O-C TOPOLOGY AND SIERPINSKI SPACE

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Abstract
In this paper we choose a different approach and propose a generalization of Sierpinski space by defining a topology analogous to Sierpinski topology with the general collection τ which is form a topology on a non empty set X , and the definition of K. Khan [5] will be only special case , so the (O-C topology) is introduced . We investigate some of the general topological characterization of it , and this has a very interesting applications especially the Sierpinski space which is result from the finite set X which containing only two points .

keywords : Sierpinski space , generalization Sierpinski space ,connectedness, separation axioms , discrete set

Introduction
Many authors took up the generalizing of Sierpinski space, , in 1994 [2] F.J. Craveiro and D'Azevedo Breda work to a generalization , they   introduced the concept of locally Sierpinski space and called a topological space X locally Sierpinski space, if every point x $\in$ X has a neighbourhood homeomorphic to the Sierpinski space .   In 2012 , K. Khan [5] deal with the generalization of Sierpinski space by defining nested open sets on any arbitrary non empty set, but in other viewpoint the importance of sierpinski space came from the only two points which has false or true (0 , 1) .
It is known from the properties of the T_1 spaces that the finite T_1 space is discrete ,so the spaces that we are study is T_0 but not T_1, Sierpinski space is a finite topological space with two points which is a smallest example of a topological space which is neither discrete nor indiscrete , and ascribe to the polish scientist Waclaw Sierpinski . Willard refer to Sierpinski space [8] , later Sierpinski space has a very famous applications , Escardo reformulate classical topological concepts as continuity notions with aid of the Sierpinski space which has one open point true and one closed point false and plays the role of a space of results of observations or semi decisions, See synthetic topology [6].
We focus our attentions upon the whole power set P(X) for any non empty set X   but this not mean the discrete topology . In a topological space X it's may happened that there are some subsets which are not open nor closed such as a half open interval in the usual topology , but in a discrete space every subset is open and closed , so we consider a topological space which has the property that every non empty proper subset is open or closed but not both (according to the exclusive usage of OR) except the sets X and ϕ , we adopt this since it will be the general case to an application in topological computational theory and a synthetic topology, in another point of view every subset from the set X has the expression   0 or   1 (exclusive –OR)

We introduce the following definition

Definition 1-1:   let(X , τ) be a topological space where, X a non singleton set which satisfying that for any non empty proper subset u ⊂ X is either open or closed, but not both (exclusive using of OR)   , we refer to this topology by , O-C topology .

Clearly from the above definition that the O-C topology is non discrete nor indiscrete topology, in fact if we ignored   the repeating of the sets X and φ we partition* the discrete topology with two parts the first containing the open sets refer it by 1   and second the closed sets refer it by 0 and so we can represent every subset by the symbol 0 or 1 . The definition 1-1 , differ from the definition of the door space by the meaning of OR , recall that a topological space is called a door space if every subset of X is either open or closed (or both) [4] .

Example1-2:   The Sierpinski space S , where S = { 0 ,1 } and τ = { φ . X . {1} }   is O-C topology . It is clear that the Sierpinski space is T_0 but not T_1 space then it is not (T_2 , T_3 , T_4)

Note1-3:   there exist an open singletons set in any O-C topology since if X consists of two points then the O-C topology must be Sierpinski space which has an open singletons set otherwise it is discrete topology, and this is true for any other set

Example 1-4:   Sierpinski space is not regular, but it is vacuous normal unfortunately the O-C topology is not regular nor normal as we explaining let   X = { a , b , c } and let   τ = {X , φ { a , c } , { b . c } , { c } } then X is O-C space but it is not normal since the disjoint closed sets { b } and {a } have no any disjoint open neighborhoods .

Proposition1-5:   If X is   O-C topology then X is T_0 space

Proof: for any two distinct points x , y in X then if {x} is open so is done , or $\{x\}^c$ is open which containing y , so X is T_0 space .

Example 1-6 : let X be a discrete topology then X is a door space and $T_{1/2}$ space (a topological space X is called $T_{1/2}$ if every singleton { x } is either open or closed set [3]) , X is not O-C space , on the other hand the O-C space is connected but the door space X   disconnected when X is a non singleton set .

Proposition 1-7 : the O-C topology is connected since the only open and closed sets are X and φ, by definition .

Proposition 1-8: let   f be a one to one open (or closed) function from the topological space (X , τ) onto f(X), where X an O-C space , then f(X) is O-C topology

Proof: for any proper subset E from f(X) then $f^{-1}(E)$ is either open or closed but f is open and closed (since f is a bijection open function) so $f\,f^{-1}(E) = E$ is either open or closed set and then f(X) is O-C topology .

proposition 1-9: Every non singleton subspace Y of O-C space X, is an O-C space

proof : for any non empty proper subset E of Y, then E is either open in X and so $E = E \cap Y$ which is open in Y or E is closed in X then $E = E \cap Y$ is closed in Y .
note that Y is either open or closed in X ,so if Y is open then $\tau_Y \leq \tau_X$

the following proposition is well known ,
Proposition 1-10

let u be any subset of a topological space X and let the mapping

χ_u: X→S (S, Sierpinski space) defined by:

$$\chi_u(x) =: \begin{cases} 1 & \text{if } x \in u \\ 0 & \text{if } x \notin u \end{cases}$$

Then u is open iff χ_u is continuous

So if the topological space is O-C space we can prove the following corollary ,

Corollary 1-11: let u be any proper subset of an O-C topological space X and let the mapping

χ_u: X→S defined by :

$$\chi_u(x) =: \begin{cases} 1 & \text{if } x \in u \\ 0 & \text{if } x \notin u \end{cases}$$

then u is closed iff χ_u is discontinuous .

Proof : let u be any proper closed subset , so we assert that χ_u is discontinuous, since $\chi_u^{-1}\{1\} = u$.
Conversely , if χ_u is discontinuous this means that $\exists$, an open set which has a

pre image not open but the only open set is {1} so $\chi_u^{-1}\{1\} = u$ not open, but X is O-C space then u is closed.

Definition 2-1: A subset D of a topological space (X, τ) is aid to be discrete if every point $d \in D$ is of the form $\{d\} = U \cap D$, for some open set U.[1]

Theorem 2-2: let: (X , τ) be a topological space such that $X = \{ x_1 ,x_2 ,x_3, …,x_n \}$ is a non singleton finite set, **,** then the topology is O-C space if it is containing only a discrete subset D such that $D = \{ x_1 ,x_2 ,x_3, …x_{n-1} \}$.

Proof: let D be an open discrete subset which contained in the space X , then the topology containing the open singleton subsets $\{x_1\}$, $\{x_2\}$, $\{x_3\}$, …$\{x_{n-1}\}$ and all its

unions by(2-1) and topology conditions , where $x_1, x_2, x_3 \ldots, x_{n-1} \in D$, so the space is not discrete since the set D^c is closed which is not open , and clearly it is not indiscrete topology, on the other hand any proper subset from X is either open or closed (see note 2-3, 1) ,and hence the topology is O-C space .

Note 2-3 : (1) it is clear that the above topology containing exactly ($2^n / 2$) +1 open subsets with the same number to the closed sets, and this is true for any other
O-C topology since the number of the open and closed sets must be identical, and we speaking upon every subset from the power set must be either open (exclusive) OR closed . (2) there is an O-C space which is not containing a discrete subset see example (1-4).

Definition 2-4: a point of a topological space is called focal if its only neighbourhood is the entire space [7]

Note   2-5 :   the closed point of sierpinski space is a focal point , the point x_n in the above theorem (2-2) is focal

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