Constructing of Multi-patches B-spline Surfaces

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ABSTRACT

Surface modeling utilizing B-spline technique is one of the most important tool in computer aided geometric design (CAGD). The aim of this paper is to build and develop an algorithm to design and implement the multi-patches of B-spline free-form surfaces. The technique has an effective contribution in technology domains and in aircrafts, ships, and cars industry, moreover for its wide utilization in making the molds. Blending the patches together in order to build the entire surface have been implemented to represent the different types of B-spline surfaces. This work includes the synthesis of these patches in a way that allows the participation of these control points for the merged patches, and the confluence of these patches at similar weights sides due to weights variation per patch.

Keywords: Blending method, Multi-patches, B-spline surfaces.

بناء أسطح الـ B-spline ذات الرقع المتعددة

الخلاصة

نمذجة السطوح باستخدام تقنية الـ (B-spline) تعد من اهم الوسائل المستخدمة في التصميم الهندسي المعان بالحاسوب (CAGD). ان الهدف من البحث هو بناء نظام لتصميم وتشغيل الرقع المتعددة لأسطح الـ (B-spline) الحرة, لما لهذه الطريقة من مساهمة فاعلة في المجالات التكنولوجية كذلك في نطاق الطائرات, السفن وصناعة السيارات, علاوة على ذلك استخدامها الواسع في صناعة القوالب. تم اجراء عملية دمج هذه الرقع فيما بينها من اجل بناء السطح الكامل لتمثيل الانواع المختلفة لأسطح الـ B-spline. هذا البحث يتضمن عملية ربط و توليف هذه الرقع فيما بينها بطريقة تضمن عملية اشتراك نقاط السيطرة للرقع المدمجة, كذلك التقاء هذه الرقع من جوانب الاوزان المتشابهة نظراً لاختلاف الاوزان بالنسبة للرقعة الواحدة.

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INTRODUCTION

Computer-aided modeling techniques have been developed since the advent of NC milling machines in the late 40's, since the early 60's Bezier and B-Spline representations have evolved as the major tool to handle curves and surfaces. Over the last two decades, the scene of computer graphics has literally exploded with progress in all directions: the arrival of dedicated 3D hardware, computer generated animation, and faster computers to name a few key events [1]. One of these directions focuses on displaying smooth curves and surfaces, suitable for modeling landscapes, faces and other topologies of interest, resulting arises of the need for B-Spline curves. In this field there were a number of researchers who treated with the issues of building the non-uniform B-spline surfaces, such as H. Pungotra, and R. Canas who presented an approach for merging arbitrary B-spline surfaces within a very low tolerance limit. The technique exploits blending matrices that are independent of the control point positions and, hence, can be pre-calculated prior to haptic interaction.

In 2011 Wei Sh.and Zhao C. presented a method on how to generate traditional quadrilateral NURBS surfaces, non-quadrilateral NURBS surfaces, such as triangle NURBS (Non-uniform Rational B-spline) NURBS surfaces and their equidistant surfaces. With their method, if a series of values of the two parameters can be obtained in sequence, surface points and the equidistant surfaces points based on these values of the two parameters can be obtained.

Jiang Li in the same year showed that the shape control of NURBS surface is through the control point matrix. By adjusting the appropriate critical control point matrix, some quadrilateral NURBS surfaces can even constructing circular surface. It is a simple way to get the quadrilateral NURBS surfaces, though sometimes are not satisfied with precision.

This research will focus on the issue of building the non-uniform B-spline surfaces, consisting of more than one patch, dealing with the weight differences of each patch and blending them in a way that ensures surface continuity.

Multi patches B-spline surfaces

Representing the complex surfaces by using one single patch is impractical way in modeling process, where the need arises to connect more than one patch together. Since the surfaces used in modeling techniques such as Bezier consisting of one patch, so it requires a blending process among more than one patch. The blending process requires a term of continuity for the connected patches. Maintaining continuity in the derivatives of the desired order at the connection point is not easy or may be tedious and undesirable. B-spline surface makes a solution for this problem, where the surface can be composed of several patches. The number of patches is determined by controlling its own variables. It also solves the problem of continuity, where the structure of the surface includes the synthesis of these patches together in a way that allows the participation of these control points for the merged patches.

B-Spline Curve Definition and Construction

A B-Spline curve differs from Hermite or Bezier curve in that it usually consists of multi segments. Each segment is defined and influenced by only a few control points,

which are the coefficient of B-Spline basis function polynomials. The degree of curve is independent of the total number of control points. These characteristics allow changes in shape that do not propagate beyond one or only few local segments [5].
The general form of B-Spline curves can be represented in terms of their blending function is:

$$P(u) = \sum_{i=0}^{n} P_i N_{i,k}(u) \qquad \ldots\ldots\ (1)$$

Where
(Pi) is the set of control points and ($N_{i,k}$) represents the appropriate basis function for B-Spline representation.
For B-Spline curve, a parameter (k) controls the degree of basis polynomials, and it's usually independent of the number of control points [5].
The basis function ($N_{i,k}$) is defined by following recursive equation:

$$N_{i,k} = \frac{(u - t_i)N_{i,k-1}(u)}{t_{i+k-1} - t_i} + \frac{(t_{i+k} - u)N_{i+1,k-1}(u)}{t_{i+k} - t_{i+1}} \qquad \ldots\ldots..(\ 2)$$

The (t_i) are knot values, and a set of knot values comprises a knot vector. The knot vector has a significant influence on the blending function $N_{i,k}$ (t) and theory on the B-Spline curve itself. These knot vectors can be classified as periodic and non-periodic knots.
Since the knot vectors influence the shape of the B-spline, it can be said, in general, that B-Spline curves are classified into uniform B-spline and non-uniform B-Spline curves [6].

B-Spline surfaces

B-Spline surface is a direct extension of a B-Spline curve. The underlying principle in defining a B-Spline surface is that a point trace out a B-Spline curve and then let this curve sweep out a B-Spline surface. The simple extension for three-dimensional free-form surface from 3-dimensional free- form curve is by incorporating another parameter (w) to the vector equation of the curve. B-spline surface is defined by [7]:

$$P(u, w) = \sum_{i=0}^{m} \sum_{j=0}^{n} P_{i,j} N_{i,k}(u) N_{j,l}(w) \qquad \ldots\ldots..\ (3)$$

The $\boldsymbol{P_{i,j}}$ are control points and vertices of the characteristic polyhedron.
Ni, k (u): **Nj, l** (w) are the basis functions and they are the same as those of B-spline curves.
Where *u* and *w* are independent variables . A Cartesian or tensor product B-Spline surface is given by: [8].

$$P_{s,t}(u, w) = U\, M_S\, P\, M_S^T\, W^T \qquad \ldots\ldots\ (4)$$

$[1 \le s \le m - k + 2]$
$[1 \le t \le n - l + 2]$
$0 \le u, w \le 1$

M_S = B-spline basis function matrix

Uniform B-Spline surfaces

The knot vectors for B-spline surfaces in the two directions of parameterization are classified as periodic and Non-periodic, and it is the same classification as the B-Spline curves. The surfaces lie within the convex hull formed by the control points. Uniform B-Spline surfaces are generated by making use of periodic/uniform knot vectors and an array of control points [9].

In addition to the initial data used for generating the desired surface the proposed program represents the surface in a graphical mode with the aid of MATLAB (V 7.10) software. Figures (1) and (2) show a second and third degree of uniform B-spline patches respectively.

It is obviously noted from figure (1) and (2), that the uniform B-spline surface doesn't interpolate the boundary control points. This result is an inherent property of B-spline surface.

Figure (3) shows an example of multi-patches closed uniform B-spline surface which can be obtained by using multiple control points.

Non uniform B-Spline surfaces

Non uniform B-spline surface solves the problem of interpolating the surface with the vertices of control points, making it the preferred method in design, especially for the open parts where the uniform B-spline shows a deficiency in representation [10]. As in the non-uniform B-spline curve, the surface also consists of number of patches depending on the values of the n and k parameters for each direction of (u, w). The basis functions that generate multiple patches, each one is different from the other but complement each other. As an example, the surface with (k=4 and n=4) will have 4 patches to generate this surface. In this proposed model, it can be seen that the first patch in figure (4) which starts with the vertices of control points but it doesn't end with them.

On the contrary, the second patch interpolates control points at the end but it doesn't start with it. The second patch starts from the end of the previews patch. It is necessary to note that the meshes of the two embedded patches have progressively got smaller at the region of meeting of these patches, as shown in figure (5).

While the third and fourth patches take an opposite diagonal position to the first and second patch composing the entire surface through blending the four patches. Because of the asymmetry of the weights per patch, where it is obviously noted that each patch doesn't have the same weight from its aspects, necessitating meeting of these patches is done at similar weighted side.

Figure (6) shows the blending of the 4 patches to form the entire surface.

Designing of multi patches B-spline surfaces using Blending method

A model has been designed representing the blending method of multi patches, where a non-uniform B-spline surface consisting of 4 patches has been designed. Using the equation (2) that has been clarified previously, B-spline variables were compensated and set of equations were obtained for each ***k***, as a result for compensating these equations between each other we got the final equations which is:

$$N_{1,4} = \boxed{(3u - \tfrac{9}{2}\, u^2 + \tfrac{7}{4} u^3)\, N_{3,1} + (2 - 3u + \tfrac{3}{2} u^2 - \tfrac{1}{4} u^3)\, N_{4,1}} \quad \ldots\ldots (5)$$

$$N_{2,4} = \boxed{(\tfrac{3}{2} u^2 - u^3)\, N_{3,1} + (-2 + 6u - \tfrac{9}{2} u^2 + u^3)\, N_{4,1}} \quad \ldots\ldots (6)$$

$$N_{3,4} = \boxed{\tfrac{1}{4} u^3\, N_{3,1} + (2 - 6u + 6u^2 - \tfrac{7}{4} u^3)\, N_{4,1}} \quad \ldots\ldots (7)$$

$$N_{4,4} = \boxed{(u3 - 3u2 + 3u - 1)\, N_{4,1}} \quad \ldots\ldots (8)$$

From the equations above, two basis functions have been extracted, the first basis function is differing from the second one but they complement each other.

$$\mathbf{M_1} = \frac{1}{4}\begin{bmatrix} -4 & 7 & -4 & 1 \\ 12 & -18 & 6 & 0 \\ -12 & 12 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{M_2} = \frac{1}{4}\begin{bmatrix} -1 & 4 & -7 & 4 \\ 6 & -18 & 24 & -12 \\ -12 & 24 & -24 & 12 \\ 8 & -8 & 8 & -4 \end{bmatrix}$$

These basis functions have been employed for building the first and second patch using MATLAB software, As well as for the third and fourth patch were the third patch has the same basis function of the first one, and the same thing with regard to the second and fourth patch. The detailed explanation of the proposed program for each patch has been given in the flowchart that is shown in Figure (7) with changing the increment values of (u, w) for the other patches, where it will be {($1 \leq u \leq 2$), ($1 \leq w \leq 2$)} for the second patch, as well as is the case for the third and fourth patch In addition to the association of these patches by its internal control points.

The control point's values of the proposed surface are as follows:

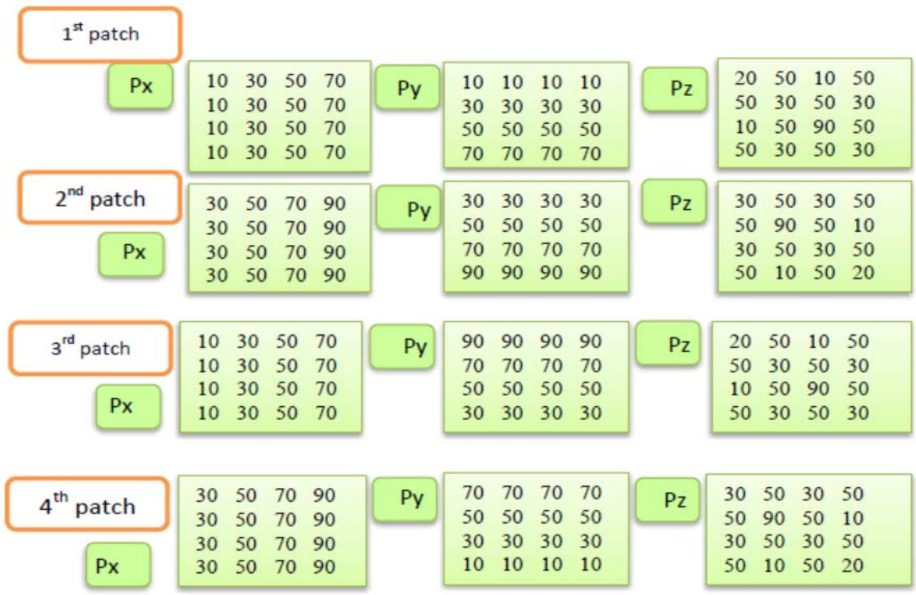


Figure (8) shows the proposed designed surface consisting of 4 patches.

DISCUSSION

Within the context of multi-patches B-spline surfaces construction the following attributes can be discussed:

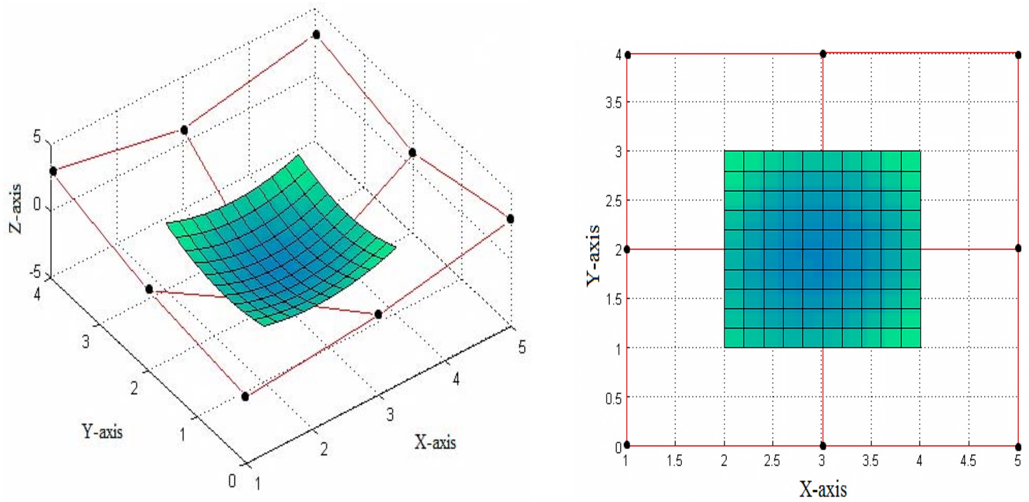
1- The patches of non-uniform B-spline surface have a dissimilar weight, therefore the process of merging these patches in order to build the entire surface associated with the problem of synthesis of these patches with each other. But at the end of the patch, the control points develop beyond the patch itself. In designing the patches of the non-uniform B-spline surface using MATLAB software, the problem of blending these patches become clear, where the first patch (as mentioned before), the vertices of control points coincide with patch itself at the start of the patch. Accordingly the second patch shares the last control points of the first patch and the control points at the end of this patch coincide with patch itself, but the control points dispatch at the start of the patch. Figure (9) shows the problem of wrong merging for two patches.

Figure (9.a) shows the variation of weights between the two patches, while the figure (9.b) shows the unequal patches heights despite the equal values of the control points in Z-axis. Blending these patches requires meeting of the patches from the similar weight sides; therefore the beginning of the control points for the second patch shares the end control points for the first patch in diagonal direction. While the third and fourth patches take an opposite position to them, through building an algorithm involving the reverse patching process combines these patches with each other to form the entire surface using MATLAB software, as shown in figure (10).

2- It is noted that with increasing the number of patches depending on n, k variables of the non-uniform B-spline surface, the patches area get smaller around the center of the surface. Figure (11) shows effect of increasing the number of patches on its area using equal values of control points.

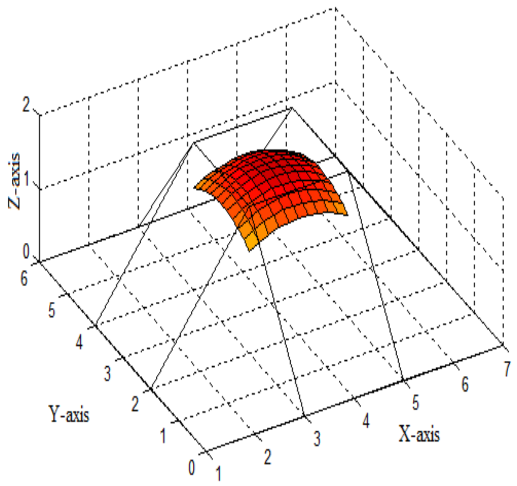
CONCLUSION

- The first patch of the non-uniform B-spline surface start with the vertices of control points but it doesn't end with it, in opposite to the last patch which is end with the vertices of control points but it doesn't start with it.
- All the curves and surfaces of uniform and non-uniform B-spline surfaces are able for changing its shape by moving one control points or more. NURBS has the weight property in addition to changing the control points making it possesses a very high design efficiency to generate any surface whatever it was complicated.
- Blending method for more than one patch requires the confluence of these patches at similar weights sides due to weights variation per patch, where it's obviously noted that the weights for the entire surface are not the same, thus the weight in the external sides is normally higher than middle of the surface.

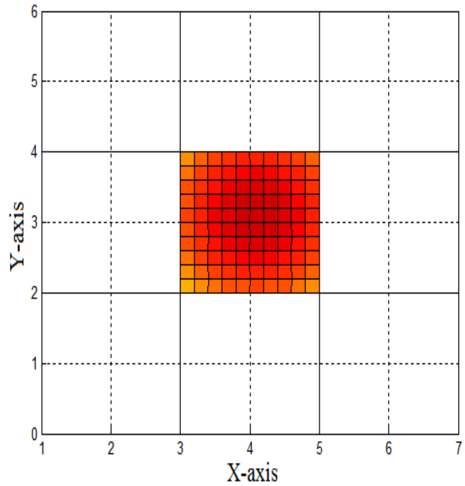


(a) (b)

Figure (1): Uniform B-spline surface k=3, (a) 3D space, (b) XY plane.



(a)



(b)

Figure (2): Uniform B-spline surface k=4, (a) 3D space, (b) XY plane.

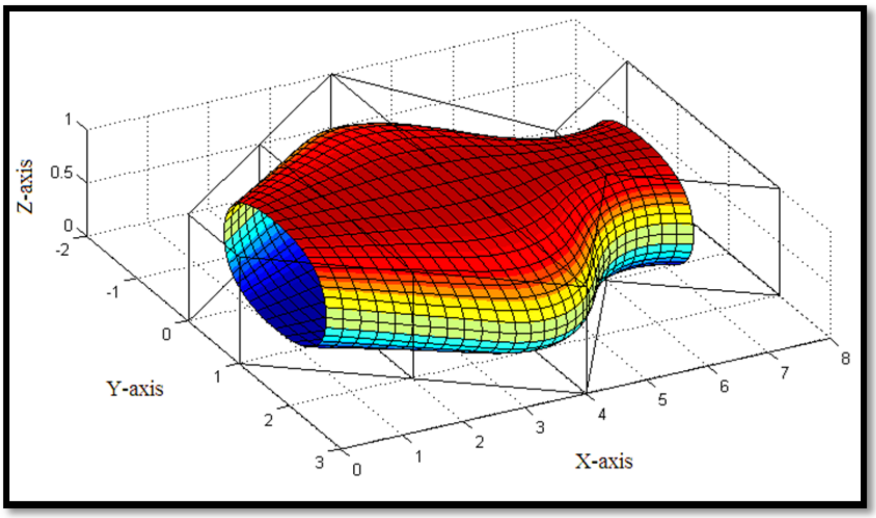
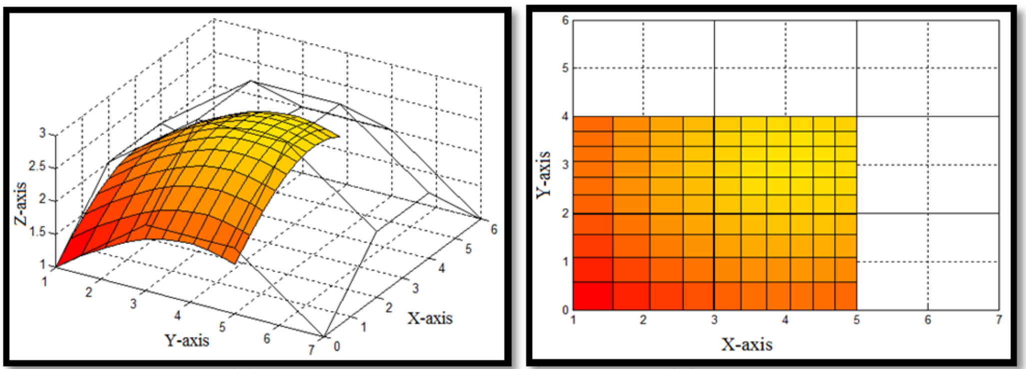
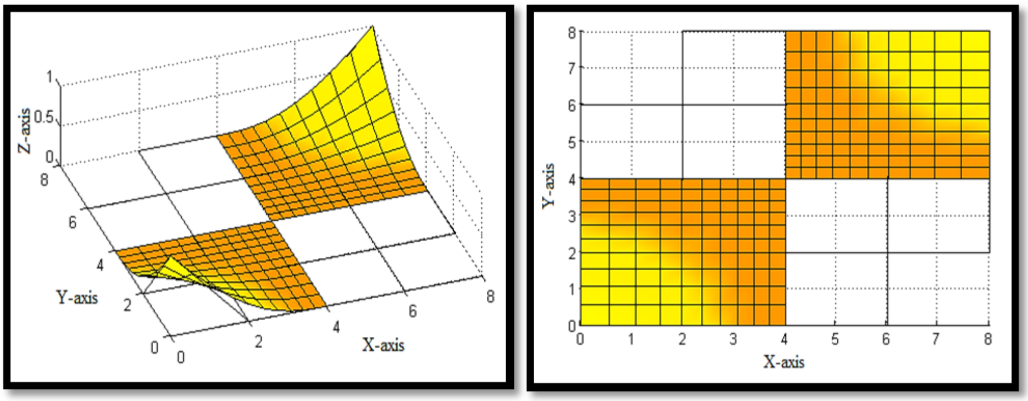


Figure (3): Multi-patches of uniform B-spline surface.



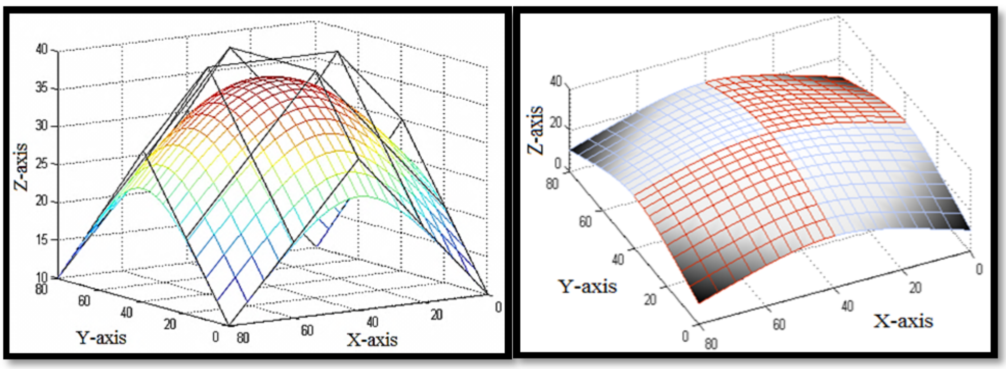
(a) **(b)**

Figure (4): First patch of non-uniform B-Spline surface (n=4, k=4), (a) 3D space, (b) XY plane.



(a) **(b)**

Figure (5): Blending (1st and 2nd) patches, (a) 3D space, (b) XY plane.



(a) **(b)**

Figure (6): Non uniform B-Spline surface consist of 4 patches.

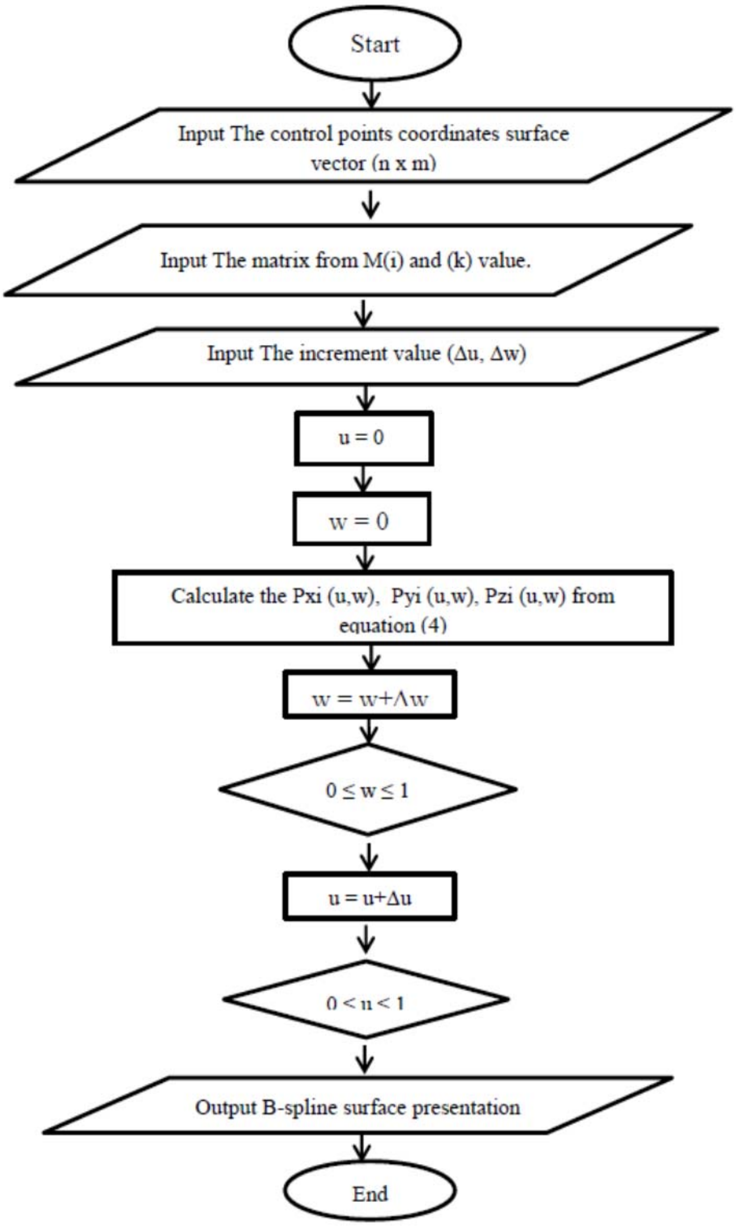
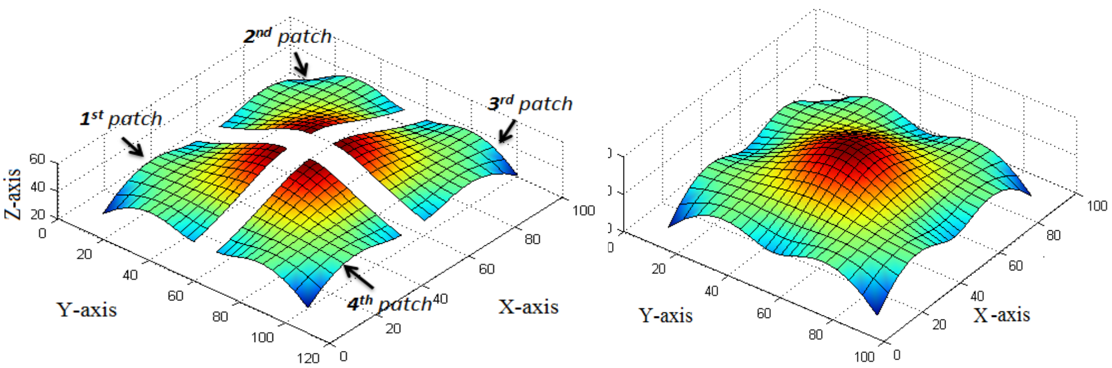
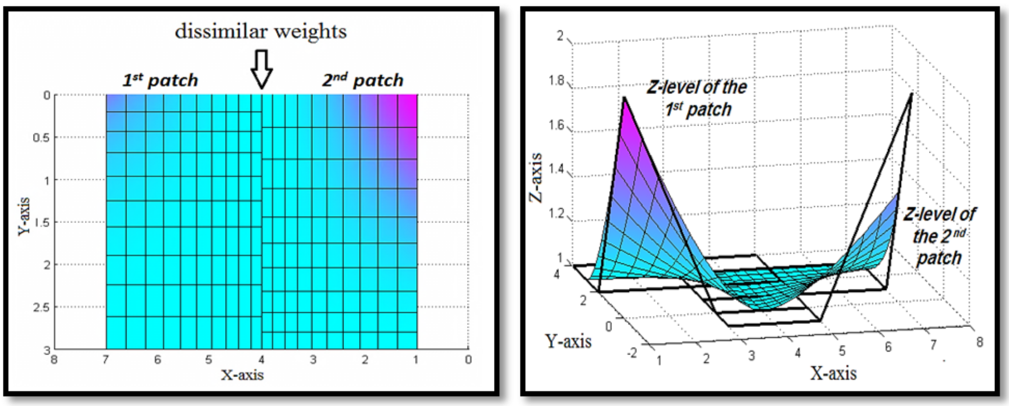


Figure (7): Flowchart of the proposed program for building B-spline surface



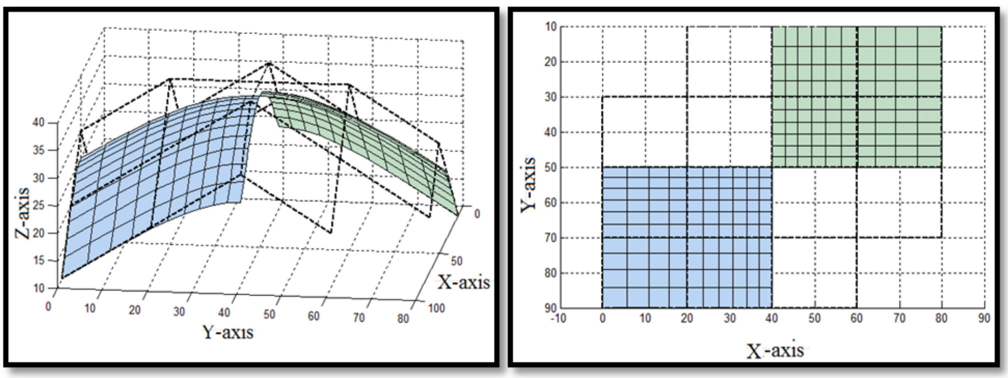
(a) **(b)**

Figure (8): Blended surface using MATLAB software.

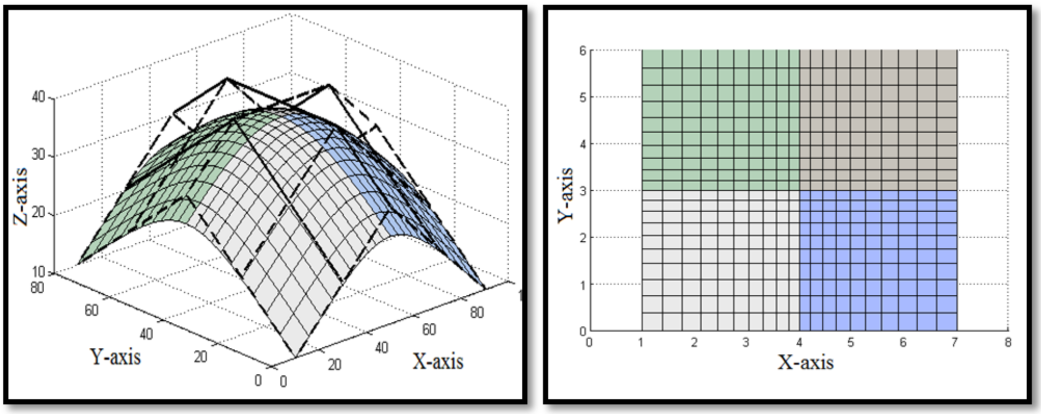


(a) **(b)**

Figure (9): Incorrect blending between two patches, (a) dissimilar weights, (b) Different patches heights

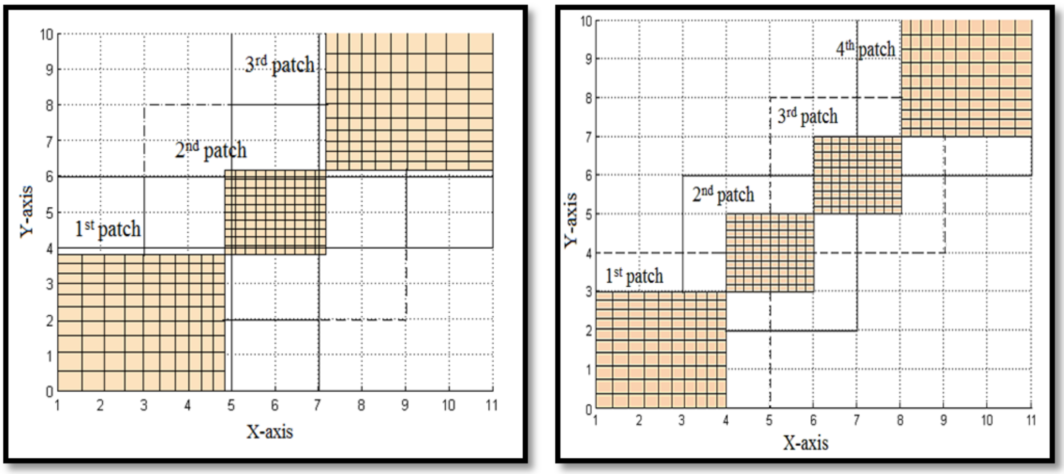


(a) **(b)**



(c) (d)

Figure (10): Blending the patches of non-uniform B-spline surface, (a) 3D space for blending two patches, (b) X-Y plane for blending two patches, (c) 3D space for blending four patches, (d) X-Y plane for blending four patches.



(a) (b)

Figure (11): The effect of increasing the number of patches, (a) three patches of B-spline surface (n =4, k= 3), (b) four patches of B-spline surface (n=5, k=3).

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