On g^*s-Closed Functions in Topological Spaces

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ABSTRACT

Pushpalatha and K. Anitha [1] introduce a new concept of functions which is g^*s-closed function in topological spaces. In this paper, we modify this concept and study the properties of some types of these functions.

Keywords:- ((g^*s -closed set, g^*s –closed function , strongly g^*s –closed function irresolute g^*s-closed function))

الدوال المغلقة – g*s في الفضاءات التبولوجية

الخلاصة

بوشبلاذا و انيثا قدما مفاهيم جديد من الدوال سميت بالدوال المغلقة –g^*s في الفضاءات التبولوجية . في هذا البحث قمنا بتعميم هذه المفاهيم ودراسة خصائص بعض انواع هذه الدوال .

INTRODUCTION

G.B. Navalagi in [3] introduce new class of closed sets is said to be g^*s-closed set in topological spaces. In 2011 Pushpalatha and K. Anitha [1], are studying the properties of these class of closed sets and investigated new types of continuous functions which are g^*s-continuous functions upon g^*s-closed sets with give some properties and relationships of these functions as well as g^*s-closed functions

In this work, we modify the definition of g^*s-closed function appear in [1] by give anther types of these functions with study the relationship of these types and some properties.

Definition (1.1),[2]:-

Let (X,T) be a Topological space a subset A of X is said to be semi closed set if $A \subseteq cl(\text{int}(A))$. The complement of semi closed is said to be semi open.

Definition (1.2),[2]:-

Let (X,T) be a Topological space. The semi closure of A is denoted by Scl(A) and defined the intersection of all semi closed sets containing A.

Definitions (1.3):-

Let (X,T) be a Topological space, a subset A of X is said to be:

(1) generalized closed [4] (briefly, g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$, where U is open set in

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X . The complement of g-closed set is g-open set.
(2) generalized semi closed [5] (briefly, gs-closed) if $s\,cl(A) \subseteq U$ whenever $A \subseteq U$, where U is open set in X . The complement of gs-closed set is gs-open set.
(3) semi generalized closed [3] (briefly, sg-closed) if $s\,cl(A) \subseteq U$ whenever $A \subseteq U$, where U is semi open set in X . The complement of sg-closed set is sg-open set.
(4) s^*g-closed [5], if $cl(A) \subseteq U$ whenever $A \subseteq U$, where U is semi open set in X . The complement of s^* g-closed set is s^* g-open set.
(5) g^*s-closed [1], if $s\,cl(A) \subseteq U$ whenever $A \subseteq U$, where U is gs- open set in X . The complement of g^*s-closed set is g^*s-open set.

Remarks (1.4),[1], [5]:-
We summarize the fundamental relationships between several types of generalized closed set in the following diagram.

$$\begin{array}{ccccc} \text{closed} & \longrightarrow & s^*\text{g-closed} & \longrightarrow & \text{g-closed} \\ \downarrow & & & & \downarrow \\ \text{sg-closed} & \longleftarrow & g^*\text{s-closed} & \longrightarrow & \text{gs-closed} \end{array}$$

all examples of this diagram can be seen in [1], [5].

Some Types Of g^*s-Closed Functions:
In this section, we introduce some types of g^*s-closed functions which are strongly g^*s-closed functions and irresolute g^*s-closed functions with some properties of these types of functions and shows the relationships between these types of functions. These types are a modification of definition appears in [1].

Definition (2.1), [1]:-
$f:(X,T)\longrightarrow(Y,\sigma)$ Let (X,T) and (Y,σ) are topological spaces. A function is said to be g^*s-closed function if for each closed set F in (X,T) then $f(F)$ g^*s-closed in (Y,σ).

Now, the following remark shows the relation between g^*s-closed function and closed function.

Remark (2.2), [1]:-
Every closed function is g^*s-closed function, but the converse is not necessary to be true. To illustrate that consider the following example.

Let $X=Y=\{a,b,c\}$ and defined the topologies $(X,T)=\{X,\phi,\{b,c\}\}$ and $(Y,\sigma)=\{X,\phi,\{a\}\}$. The function $f:(X,T)\longrightarrow(Y,\sigma)$ defined by f(x)=x is g^*s-closed function but not closed since {a} is closed in (X,T) and f({a})={a} is not closed in (Y,σ).

Now, we give other type of g^*s-closed functions from modification of definition (2.1) which call strongly g^*s-closed function.

Definition (2.3):-

Let (X,T) and (Y,σ) are topological spaces. A function $f:(X,T)\longrightarrow(Y,\sigma)$ is said to be strongly g^*s-closed function if for each g^*s- closed set F in (X,T) then $f(F)$ closed in (Y,σ).

Next, the following proposition give the relation between strongly g^*s-closed function and closed function.

Proposition (2.4):-

Let $f:(X,T)\longrightarrow(Y,\sigma)$ be a function. If f is strongly g^*s-closed function then f is closed function.

Proof:-

Let f is strongly g^*s-closed function and F is closed set in (X,T) by using remark (1.4), we get F is g^*s-closed set, thus f(F) is closed set in (Y,σ), then f is closed function.

But, the converse of proposition above is not necessary to be true. To illustrate that consider the following example.

Example (2.5):-

Let $X=Y=\{a,b\}$, $(X,T)=\{X,\phi\}$ and $(Y,\sigma)=\{X,\phi,\{b\}\}$, then the function $f:(X,T)\longrightarrow(Y,\sigma)$ is defined by f(x)=x is closed function but not strongly g^*s-closed function since {b} is g^*s-closed in (X,T) but not closed in (Y,σ).

From proposition (2.4), we can get the following corollary. The proof is easy thus we its omitted

Corollary (2.6):-

Let $f:(X,T)\longrightarrow(Y,\sigma)$ be a function. If f is strongly g^*s-closed function then f is g^*s- closed function.

also, the converse of above corollary is not necessary to be true. To show that see the following example.

Example (2.7):-

Let $X=Y=\{a,b\}$, (X,T) an indiscrete topology on X and $(Y,\sigma)=\{X,\phi,\{a\}\}$, then the function $f:(X,T)\longrightarrow(Y,\sigma)$ the identity function is g^*s- closed function but not strongly g^*s-closed function since {a} is g^*s-closed in (X,T) but not closed in (Y,σ).

More, properties of strongly g^*s-closed function have been given by the following theorem

Theorem (2.8):-

Let $f:(X,T)\longrightarrow(Y,\sigma)$ be a surjctive function. Then f is strongly g^*s-closed function if and only if for each subset S of Y and each g^*s-open U in (X,T) there is an open set V in (Y,σ) such that $S\subseteq V$ and $f^{-1}(V)\subseteq U$.

Proof:-

Necessity. Suppose that f is strongly g^*s-closed function and let S subset of Y and each g^*s-open U in X containing $f^{-1}(S)$. PUT $V=Y-(f(X-U))$, then $S\subseteq V$ and $f^{-1}(V)\subseteq U$.

Sufficiency. Let F be g^*s-closed set in X, then $f^{-1}(Y-f(F))\subseteq X-F$ and $X-F$ is g^*s- open then by hypothesis there is open set V in (Y,σ) such that $Y-f(F)\subseteq V$ and $f^{-1}(V)\subseteq X-F$. Therefore , we have $Y-V\subseteq f(F)$ and $F\subseteq f^{-1}(Y-V)$, hence we obtain $f(F)=Y-V$, thus $f(F)$ is closed set in (Y,σ) then f is strongly g^*s-closed function.

As well as these types can be modifinied g^*s-closed function into the following definition

Definition (2.9):-

Let (X,T) and (Y,σ) are topological spaces. A function $f:(X,T)\longrightarrow(Y,\sigma)$ is said to be irresolute g^*s-closed function if for each g^*s- closed set F in (X,T) then $f(F)$ is g^*s- closed in (Y,σ).

Next, the following proposition shows the relationship of irresolute g^*s-closed with other types of these functions

Proposition (2.10):-

Let $f:(X,T)\longrightarrow(Y,\sigma)$ be a function. If f is strongly g^*s-closed function then f is irresolute g^*s- closed function.

Proof:-

Let f is strongly g^*s-closed function and F is g^*s- closed set in (X,T) the using definition (2.3), we get f(F) is closed set, thus f(F) is g^*s-closed set in (Y,σ), then f is irresolute g^*s- closed function.

But, the converse of above proposition above is not necessary to be true. To illustrate that consider the following example.

Example (2.11):-

Let $X=Y=\{a,b\}$, $(X,T)=\{X,\phi\}$ and $(Y,\sigma)=\{X,\phi,\{b\},\{a\}\}$, then the function $f:(X,T)\longrightarrow(Y,\sigma)$ is defined by f(x)=x is closed function but not irresolute g^*s-closed function since {b} is g^*s-closed in (X,T) but not g^*s -closed in (Y,σ).

also, give the relation between irresolute g^*s-closed function and g^*s-closed function by the following proposition.

Proposition (2.12):-

Let $f:(X,T)\longrightarrow(Y,\sigma)$ be a function. If f is irresolute g^*s-closed function then f is g^*s- closed function.

Proof:-

Let f is irresolute g^*s-closed function and F is closed set in (X,T) the using remark (1.4), we get f(F) is g^*s- closed set, thus f(F) is g^*s-closed set in (Y,σ), then f is g^*s- closed function.

also, the converse of above proposition above is not necessary to be true. To illustrate that consider the following example.

Example (2.11):-

Let $X=Y=\{a,b\}$, $(X,T)=\{X,\phi,\{a\}\}$ and $(Y,\sigma)=\{Y,\phi,\{b\}\}$, then the function $f:(X,T)\longrightarrow(Y,\sigma)$ is defined by f(x)=x is closed function but not g^*s-closed function since {b} is closed in (X,T) but not g^*s- closed set in (Y,σ).

The following theorem give anther properties of irresolute g^*s-closed function.

Theorem (2.12):-

Let $f:(X,T)\longrightarrow(Y,\sigma)$ be a surjctive function. Then f is irresolute g^*s-closed function if and only if for each subset S of Y and each g^*s-open U in (X,T) there is an g^*s- open set V in (Y,σ) such that $S\subseteq V$ and $f^{-1}(V)\subseteq U$.

Proof:-

Necessity. Suppose that f is irresolute g^*s-closed function and let subset S of Y and each g^*s-open U in X containing $f^{-1}(S)$. PUT $V=Y-(f(X-U))$, then $S\subseteq V$ and $f^{-1}(V)\subseteq U$.

Sufficiency. Let F be g^*s-closed set in X, then $f^{-1}(Y-f(F))\subseteq X-F$ and $X-F$ is g^*s- open then by hypothesis there is open set V in (Y,σ) such that $Y-f(F)\subseteq V$ and $f^{-1}(V)\subseteq X-F$. Therefore, we have $Y-V\subseteq f(F)$ and $F\subseteq f^{-1}(Y-V)$, hence we obtain $f(F)=Y-V$, thus $f(F)$ is g^*s- closed set in (Y,σ) then f is irresolute g^*s-closed function.

The Composition of Some Types Of g^*s-Closed Functions:

In this section, we introduce the composition of some types of g^*s-closed function which studied in previous section.

Theorem (3.1):-

Let $(X,T),(Y,\sigma)$ and (Z,η) topological spaces and $f:(X,T)\longrightarrow(Y,\sigma)$ be g^*s-closed function.

(1) if $h:(Y,\sigma)\longrightarrow(Z,\eta)$ is strongly g^*s- closed then hof is closed function.

(2) if $h:(Y,\sigma)\longrightarrow(Z,\eta)$ is irresolute g^*s- closed then hof is g^*s- closed function.

Proof:-

(1) Let F be a closed set in (X,T) and since f is g^*s-closed function then f(F) is g^*s-closed set in Y also, since h is strongly g^*s – closed function thus, $h(f(F))$ is closed set in Z , but $h(f(F))=(hof)(F)$ Therefore; hof is closed function.

(2) Let F be a closed set in (X,T) and since f is g^*s-closed function then f(F) is g^*s-closed set in Y also, since h is irresolute g^*s – closed function thus, $h(f(F))$ is g^*s- closed set in Z , but $h(f(F))=(hof)(F)$ Therefore; hof is g^*s- closed function.

Theorem (3.2):-

Let $(X,T),(Y,\sigma)$ and (Z,η) topological spaces and $f:(X,T)\longrightarrow(Y,\sigma)$ be closed function.

(1) if $h:(Y,\sigma)\longrightarrow(Z,\eta)$ is g^*s- closed function then hof is g^*s- closed function.

(2) if $h:(Y,\sigma)\longrightarrow(Z,\eta)$ is strongly g^*s- closed function then hof is closed function.

(3) if $h:(Y,\sigma)\longrightarrow(Z,\eta)$ is irresolute g^*s- closed function hof is g^*s- closed function.

Proof:-

(1) Let F be a closed set in (X,T) and since f is closed function then f(F) is closed set in Y also, since h is g^*s – closed function thus, $h(f(F))$ is g^*s- closed set in Z , but $h(f(F))=(hof)(F)$ Therefore; hof is g^*s- closed function.

(2) Let F be a closed set in (X,T) and since f is closed function then f(F) is closed set in Y and by remark (1.4) we get f(F) is g^*s-closed set also, since h is strongly g^*s – closed function thus, $h(f(F))$ is closed set in Z , but $h(f(F))=(hof)(F)$ Therefore; hof is closed function.

(3) Let F be a closed set in (X,T) and since f is closed function then f(F) is closed set in Y and by remark (1.4) we get f(F) is g^*s-closed set also, since h is irresolute g^*s – closed function thus, $h(f(F))$ is g^*s- closed set in Z , but $h(f(F))=(hof)(F)$ Therefore; hof is g^*s- closed function.

Theorem (3.3):-

Let $(X,T),(Y,\sigma)$ and (Z,η) topological spaces and $f:(X,T)\longrightarrow(Y,\sigma)$ be strongly g^*s- closed function.

(1) if $h:(Y,\sigma)\longrightarrow(Z,\eta)$ is g^*s- closed function then hof is irresolute g^*s- closed function.

(2) if $h:(Y,\sigma)\longrightarrow(Z,\eta)$ is closed function then hof is strongly g^*s- closed function.

(3) if $h:(Y,\sigma)\longrightarrow(Z,\eta)$ is strongly g^*s- closed function then hof is strongly g^*s- closed function.

(4) if $h:(Y,\sigma)\longrightarrow(Z,\eta)$ is irresolute g^*s- closed function then hof is irresolute g^*s- closed function.

Proof:-

(1) Let F be a g^*s – closed set in (X,T) and since f is strongly g^*s – closed function then f(F) is closed set in Y also, since h is g^*s – closed function thus, $h(f(F))$ is g^*s- closed set in Z , but $h(f(F))=(hof)(F)$ Therefore; hof is irresolute g^*s- closed function.

(2) Let F be a g^*s – closed set in (X,T) and since f is strongly g^*s – closed function then f(F) is closed set in Y also, since h is closed function thus, $h(f(F))$ is closed set in Z , but $h(f(F))=(hof)(F)$ Therefore; hof is strongly g^*s- closed function..

(3) Let F be a g^*s – closed set in (X,T) and since f is strongly g^*s – closed function then f(F) is closed set in Y and by remark (1.4) we get f(F) is g^*s-closed set also, since h is strongly g^*s- closed function thus, $h(f(F))$ is closed set in Z , but $h(f(F))=(hof)(F)$ Therefore; hof is strongly g^*s- closed function.

(4) Let F be a g^*s – closed set in (X,T) and since f is strongly g^*s – closed function then f(F) is closed set in Y and by remark (1.4) we get f(F) is g^*s-closed set also, since h is irresolute g^*s- closed function thus, $h(f(F))$ is g^*s- closed set in Z , but $h(f(F))=(hof)(F)$ Therefore; hof is irresolute g^*s- closed function.

Theorem (3.4):-

Let $(X,T),(Y,\sigma)$ and (Z,η) topological spaces and $f:(X,T)\longrightarrow(Y,\sigma)$ be irresolute g^*s-closed function.

(1) if $h:(Y,\sigma)\longrightarrow(Z,\eta)$ is strongly g^*s- closed function then hof is strongly g^*s- closed function.

(2) if $h:(Y,\sigma)\longrightarrow(Z,\eta)$ is irresolute g^*s- closed function then hof is irresolute g^*s- closed function.

Proof:-

(1) Let F be a g^*s- closed set in (X,T) and since f is irresolute g^*s-closed function then f(F) is g^*s-closed set in Y also, since h is strongly g^*s – closed function thus,

$h(f(F))$ is closed set in Z , but $h(f(F)) = (hof)(F)$ Therefore; hof is strongly g^*s – closed function.

(2) Let F be a g^*s- closed set in (X,T) and since f is irresolute g^*s-closed function then f(F) is g^*s-closed set in Y also, since h is irresolute g^*s – closed function thus, $h(f(F))$ is closed set in Z , but $h(f(F)) = (hof)(F)$ Therefore; hof is irresolute g^*s – closed function.

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