A Cumulative Damage Model for Fatigue Life Prediction Based on Dynamic and Static Deflections

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Received on:4/9/2014 & Accepted on:2/4/2015

Abstract

The main goal of this study is to report experimental evidence about the accumulative fatigue damage behavior of CK35 steel alloy at room temperature and zero main stress (R = -1) . A non-linear accumulative damage model based on static and dynamic deflection is proposed, considering the loading sequence into account and the constant S-N curve. Satisfactory predictions of cumulative fatigue life have been observed when applying the proposed model to the two block loading sequence. Comparison between LDR and proposed model has been made. It was found that LDR can underestimate the fatigue damage and the proposed model showed good agreement with the experimental results. A non-linear damage model seems to be proper choice for predicting the cumulative fatigue two block loading histories.

Key words: fatigue damage accumulative, non-liner model, static and dynamic deflection, CK35 steel alloy.

انموذج للضرر التراكمي لتخمين اعمار الكلال معتمدا على الانحرافات المستقرة والداينميكية

الخلاصة:

الهدف الاساسي لهذه الدراسة هو لتوثيق تصرف سبيكة الفولاذ CK35 تحت ضرر الكلال عند درجة حرارة الغرفة و نسبة اجهاد (R=-1). تم اقتراح انموذج لا خطي للضرر التراكمي يعتمد على الانحرافات المستقرة والداينميكية . اخذ بنظر الاعتبار تتابع الحمل و منحنيات S-N . تم الحصول على تخمينات مقنعة عند استخدام الانموذج المقترح على تتابع الاحمال ذات الحزمتين للفحوصات العملية . تمت المقارنة بين LDR مع الانموذج المقترح وقد اعطى الانموذج المقترح توافق جيد مع النتائج العملية . لذلك يبدو ان الانموذج اللاخطي للضرر هو الاختيار الافضل لتخمين الكلال التراكمي من نوع الحزمتين .

الكلمات المرشدة : تراكم الضرر الكلالي , انموذج الضرر اللاخطي , انحرافات مستقرة و داينميكية , سبيكة الفولاذ CK35 .

https://doi.org/10.30684/etj.33.4A.1
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Notation:

Symbol	Title
σ_b	bending stress in (MPa)
P	Force (N)
l	The arm of load equal to 135.7 in (mm)
d_{min} , d	The minimum diameter of the specimen
a,b	The fitting coefficient
I	The second moment of inertia
R^2	correlation coefficient
M	Bending moment
E	Module of elasticity
R	Radius of curvature
N	The applied cycles at σ_i .
D	Accumulative fatigue damage
δ_{static} , δ_s	Static deflection
σ_L , σ_H	The applied stress at low and high stress levels respectively.
$\delta_{dynamic}$, δ_d	Dynamic deflection
X	Variable quantity related to the loading stress level
α	Function of the applied load
B	The inverse slope of The S-N curve
S_s	Slope of static deflection No. of cycles to failure
S_d	Slope of dynamic deflection No. of cycles to failure
N_f	Number of cycles to failure
x_i	Function of loading sequence
L	The distance from the applied load p to fixed support
D exp	Experimental Damage
D miner	Miner Prediction Damage
D model	Proposed model Damage

INTRODUCTION

Fatigue cracks occur when the material is subjected to dynamic loading (cycle or fluctuating loading) although the stress produced is below the yield stress of the material. A method of determining the cycle stress based on its deflection is presented in this work. The stress gradients contribute to both axial load and bending moment. The accurate measurement of deflection has become an important consideration in the design of the fatigue components [1, 2]. This paper examines the fatigue behavior of CK35 steel alloy by investigating fatigue life and strength depending on the dynamic deflections.

Recently, the fatigue behavior of metallic materials in high cycle fatigue (HCF) region has drawn great attention because the development of modern industry often needs metal structure such as railway wheels, turbine disks in aerospace [3].

Aid et al [4] proposed a new damage model based on S-N curve and the loading history. They concluded that the model results are in good agreement with experimental data.

Zhang et al [5] proposed a nonlinear fatigue damage model was applied to the vehicle bridges life cycle. This model was providing to be an effective method in predicting the bridges progressive fatigue damage due to random variables.

Al – Garni et al [6] tested AISI304 stainless steel under variable loadings experimentally and using finite element analysis. It was found that, as the load step increased the summation of damage become around to unity. The finite element (FE) used in above work gave good correlation with the experimental results.
Zengah S. et al [7] proposed damage stress model to evaluate the cumulative fatigue data and they concluded that the application of the proposed model showed safe prediction of fatigue compared to the experimental results.

Fatigue and fatigue damage accumulation of P335N.L1 steel were assessed using miner rule and non-linear damage models. It was found that miner rule produced inconsistent result while the non-linear theory gave reasonable result and final conclusion was that the non-liner damage theory proposed by Marco and Starkey seem to be a proper choice [8].

A comprehensive over view of research activities of accumulative fatigue damage theories has been presented by Yang and Fatemi [9].
The main object of this work is to propose a non-linear fatigue damage accumulation and to report experimental evidence about this model.

Experimental Work:

Material selection:

Medium carbon steel CK35 is selected because it has a widely used in many applications where better properties than those for mild steel are required. The material was received from a state company – Mechanical Industries AL-Ascandarya – and tested to determine its chemical composition and mechanical properties. Table (1) gives the chemical analysis which is done in engineering center for testing and recondition. The relevant mechanical properties are listed in Table (2).

Table (1) chemical analysis of CK35 steel alloy

CK 35	C	Mn	Si	S	p
Standard DIN50114	0.32-0.39	0.5-0.8	0.15-0.35	0.035	0.035
Measured	0.33	0.75	0.25	0.025	0.013

Table (2) mechanical properties of CK35 steel alloy

CK 35	σ_u (MPa)	σ_y (MPa)	E (GPa)	G (GPa)	Poissions ratio
Standard DIN50114	550-700	> 392	210	81	0.3
Measured	660	400	205	80	0.28

The mechanical properties listed in Table (2) was conducted in the Production and Metallurgy Eng. Dep. University of technology using the tensile test rig (WDW – 200E) . The specifications of the tensile test and specimen configuration have been restricted according to the American Society for testing and Materials (AST81-8). The above results are the average of three readings.

Fatigue test:

The specimens of the fatigue test were prepared according to the Machines manual as shown in Figure (1). 20 specimens were manufactured and tested to generate the S-N curve by an alternating bending specification of fatigue test machine (rotating bending fatigue testing machine) shown in Figure (2)

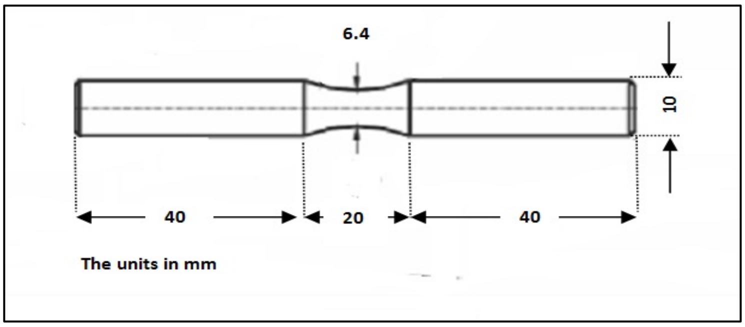

Figure (1) Rotating bending fatigue specimen (cantilever beam) according to the American Society for testing and Materials (AST81-8).

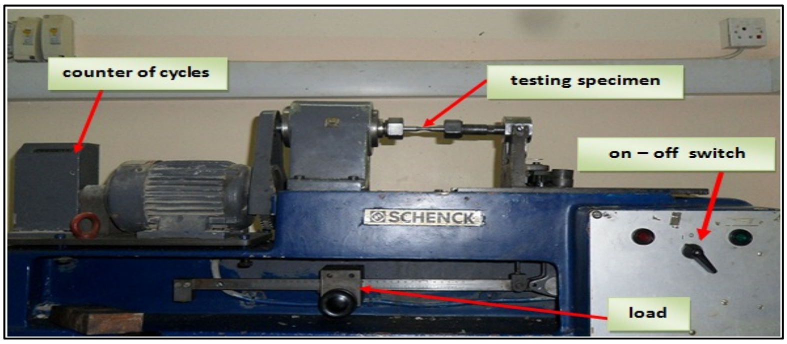

Figure (2) rotating bending testing machine

The application stress is calculated from the applied moment according to the simple theory of a cantilever beam as:[10]

$$\sigma_b = \frac{l * 32 * P}{\pi * d^3} \qquad \dots (1)$$

Where:

σ_b is the bending stress in (MPa). P is the force in (N.). l is the arm of load P 135.7 mm. d is the minimum diameter of the specimen(mm).

Figure (3) shows the application of load with its arm.

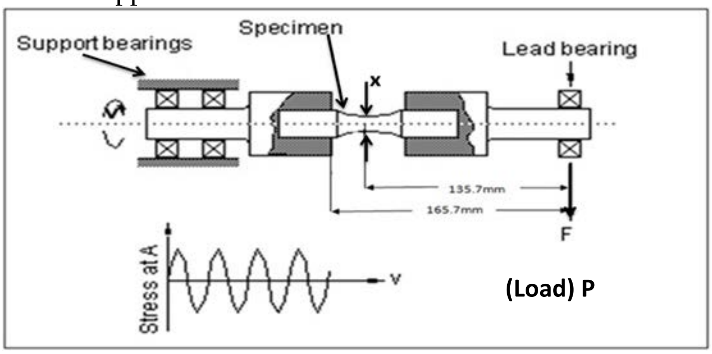

Figure (3) schematic diagram shows the application of load with its arm.[11]

The number of cycles was recorded by using digital counter and the relation between bending and the number of cycles can be expressed in power law regression as the following equation [12]. The results of Ref [12] were observed that the values of R^2 located between 0.99 to 0.97.

$$\sigma_b = aN_f{}^b \qquad \dots (2)$$

Where

a, b are the fitting coefficient [13]. The parameter (a) is related to static bending strength, while the parameter (b) is related to fatigue degradation and describes the fatigue sensitivity .A correlation coefficient ($R^{2)}$ was used to verify whether the experimental data are explained by power formula. The correlation coefficient can be calculated by equations mentioned in ref [12]. The closer is R^2 to unite the stronger is the relationship between stress or deflection and number of cycles to failure.

Measurement of Deflection:

The experimental evaluation of deflection measurement was performed using the electronic sensor (digital dial gauge) holded with the fatigue test. The sensor was fixed at the mid span of the specimen. The properties of the electronic sensor (digital dial gauge) are:

- time response 0.5 m/s
- resolution digital indictor 0.01mm
- measuring range 0-12.7 mm
- power , one silver oxide battery 1.5v
- operation temperature 0C°-40C°

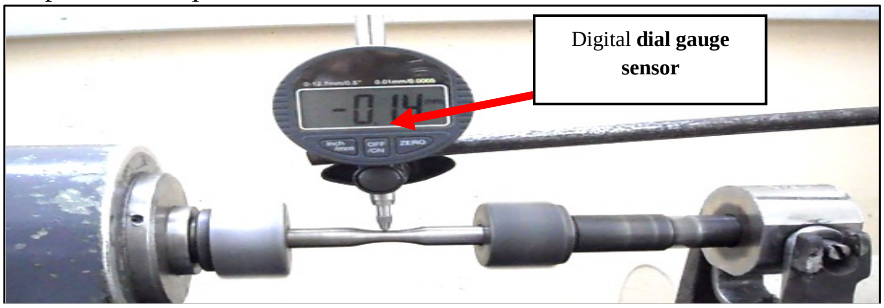

Figure (4) the sensor holding on the test rig

Theoretical consideration:

Consider a shaft (fatigue specimen) of minimum cross section area ($\frac{\pi}{4}d^2$min), where d_{min} is minimum diameter of specimen. It is subjected to a vertical load (point load) as shown in Figure (3).

The simple theory of elastic bending states that [14] .

$$\frac{M}{I} = \frac{\sigma b}{y} = \frac{E}{R} \qquad \dots (3)$$

Where

M is bending moment.

I is the second moment of inertia.

σb is the bending stress .

E is the module of elasticity and R is radius of curvature.
Using the Bernoulli – Euler's elastic equation which states that:

$$EI\frac{dy^2}{dx^2} = bending\ moment\ (M) \qquad \ldots (4)$$

The bending moment equation at section X (minimum diameter of specimen) for the action of P to the min. diameter which is equal to 135.7 mm.
Equation (4) becomes:

$$y = \frac{P}{EI}\left(-\frac{x^3}{6} + \frac{L^2x}{2} - \frac{L^3}{3}\right) \qquad \ldots (5)$$

Where
L is the distance from the applied load p to fixed support.
On integrating and solving the above equation with required boundary conditions, we get the down word deflection of the beam as:

$$\delta_{static} = -P * 4.83 * 10^{-3} \qquad \ldots (6)$$

$in\ mm\ at\ the\ min.diameter\ of\ the\ specimen$
L is equal to (156.7) mm based on the design of the fatigue testing. E = 200 GPa for CK 35 steel alloy, I = $\frac{\pi}{36}(6.2)^4$ = 72.5 mm^4

Results and Discussion:
The results obtained from the experimental work are listed and discussed in details through this section. The experimental results include results of fatigue under constant amplitude loading, fatigue under variable amplitude loading and deflection measurement in static and dynamic loading test.

Constant fatigue test results:
The constant fatigue test was carried out at room temperature and stress ratio R = -1. The results are graphically display in the form of S-N curve. These curves are obtained by curve fitting the experimental data of fatigue test. The fatigue results with the power law equation constants which express the fatigue behavior of CK35 steel alloy and its correlation coefficient (R^2) are given in Table (3). It is noted that these equation have relatively high correlation coefficient which indicated that are well described by power law formula. This finding is in good agreement with ref [12]. Table (4) gives the analytic static deflection and the measured static deflection using the electronic sensor (digital dial gauge) corresponding to the load in (N) and bending stress in MPa

Table (3) constant fatigue results with the regression parameters

Specimen no.	σ_f (MPa)	N_f cycles	N_f cycles average	a	b	R^2
1,2,3,4,5	175	164000,198000,210000 230000, 180000	190666	1169	-0.156	0.9992
6,7,8,9,10	200	95000,86000,90000	90333			

		92000, 89000				
11,12,13,14,15	300	4000,5000,9000 4500.8000	6000			
16,17,18,19,20	325	3000,5000,3500 3500,5500	3833			

Table (4) analytic and measured static deflection.

P(N) load	σ_b N/mm^2	δ_{static} mm (analytic)	δ_{static} mm (measured)
86.15	500	0.41	0.46
69	400	0.33	0.37
56	325	0.27	0.24
51.7	300	0.25	0.20
34.5	200	0.17	0.19
30.15	175	0.15	0.165
17.23	100	0.08	0.13

Table (5) the static and dynamic deflection corresponding to the values of bending stress listed in Table (3).

Table (5) the static and dynamic deflection corresponding to bending stress

σ_b (MPa)	δ_{static} (mm)	$\delta_{dynamic}$ (mm)	N_f cycles Av.
175	0.13	0.18	190666
200	0.19	0.22	90333
300	0.20	0.27	6000
325	0.24	0.33	3833

Note that the above data of deflections are the average of five readings.

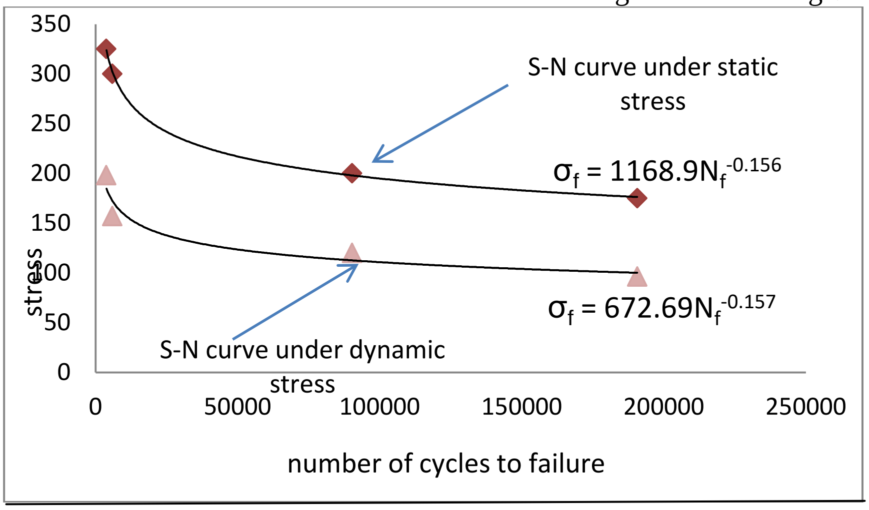

Figure (5) the typical S-N curves

The proposed model:

Marco and Starkey [15] proposed the first non linear load depending damage theory, written by a power law:

$$D\ (damage) = \sum_{i=1}^{n} \left[\frac{n}{Nf}\right]^{xi} \qquad \ldots (7)$$

Where

x_i is a variable quantity related to the loading stress level.

Pereira et al [16] proposed a non linear model for low-high and high-low loading sequence.

$$\frac{n_H}{N_H} = 1 - \left[\frac{n_L}{N_L}\right]^{\frac{\alpha_L}{\alpha_H}} \text{ For (low - high) loading stress level} \qquad \ldots (8)$$

$$\frac{n_L}{N_L} = 1 - \left[\frac{n_H}{N_H}\right]^{\frac{\alpha_H}{\alpha_L}} \text{ For (high – low) loading stress level} \qquad \ldots (9)$$

Where

α is a function of the applied load, to be obtained from the experimental data.

Alalkawi et al [17] presented a non linear damage model taking in to account the effect of loading sequence and shot peening treatment using medium carbon steel.

$$D = \frac{\left[\sum \frac{n_i}{N_{fi}}\right]^x}{x} \qquad \ldots (10)$$

Where

x represented the effect of loading sequences $\frac{\sigma_L}{\sigma_H}$ and shot peening treatment (β). x can be defined as :

$x = \frac{\sigma_L}{\sigma_H}\beta$ For low to high loading sequence … (11)

And $x = \frac{\sigma_H}{\sigma_L}\beta$ For high to low loading sequence …(12)

Where

β is the inverse slope of the S-N curve.

Following the above researchers, damage can be defined for two loading sequence as:

For low-high loading

$$D = \frac{\delta_s}{\delta_d} * \frac{S_d}{S_s} \qquad \ldots (13)$$

Where

δ_s , S_s are static deflection and slope of static deflection and No. of cycles to failure curve respectively.

Figure. (6) Shows the δ_d-N_f curve , the best fit equation which describers the experimental data may be taken the form :

$$\delta_d = 0.904\, N_f^{-0.129} \qquad \text{... (14)}$$

δ_d , S_d are dynamic deflection and slope of dynamic deflection No. of cycles to failure respectively. Figure (6) gives the relation between the δ_s and N_f , the experimental data may be described the following equation :

$$\delta_s = 0.443 N_f^{-0.082} \qquad \text{... (15)}$$

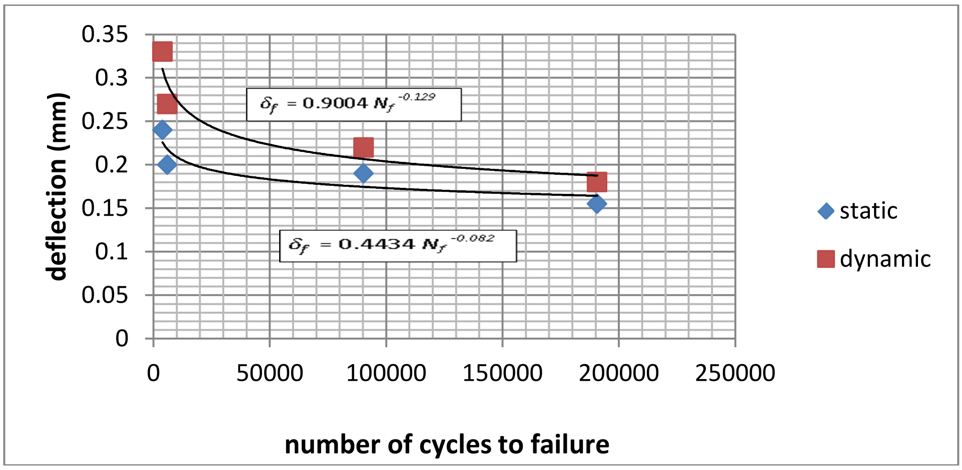

Figure (6) deflection-number of cycles to failure curve (static &dynamic)

Equation (13) can be equated to the ratio of number of cycles considering the effect of loading sequence as:

$$\sum_{i=1}^{n} \left(\frac{n}{N_f}\right)_x^{x_i} = D \qquad \text{...(16)}$$

Where

x_i is a function of loading sequence.

n is the applied cycles at σ_i .

And N_f is the cycles to failure at σ_i .

Combine equation (13) and (16) to get:

$$\left[\left[\frac{n}{N_f}\right]_H^{\frac{\sigma_H}{\sigma_L}} + \left[\frac{n}{N_f}\right]_L^{\frac{\sigma_L}{\sigma_H}}\right] R = \frac{\delta_s\, S_d}{\delta_d\, S_s} \qquad \text{... (17)}$$

Where

R is the number of programs and σ_L , σ_H are the applied stress at low and high stress levels respectively.

For high – low loading:

Here, damage may be defined as the inverse of equation (13)

$$D = \frac{\delta_d}{\delta_s} * \frac{S_s}{S_d} \quad \ldots (18)$$

Equation (17) can be written in form:

$$\left[\left[\frac{n}{N_f}\right]_H^{\frac{\sigma_H}{\sigma_L}} + \left[\frac{n}{N_f}\right]_L^{\frac{\sigma_L}{\sigma_H}}\right] R = \frac{\delta_d S_s}{\delta_s S_d} \quad \ldots (19)$$

The application of proposed model to experimental results:

Table (6) gives the fatigue life prediction using the proposed model compared to the experimental data.

Table (6) fatigue life prediction using the proposed model*

Spec. No.	Loading sequence(MPa)	N_f Ava.	D exp.	R exp.	N_f model	D model	R model
1,2,3,4,5	200-250	39000	1.245	3.9	35836	1.05	3.5831
6,7,8,9,10	250-200	34667	1.107	3.46	32757	0.9598	3.2757
11,12,13,14,15	175-300	15334	1.293	1.53	12795	1.0573	1.2795
16,17,18,19,20	300-175	14334	1.208	1.43	11684	0.9655	1.1684

*For more details, see appendix (A) and calculation.

The most common methods to investigate the accumulative fatigue damage is the block loading experiments. The accumulative damage under two blocking usually evaluated by miner rule or linear damage rule (LDR). The experimental damage (D exp.) in Table (6) is greater than unity while Miner damage is equal to unity. But the non linear damage model proposed give D model greater than unity for low-high loading sequences and slightly less than unity for high-low loading sequences. This means that, the LDR produces clearly inconsistent prediction. For the present model , the loading histories beginning with a low (L) block loading are more damaging than sequences starting by high (H) blocks [18] while miner prediction states than D=1 for all cases of loading histories .

Conclusions:

1- The fatigue damage accumulation for two block loading sequences of CK35 steel alloy was investigated at room temperature and stress ratio R = -1 .

2- The LDR rule trend underestimated the accumulative fatigue life.

3- A non-linear proposed model based on static and dynamic deflections was suggested and it give satisfactory fatigue life predictions compared to the experimental results .

Appendix (A):

For low-high loading sequences

$$D = \frac{\delta_s S_s}{\delta_d S_s} \quad \ldots (1A)$$

δ_s *and* δ_d where calculated from the deflection equations which are obtained experimentally (see Figure. (A1))

$$\delta_s = \frac{\sigma + 32}{1026} \text{ and } \delta_d = \frac{\sigma + 21}{653} \qquad \ldots(2A)$$

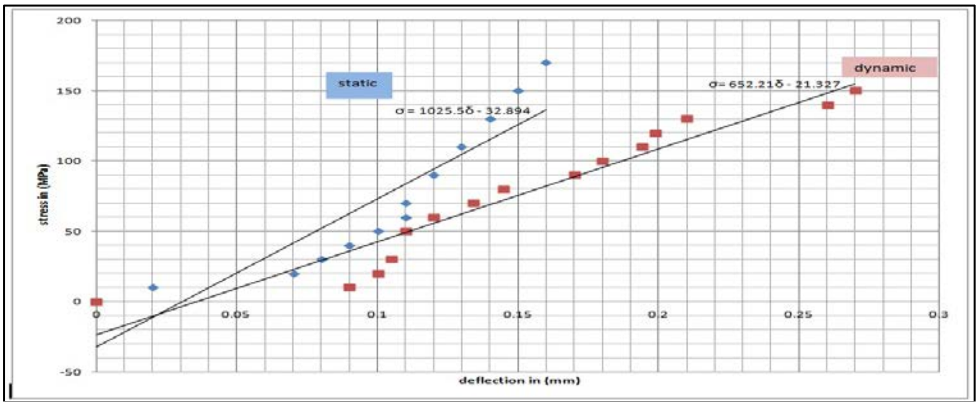

Figure (1A) Relation between stress and deflection

Table (A_1) gives the values of δ_s *and* δ_d corresponding to applied stress used in cumulative fatigue damage testes

Table (A_1) δ_s *and* δ_d against applied stress

$\sigma\ (MPa)$	$\delta s\ (mm)$	$\delta_d\ (mm)$
200	0.226	0.3384
250	0.2748	0.415
175	0.2017	0.3001
300	0.3235	0.4915

While S_s and S_d where obtained from deflection-No. of cycles to failure . see Figure. (2A)

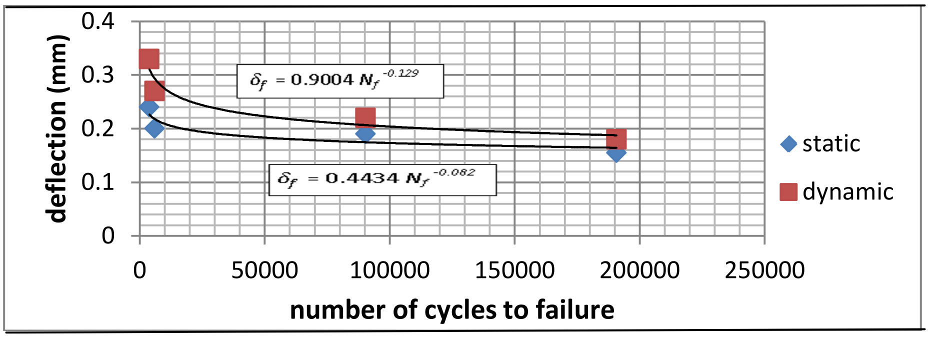

Figure (2A) the deflection – NO. of cycle to failure curve

The value of D corresponding to applied stress are given in Table (A2)

Table (A2) damage D corresponding to applied stress level

$\sigma\ (MPa)$	$D = \frac{\delta_s S_d}{\delta_d S_s}$	$\left(\frac{\delta_s S_d}{\delta_d S_s}\right)^{-1}$
200	1.05	0.951
250	1.041	0.9598
175	1.057	0.9452
300	1.035	0.9655

Table (A3) shows a comparison between LDR and proposed model based on the experimental results

Loading sequence(MPa)	N_f exp. cycles	N_f miner (cycles)	N_f Proposed model (cycles)	D miner	D model
200-250	39000	31318	35836	1	1.05
250-200	34667	31318	32757	1	0.9598
175-300	15334	11858	12795	1	1.0573
300-175	14334	11858	11684	1	0.9655

It has been showed that the damage expressed by the right hand side of equation (11) (proposed model) is greater than unite , see Table (A3) , because the accumulative of damage at low stress level is much greater than at high levels [19][20] while the damage is less than unity if the test stars at high loading sequence equation (13) . For this reason, LDR is sometimes dangerously optimistic [5]

Calculation:
200-250 (MPa)

$$D = \frac{\delta_s S_d}{\delta_d S_s} \quad \Longrightarrow \quad D = \frac{0.226*0.129}{0.338*0.082} = 1.05$$

Appling equation (11)

$$\left[\left[\frac{n}{N_f}\right]^{\frac{\sigma_L}{\sigma_H}} + \left[\frac{n}{N_f}\right]^{\frac{\sigma_H}{\sigma_L}}\right] R = \mathrm{D} \quad \Longrightarrow \quad \left[\left[\frac{5000}{76556}\right]^{\frac{200}{250}} + \left[\frac{5000}{19682}\right]^{\frac{250}{200}}\right] R = 1.05$$

$[(0.06531)^{0.8} + (0.254)^{1.25}\ ]\ R = 1.05$
$0.293R = 1.05 \Longrightarrow R = 3.5831$
$N_f = R\ [\ 10000] = 35831$ cycles

250-200(MPa)
$D^{-1} = 0.9598$ Appling equation (13) gives $0.293\ R = 0.9598$
$R = 3.2757$
$N_f = R\ [\ 10000] = 32757$ cycles

175-300 (MPa)

$$D = \frac{\delta_s S_d}{\delta_d S_s} \quad \Longrightarrow \quad D = \frac{0.2017*0.129}{0.3001*0.082} = 1.0573$$

Appling equation (11)

$$\left[\left[\frac{5000}{193655}\right]^{\frac{175}{300}} + \left[\frac{5000}{6116}\right]^{\frac{300}{175}}\right] R = 1.0573$$

$[(0.0258)^{0.5833} + (0.8175)^{1.714}]\,R = 1.0573$
0.8263R = 1.0573 $\longrightarrow$ R = 1.2795
N_f = R [10000] = 12795 cycles

300-175(MPa)
D^{-1} = 0.9655 Appling equation (13) gives 0.8263R = 0.9655
R = 1.1684 N_f = R [10000] = 11684 cycles

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