.034 mm
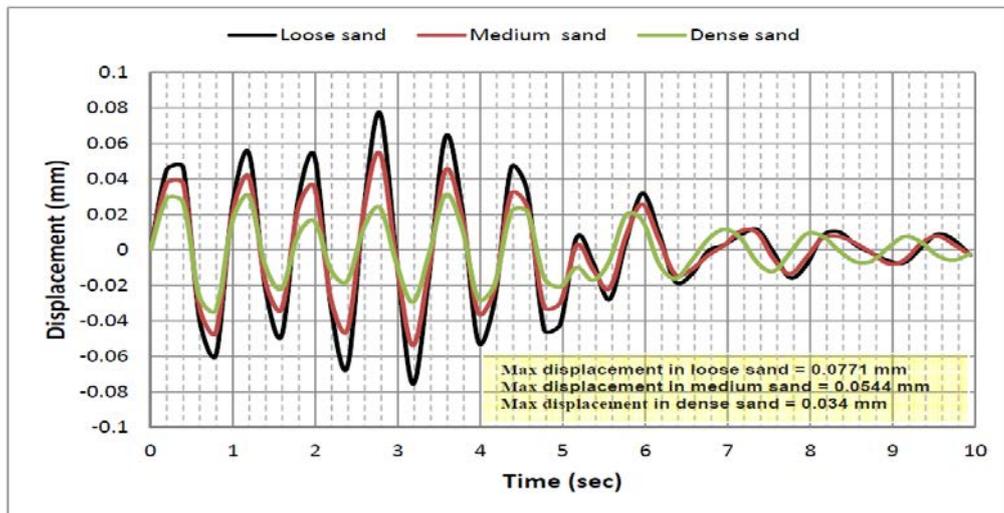
Figure (4): Predicted vertical displacement at the center of foundation (Point A)

Effect of pile length on the dynamic response

In this section, two different lengths of pile are considered (7.0 m and 9.0 m). The first model consists of piles with length of 7.725 m; the embedded length of the pile is 7.0 m. The second model consists of piles with length of 9.725 m, and embedded length of the pile is 9.0m. The pile cap for all cases in this application is constant (0.3). The results of the displacement response are shown in Figures (5) and (6).

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The reduction factor of the displacement of deep foundation due to change in
length of pile is shown in Table (3).From these results, it can be seen that, when the
pile length increase this will lead to decrease in the amplitude of displacement due to
the increase in the mass of the foundation.
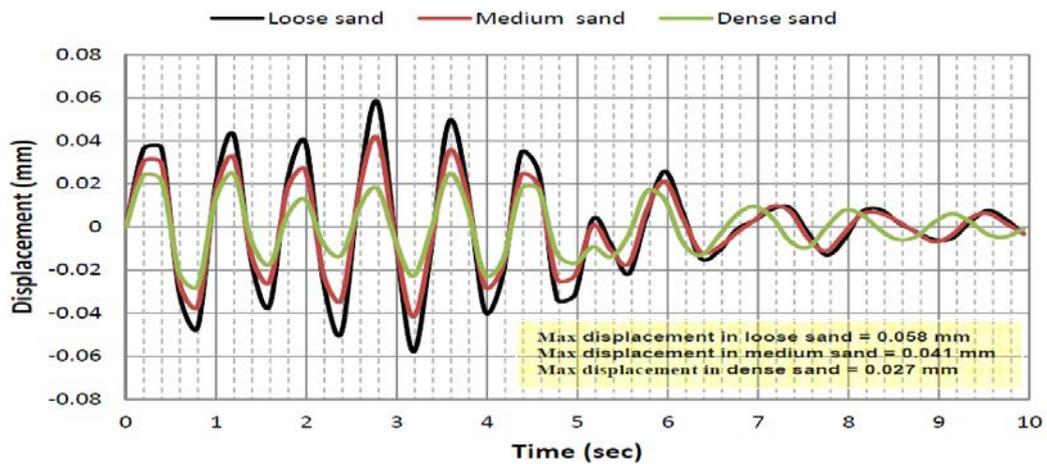
**Figure (5): Predicted vertical displacement at the center (Point A) with piles
length of 7.725 m.**
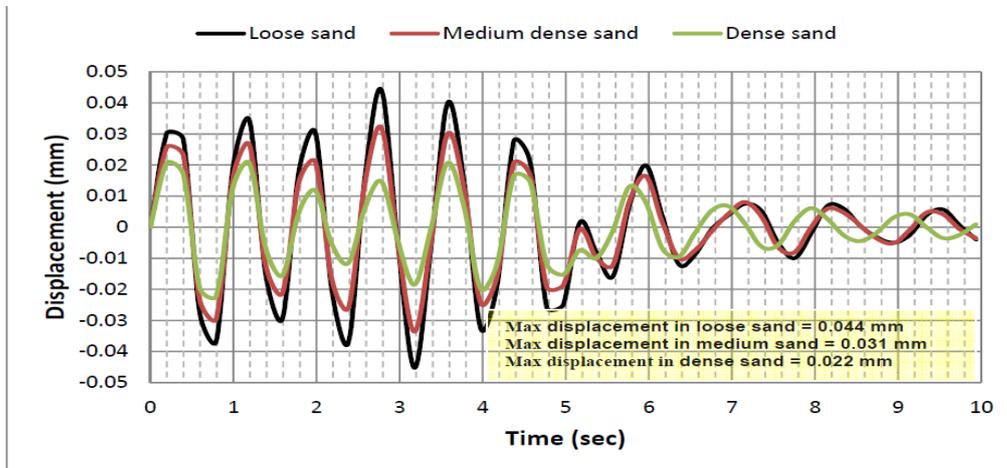
**Figure (6): Predicted vertical displacement at the center (Point A)
with pile length of 9.725 m**

Table (3): Reduction factor of the displacement response of deep foundation due to change in length of pile.

Type of Soil	Amplitude of Displacement (mm)			Reduction Factor [1]	
	Pile length 5.725 m	Pile length 7.725 m	Pile length 9.725 m	For pile with length of 7.725 m	For pile with length of 9.725 m
Loose sand	0.077	0.058	0.044	0.246	0.428
Medium sand	0.054	0.041	0.031	0.240	0.425
Dense sand	0.034	0.027	0.022	0.205	0.352

(1)The data base of the reduction factor is the value of displacement for piles with length of 5.725m.

Effect of spacing between piles on the dynamic response

When a group of piles is pushed vertically, it is possible that the vertical stiffness of the system will be less than the sum of the stiffness of individual soil-pile systems. This is due to the pile-soil interaction which occurs when the spacing between piles is small. For pile design, the group effects vanish for in-line piles at spacing equal to five diameters of the pile or more (Chowdhary and Dasgupta, 2009).

In this case, the dimensions of the pile cap are (2.75×2.75) m with thickness of 0.3 m while two values of spacing are considered which is 1.5 m and 2.0m. The cross-section area of the pile is chosen to be (0.25× 0.25) m, and the length of piles is kept constant and equal to 5.725 m. The material properties of the soil and concrete, and the dynamic force which is applied at the surface of the foundation as explained in the description of the problem. Figures (7) and (8) show the displacement response of pile foundation with spacing between piles 1.5 m and 2.0 m, respectively. In addition, Table (4) illustrates the reduction factor due to change in the spacing between piles.
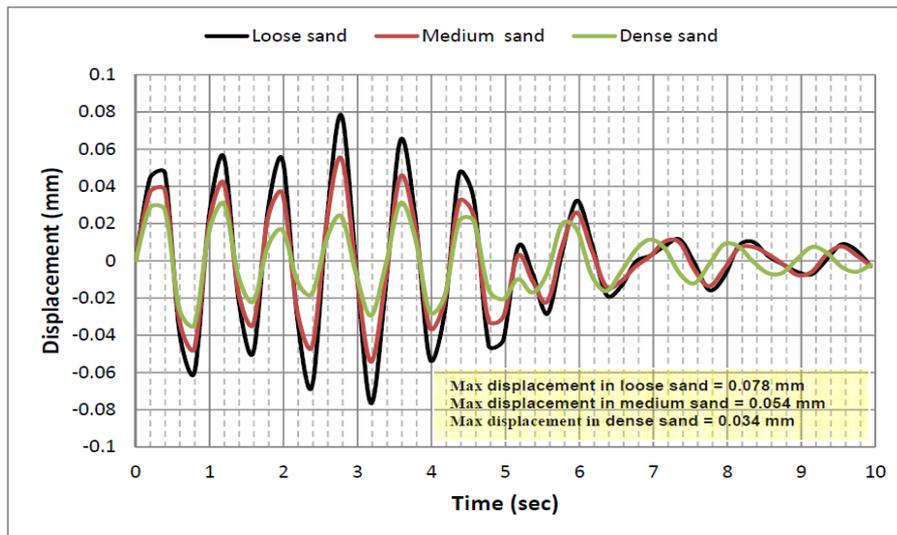
Figure (7): Predicted vertical displacement at the center (Point A) with length of pile cap 2.75 m and spacing between piles 1.5 m

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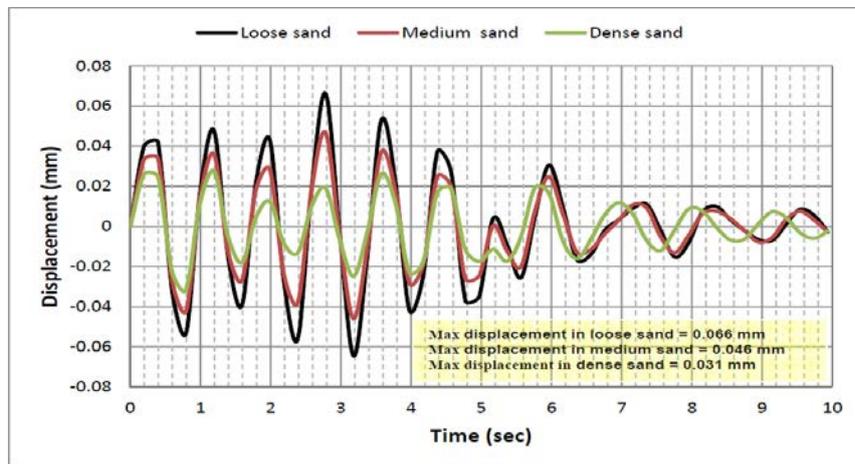
Figure (8): Predicted vertical displacement at the center (Point A) with length of pile cap 2.75 m and spacing between piles 2.0 m

Table (4): Reduction factor of the displacement response of the deep foundation due to change in spacing between piles.

Type of Soil	Amplitude of Displacement (mm)		Reduction Factor
	Space between piles 1.5 m	Space between piles 2 m	
Loose sand	0.078	0.066	0.153
Medium sand	0.054	0.046	0.148
Dense sand	0.034	0.031	0.088

Effect of pile cap size on the dynamic response

Two values of pile cap size are considered. The top view of pile cap with different length of cap is shown in Figure (9). The soil and concrete properties and the dynamic load which is applied at the surface of the foundation have been explained in the description of the problem.
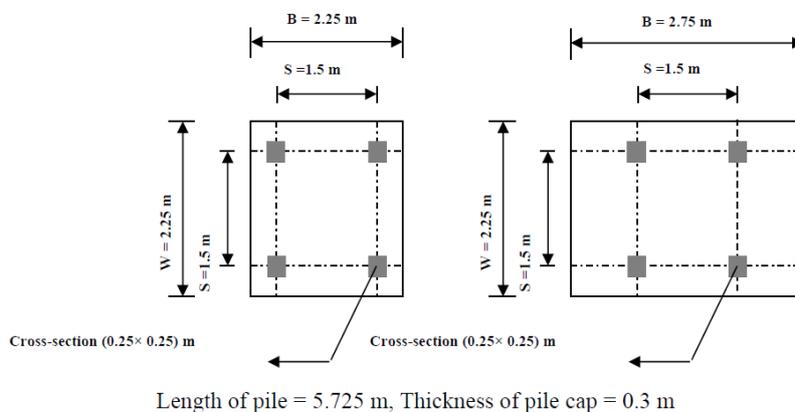
Figure (9): Top view of pile cap with different size of cap.

The result of the displacement response with the pile cap of length 2.25 m as shown in Figure (4), and the results of the displacement response with pile cap of length 2.75 m are shown in Figure (10).
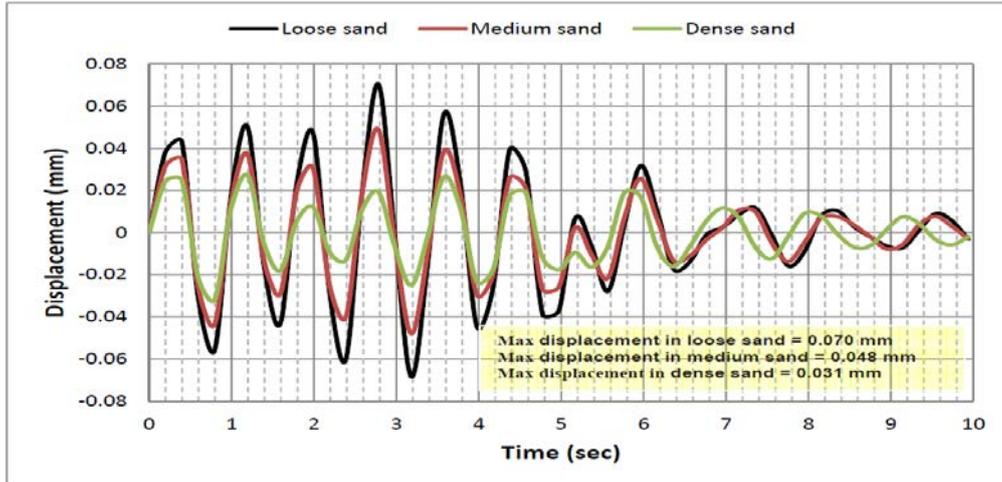
Figure (10): Predicted vertical displacement at the center (Point A) with cap of size (2.75 m × 2.25 m)

The reduction factor of the displacement response of the deep foundation due to change in size of pile cap is shown in Table (5).From these results, it can be stated that the displacement will be decreased with the increasing in pile cap size due to the excess of weight.

Table (5): Reduction factor of the displacement response of the deep foundation due to change in size of pile cap

Type of Soil	Amplitude of Displacement (mm)		Reduction Factor
	Length of pile cap 2.25 m	Length of pile cap 2.75 m	
Loose sand	0.077	0.070	0.091
Medium sand	0.054	0.048	0.111
Dense sand	0.034	0.031	0.088

Conclusions
(1) The maximum amplitude of displacement of pile foundation under dynamic loads can occur in soil of loose sand. This behavior can be attributed to the reduction in the stiffness of loose sand which is represented by the reduction in modulus of elasticity.
(1)   When the pile length increase this will lead to a decrease in the amplitude of displacement due to the increase in the mass of the foundation.
(2)   The effect of increase in spacing between piles on reduction of displacement of pile foundation will be significant. When the spacing between piles increase, the displacement of the pile foundation will be decreased.

(3)   The displacement of the pile foundation will be decreased with the increasing in pile cap size due to the excess of weight which lead to decrease in frequency of the dynamic load.

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