Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(155-170)
ISSN 1992 - 0849

Coplanar Maneuvers Transfer for Mission Design with Lowest Δυ using Series Solution

[1]Mayada J. Hamwdi , [2]Wafaa A. Zaki , [3]Ahmed K. Izzet

[1]North Technical University / Technical College / KIRKUK / department of Control and Electronic Engineering

[1]mayada_jas_ham@yahoo.com

[2] University of KIRKUK / College of Sciences / department of physics

[2]hassanf99@yahoo.com

[3]North Technical University / Technical College / KIRKUK / department of Survey Engineering.

[3]ahmed_izzet@yahoo.com

Received date: 5 / 3 / 2015 **Accepted date: 8 / 3 / 2015**

ABSTRACT

Orbital maneuver transfer time is traditionally accomplished using direct numerical sampling to find the mission design with the lowest delta-υ requirements. The availability of explicit time series solutions to the Lambert orbit determination problem allows for the total delta-υ of a series of orbital maneuvers to be expressed as an algebraic function of only the individual transfer times. Series solution was applied for Hohmann transfer and Bi-elliptic transfer and comparing between Hohmann transfer and Bi-elliptic transfer for long distance. It has been concluded that Hohmann transfer is more appropriate when the ratio of radius of final orbit to initial orbit (R) is less than 11.94.

The purpose of this work is to minimize total full requirements, as well known that no refueling station in space, then using the computed Δυ for determining the mass propellant consumed Δm, at different specific impulse of the propellants, help us to carefully plane a mission to minimize the propellant mass carried on the rocket.

Keywords: *Coplanar Maneuvers. Series Solution. Hohmann Transfer Maneuver. Orbital Maneuver Optimization.*

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(155-170)
ISSN 1992 - 0849

نقل المناورات متحدة المستوى لتصميم رحلة فضائية مع أدنى Δv باستخدام حلول المتسلسلات

ميادة جاسم حمودي [1] ، وفاء حسن علي زكي [2] ، أحمد قادر عزت [3]
[1] الجامعة التقنية الشمالية، الكلية التقنية / كركوك / قسم هندسة تقنيات الألكترونيك والسيطرة
[1]mayada_jas_ham@yahoo.com
[2]جامعة كركوك / كلية العلوم / قسم الفيزياء
[2]hassanf99@yahoo.com
[3]الجامعة التقنية الشمالية، الكلية التقنية / كركوك / قسم هندسة تقنيات المساحة
[3]ahmed_izzet@yahoo.com

تاريخ استلام البحث: 5 / 3 / 2015　　　　تاريخ قبول البحث: 8 /3 / 2015

الملخص

تم أستخدام معادلات لامبرت الزمنية لإيجاد أقل تغيير في السرعة واللازمة للأنتقال بين المدارات الواقعة في نفس المستوي، وقد تم تطبيقها في أنتقالات هوهمان والأنتقالات الثنائية الأهليجية من مدارها الأولي الى مدارها النهائي وقد تبين من النتائج أن أنتقالات هوهمان هي الأنسب عندما تكون قيمة R (النسبة بين المدار النهائي والأولي) أقل من (11.94) بينما يكون الانتقال الأهليجي هو الأمثل لقيم R^* (النسبة بين المدار الأكبر الى المدار الأولي) الأكثر من (15.58).

وكانت الغاية من البحث هي الحصول على أقل تغير في السرعة ومنها حساب نسبة الكتلة الدافعة $\Delta m/m$ (التغير في كتلة المركبة إلى كتلتها الكلية) كدالة للدفع النوعي للوقود المستخدم I_{sp} وذلك لتقليل الوقودالمحمول في الرحلة الفضائية.

الكلمات الدالة: المناورات في نفس المستوي. حلول المتسلسلات. مناورات نقل هوهمان. تحقيق أمثلية المناورة المدارية.

1. INTRODUCTION

Orbital maneuvers are transferring a spacecraft from one orbit to another. Orbital changes can be dramatic, such as the transfer from a low-earth parking orbit to an interplanetary trajectory. They can also be quite small, as in the final stages of the rendezvous of one spacecraft with another [1]. Orbital maneuvering encompasses all orbital changes after insertion required to place a satellite in the orbit that choose. Changing orbits requires the firing of onboard rocket engines [2], [3]. Orbital maneuver optimization as a function of the transfer time is traditionally accomplished using either classical calculus of variations techniques for restricted cases, or by direct numerical sampling to minimize the magnitude of the required changes in velocity vectors [4]. The orbit transfer maneuvers considered accomplished by ideal impulsive velocity changes. It was assumed that the velocity required achieving certain mission objectives could be attained instantaneously. The concept of an impulsive velocity change can be exploited to provide an excellent rocket engine steering law which is applicable for a wide variety of orbit transfers [5]. One of the important characteristics of a space maneuver (and a space mission) is the change of characteristic velocity needed to realize the maneuver/mission, the so-called delta-υ ($\Delta\upsilon$). Any rocket or spacecraft possesses its ideal velocity- the maximal change of speed it can provide to its payload using the fuel onboard. So, delta-υ of any maneuver (and any mission in total) is limited by the ideal velocity of the vehicle. As it has already been mentioned, characteristic velocity should be treated as exponential cost of the mission in terms of mass. To provide heavier payloads and more complicated mission, it is critical to use the limited reserve of ideal velocity as efficiently as possible, thus seeking for maneuvers with smaller delta-υ [6]. Orbital transfers are usually achieved using the propulsion system onboard the spacecraft. Since the propellant mass on board is limited, it is very crucial for mission planning to estimate the propellant required for every transfer. The overall need for propulsion is usually expressed in terms of spacecraft total velocity change, or (delta-υ) budget as shown in Figure (1). The propulsion was assumed is applied impulsively, i.e. the velocity change will be acquired instantaneously. This assumption is reasonably valid for high-thrust propulsion.

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(155-170)
ISSN 1992 - 0849

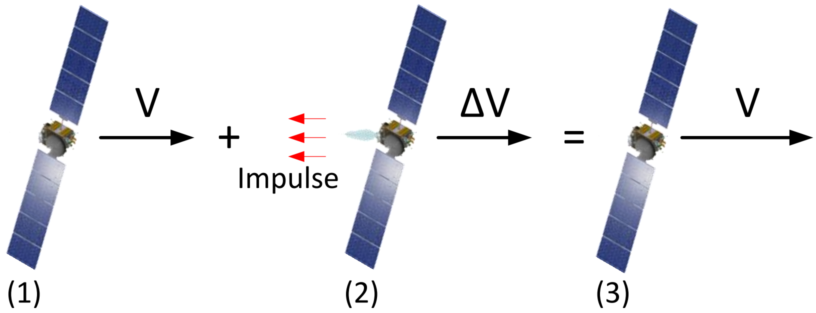

Figure (1): delta-υ budget [2].

Speed change needed for a particular change in orbit parameters. The direction and size of the delta-υ determines which orbit parameters are most affected, and by how much. The general definition of delta-υ is as follows:$\Delta \upsilon = \int_0^t |F|/m \, dt$ Where F is the instantaneous thrust, m is the instantaneous mass of spacecraft and t the time from the start of the maneuver/mission. The magnitude Δυ of the velocity increment is related to Δm, the mass of propellant consumed, by the formula $\Delta m \,/\, m = 1 - e^{(\Delta \upsilon/(I_{sp}\, g_0)}$ Where m is the mass of the spacecraft before the burn, g_0 is the sea-level standard acceleration of gravity [1].

The problem of two position vectors and the time of flight between them are usually known as Lambert's problem because Lambert first formed the solution [2]. In the Lambert problem, the initial position, final position, and the desired time for the transfer between the two positions is known. Solving Lambert’s problem should define the orbital elements of the desired transfer orbit (allowing the calculation of the velocities at the initial and final positions) [7]. The original Lambert's problem is one of the most important and popular topics in celestial mechanics. Several important authors worked on it, trying to find better ways to solve the numerical difficulties involved (Breakwell et al Battin; Lancaster et al; Lancaster & Blanchard; Herrick; Prussing; Sun & Vinh; Taff & Randall; Gooding). It can be defined as: "A Keplerian orbit, about a given gravitational center of force is to be found connecting two given points (P_1 and P_2) in a given time Δt"[8]. The Lambert problem may be expressed in terms of the Lagrange trajectory equations, which equate the transfer time t to transcendental functions of the unknown semi-major axis. Recently, time series solutions have been found to solve all orbital cases of the Lambert problem by analytically reversing the functional dependence of the Lagrange trajectory equations from a to t. The availability of the complete set of time series solutions for the Lambert orbit determination problem allows for the total Δυ

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(155-170)
ISSN 1992 - 0849

magnitude for a series of orbital maneuvers to be written as a single algebraic expression, an explicit function of only the individual transfer times [9].

The purpose of this work is to minimize total full requirements, as well known that no refueling station in space, then using the computed $\Delta \upsilon$ for determining the mass propellant consumed Δm, at different specific impulse of the propellants, help us to carefully plane a mission to minimize the propellant mass carried on the rocket.

2. COPLANAR MANEUVERS

Coplanar maneuvers don't change the orbital plane, as the name implies, so the initial and final orbits lie in the same plane. These maneuvers can change the orbit's size and shape and the location of the line of apsides. Coplanar burns are either tangential or non-tangential. The burns allowed doing two types of coplanar changes: Hohmann transfers (two tangential burns) and general transfers (two non-tangential burns). Consider the simple tangential transfer of Figure (2). The both orbits are tangent at the transfer point. As a result, the velocity vectors are parallel, and then the required change in velocity has been directly found as:

$$\Delta \upsilon = \upsilon_{final} - \upsilon_{initial} \quad \cdots\cdots\cdots (1)$$

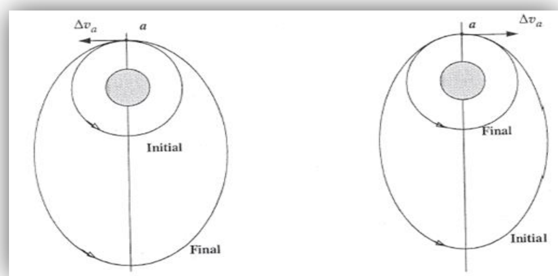

Figure (2): Tangential Orbit Transfer Theoretical approach [1]

The direction of firing can be determined by the sign of the change in velocity. For instance, the left orbit in Figure (2) has a positive $\Delta \upsilon$ because the velocity is added in the same direction as the original velocity vector. In the other orbit, the change in velocity is applied opposite to the direction of motion, and the satellite slows down to the circular orbit as shown.

The Hohmann transfer is the most energy efficient two-impulse maneuver for transferring between two coplanar circular orbits sharing a common focus [1]. The resulting Hohmann transfer orbit between two circular orbits is elliptical; the transfer between two elliptical orbits

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(155-170)
ISSN 1992 - 0849

may be circular or elliptical depending on the geometry of the initial and final orbits. Walter Hohmann proposed a theory which suggested the minimum-energy (and therefore most efficient) transfer could be achieved between orbits by using two tangential burns. Although his original work considered only transfer between circular orbits, other authors have explored transfers between coaxially aligned elliptical orbits and concluded the transfer energy was lowest using two tangential burns. The initial and final orbits will have velocities as shown from Figure (3).

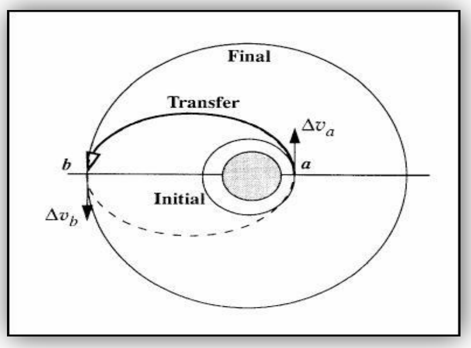

Figure (3): Hohmann transfer [1]

$$\upsilon_{initial} = \sqrt{\frac{\mu}{r_{initial}}} \quad \cdots\cdots\cdots (2)$$

$$\upsilon_{final} = \sqrt{\frac{\mu}{r_{final}}} \quad \cdots\cdots\cdots (3)$$

The velocities of initial and final transfer orbits are

$$\upsilon_{trans_a} = \sqrt{\frac{2\mu}{r_{initial}} - \frac{\mu}{a_{trans}}} \quad \cdots\cdots\cdots (4)$$

$$\upsilon_{trans_b} = \sqrt{\frac{2\mu}{r_{final}} - \frac{\mu}{a_{trans}}} \quad \cdots\cdots\cdots (5)$$

The changes in velocity for Hohmann transfer are

$$\Delta\upsilon_a = \upsilon_{trans_a} - \upsilon_{initial} \quad \cdots\cdots\cdots (6)$$

$$\Delta\upsilon_b = \upsilon_{final} - \upsilon_{trans_b} \quad \cdots\cdots\cdots (7)$$

$$\Delta\upsilon = |\Delta\upsilon_a| + |\Delta\upsilon_b| \quad \cdots\cdots\cdots (8)$$

The semi-major axis of the transfer is readily defined, the transfer time, τ_{trans} for the Hohmann transfer is simply half the orbital period of the transfer orbit

$$a_{trans} = \frac{r_{initial} + r_{final}}{2} \quad \cdots\cdots\cdots (9)$$

$$\tau_{trans} = \frac{p_{trans}}{2} = \pi\sqrt{\frac{a_{trans}^3}{\mu}} \quad \cdots\cdots\cdots (10)$$

If the pass from one circular orbit to another coplanar circular orbit is needed the radius of which is significantly larger, a more economical alternative to Hohmann transfer is the bi-parabolic transfer. It means that the spacecraft may be first send to the infinity providing it with the escape velocity, and then with an infinitely small impulse return it back along a parabolic path tangential to the target orbit [7]. A variant of the Hohmann transfer is a method which actually performs two Hohmann transfers in series. Figure (4) shows the Bi-elliptic transfer as a transfer into the transfer ellipse, τ_{trans_1}, at point a, followed by a transfer into a second transfer ellipse, τ_{trans_2}, at point b, and a transfer into the final orbit at point c. There is an intermediate circular orbit between the two elliptical transfer orbits. The middle velocity-change was simplifying calculation by determining it using the two elliptical orbits, rather than two separate circular orbits.

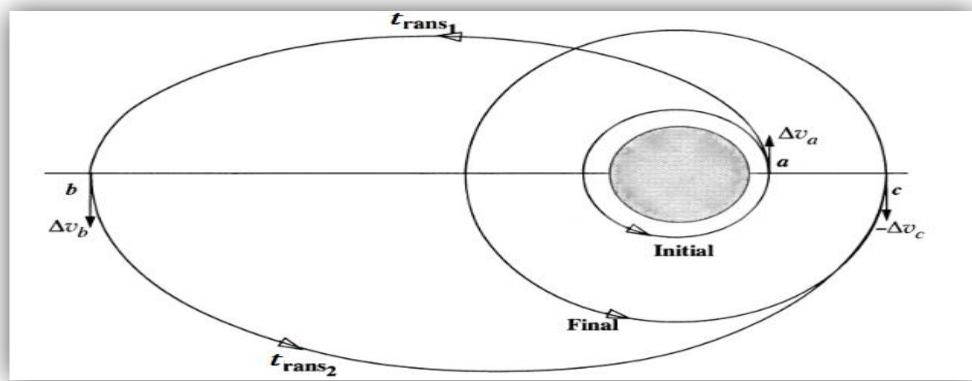

Figure (4): Bi-elliptic Transfer [1].

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(155-170)
ISSN 1992 - 0849

The transfer time for the maneuver is now the sum of the two Hohmann-like transfer times.

$$\tau_{trans} = \pi\sqrt{\frac{a_{trans_1}^3}{\mu}} + \pi\sqrt{\frac{a_{trans_2}^3}{\mu}} \quad \cdots\cdots\cdots (11)$$

Then:

$$a_{trans_1} = \frac{r_{initial} + r_b}{2} \quad \cdots\cdots\cdots (12)$$

And:

$$a_{trans_2} = \frac{r_b + r_{final}}{2} \quad \cdots\cdots\cdots (13)$$

The bi-elliptic transfer can reduce the total Δυ necessary for the transfer.

$$\Delta\upsilon = |\Delta\upsilon_a| + |\Delta\upsilon_b| + |\Delta\upsilon_c| \quad \cdots\cdots\cdots (14)$$

Where:

$$\Delta\upsilon_a = \upsilon_{trans1_a} - \upsilon_{initial} \quad \cdots\cdots\cdots (15)$$

$$\Delta\upsilon_b = \upsilon_{trans2_b} - \upsilon_{trans1_b} \quad \cdots\cdots\cdots (16)$$

$$\Delta\upsilon_c = \upsilon_{final} - \upsilon_{trans2_c} \quad \cdots\cdots\cdots (17)$$

3. SERIES SOLUTION OF LAMBERTS TIME FUNCTION

The Lambert problem may be expressed in terms of the Lagrange trajectory equations, which equate the transfer time t to transcendental functions of the unknown semi-major axis [9], [10]. Where the Lagrange coefficient functions $f_n, g_n, \dot{f}_n, \dot{g}_n$ are given by:

$$f = 1 - \frac{a}{r_0}(1 - \cos(\Delta E)) \quad \cdots\cdots\cdots (18)$$

$$g = (t - t_0) - \sqrt{\frac{a^3}{\mu}}(\Delta E - \sin(\Delta E)) \quad \cdots\cdots\cdots (19)$$

$$\dot{f} = \frac{-\sqrt{\mu a}\sin(\Delta E)}{r_0 r} \quad \cdots\cdots\cdots (20)$$

$$\dot{g} = 1 - \frac{a}{r}(1 - \cos(\Delta E)) \quad \cdots\cdots\cdots (21)$$

The change in eccentric anomaly ΔE can be found by using the Lagrange parameters α and β, depending on the type of orbit transfer. To obtain Lambert's Time Function the Integration of energy equation for a two-body orbit produces

$$t = \frac{1}{\sqrt{\mu}} \int_{s-c}^{s} \frac{rdr}{\sqrt{2r - r^2/a}} \quad \cdots\cdots\cdots (22)$$

Where r the radial distance between the two bodies, a is the semimajor axis, and t is the time. Lambert's Time Function for an elliptic trajectory depends on the transfer angle and a flight time less than the minimum energy transfer time as shown

$$t = \sqrt{\frac{a^3}{\mu}}[(\alpha - \sin\alpha) \pm (\beta - \sin\beta)] \quad \cdots\cdots\cdots (23)$$

To find the unknown semi-major axis a first define the quantity $T = \Delta t/t_p - 1$ as a non-dimensional time parameter, where Δt is the desired flight time and t_p is the known parabolic flight time between the two given position vectors

$$t_p = \frac{2}{3}\sqrt{\frac{s^3}{\mu}}\left\{1 - \left(\frac{s-c}{s}\right)^{\frac{3}{2}}\right\} \quad \cdots\cdots\cdots (24)$$

After some algebraic manipulation, Lambert's Time Function has been expressed by using hypergeometric series definition for $\sin^{-1} x$ and $\sqrt{1 - x^2}$ [7]. Then Lambert's time function could be expressed as [2]

$$T = \sum_{i=1}^{\infty} \frac{\left(1 - \left(\frac{s-c}{s}\right)^{i+\frac{3}{2}}\right)}{\left(1 - \left(\frac{s-c}{s}\right)^{\frac{3}{2}}\right)} \frac{\left(\frac{1}{2}\right)_i \left(\frac{3}{2}\right)_i}{\left(\frac{5}{2}\right)_i i!} \left(\frac{s}{2a}\right)^i = \sum_{i=1}^{\infty} A_i \left(\frac{s}{2a}\right)^i \quad \cdots\cdots\cdots (25)$$

Where:

$$A_i = \frac{\left(1 - \left(\frac{s-c}{s}\right)^{i+\frac{3}{2}}\right)}{\left(1 - \left(\frac{s-c}{s}\right)^{\frac{3}{2}}\right)} \frac{\left(\frac{1}{2}\right)_i \left(\frac{3}{2}\right)_i}{\left(\frac{5}{2}\right)_i i!}$$

The terms$(x)_i$ are Pochhammer symbols, which are defined by $(x)_i = x(x+1) \ldots (x+i-1)$

To determine the semi-major axis, the reciprocal of the series may be found [10], then

$$a = \frac{s}{2}\sum_{i=1}^{\infty} B_i T^{i-2} \quad \cdots\cdots\cdots (26)$$

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(155-170)
ISSN 1992 - 0849

The B_i coefficients can be treated as a vector, where $B = QA$

And the elements of Q could be found from the following recursive expressions [2]:

$$Q_{(1,1)} = A_1^{-1}$$

$$Q_{(1,1)} = \sum_{k=1}^{i-1} Q_{(i-k,\ j-1)} \quad (i = 2, 3, 4, \dots),\ \ 1 < j \le i$$

$$Q_{(1,1)} = \sum_{k=1}^{i-1} \left(\frac{-1}{A_1}\right) Q_{(i,\ k+1)} A_{(k+1)}$$

The total delta-υ that required for transformation is the sum of the n vector differences. The sum of the magnitudes of the total delta-υ is then given by

$$\sum_{n=1}^{total} \Delta\upsilon_n = \sqrt{(\dot{x}_{2n-1} - \dot{x}_{2n-2})^2 + (\dot{y}_{2n-1} - \dot{y}_{2n-2})^2 + (\dot{z}_{2n-1} - \dot{z}_{2n-2})^2} \cdots (27)$$

Velocity and there component is given by

$$\dot{x}_{2n-1} = \frac{1}{g_n} x_{n+1} - \frac{f_n}{g_n} x_n \quad \cdots\cdots\cdots (28)$$

$$\dot{y}_{2n-1} = \frac{1}{g_n} y_{n+1} - \frac{f_n}{g_n} y_n \quad \cdots\cdots\cdots (29)$$

$$\dot{z}_{2n-1} = \frac{1}{g_n} z_{n+1} - \frac{f_n}{g_n} z_n \quad \cdots\cdots\cdots (30)$$

$$\dot{x}_{2n} = \dot{f}_n x_n + \dot{g}_n \dot{x}_{2n-1} \quad \cdots\cdots\cdots (31)$$

$$\dot{y}_{2n} = \dot{f}_n y_n + \dot{g}_n \dot{y}_{2n-1} \quad \cdots\cdots\cdots (32)$$

$$\dot{z}_{2n} = \dot{f}_n z_n + \dot{g}_n \dot{z}_{2n-1} \quad \cdots\cdots\cdots (33)$$

The goal is how to minimize the Δυ to obtain the transfer orbit with less fuel consumed as well as Δt is the time of transfer is not very long.

Let J is a function of the velocity vector $\vec{\upsilon}$, therefore

$$J = J(\Delta\upsilon) \quad \cdots\cdots\cdots (34)$$

4. APPLICATION AND DISCUSSION

For coplanar transfer the vectors input data are (r_1= 8839.683 km and r_2= 18689.09 km) and using the Lambert's theorem at a given flight time the velocity computed and given in Table (1). There is a unique value for semi-major axis associated with the arc of conic section, express the major axis in terms of transfer time to solve orbital cases, Table (2) show the

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(155-170)
ISSN 1992 - 0849

computed velocity component and the total minimum Δυ by using the analytical optimization with assistant of Matlab program. Table (3) show the computed ($\Delta m/m$) at different I_{sp}.

Table (1): position vector for coplanar transfer

position vector components (km)
$\mathbf{r_1} = 6250.6\ \hat{\mathbf{I}} + 6250.6\ \hat{\mathbf{J}} + 0\ \hat{\mathbf{k}}$
$\mathbf{r_2} = -\ 18372\ \hat{\mathbf{I}} -\ 3428.1\ \hat{\mathbf{J}} + 0\ \hat{\mathbf{k}}$

Table (2): Velocity Computed at given flight time

velocity vector components (km/s)	flight time (sec)
$\upsilon_1 = -\ 8.135\ \hat{\mathbf{I}} + 4.05064\ \hat{\mathbf{J}} + 0.0\ \hat{\mathbf{k}}$	3600
$\upsilon_2 = -\ 3.47465\ \hat{\mathbf{I}} - 4.7942\ \hat{\mathbf{J}} - 0.0\ \hat{\mathbf{k}}$	
$\Delta\upsilon = 4.66035\ \hat{\mathbf{I}} - 8.84484\ \hat{\mathbf{J}} - 0.0\ \hat{\mathbf{k}}$	

Table (3): computed the minimum change in velocity for the transfer and the time meets it

velocity vector components (km/s)	Transfer time (sec)	$\emptyset_{fpa}$(deg)	ε (km^2/s^2)
$\upsilon_1 = -\ 8.76093\ \hat{\mathbf{I}} + 3.66464\ \hat{\mathbf{J}} + 0.0\ \hat{\mathbf{k}}$	3302	-22.875	-3.799
$\upsilon_2 = -\ 4.19057\ \hat{\mathbf{I}} - 5.00941\ \hat{\mathbf{J}} + 0.0\ \hat{\mathbf{k}}$			
$\Delta\upsilon = 4.57036\ \hat{\mathbf{I}} - 8.67405\ \hat{\mathbf{J}} - 0.0\ \hat{\mathbf{k}}$			

Table (4): Specific impulses and the change in ($\Delta m/m$)

($\Delta m/m$)	Propellant	I_{sp}(sec)
0.9936	Cold gas	50
0.5817	Solid propellant	290
0.4262	Liquid oxygen/liquid hydrogen	455
0.66679	Monopropellant	230
0.557527	Nitric acid/monomethylhydrazine	310

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(155-170)
ISSN 1992 - 0849

The purpose of this work is to minimize total full requirements, as well known that no refueling station in space, then using the computed Δυ in Table (3) for determining the mass propellant consumed Δm, at different specific impulse of the propellants, help us to carefully plane a mission to minimize the propellant mass carried on the rocket, Table (4) show the computed $(\Delta m/m)$ at different I_{sp}.

From Table (4) using the propellant Liquid Oxygen /Liquid hydrogen at I_{sp} 455 second and assumer the mass of spacecraft equal to 2000 kg , the computed.

$$\boldsymbol{\Delta m = 0.426 \times 2000\ kg = 852\ kg}$$

While if we use Cold gas with I_{sp}=50 sec

We get Δm = 0.99*2000kg = 1980kg

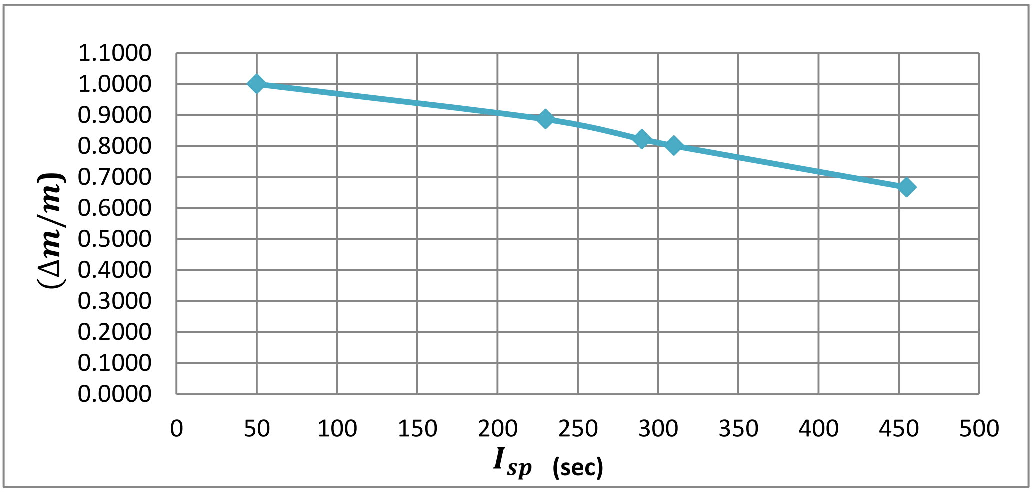

Figure (5): Relationship I_{sp}in units (sec) and $\Delta m/m$ propellant ratio at Δυ=9.8 km/sec

Figure (5) represent, the relationship between the I_{sp} Specific Impulse of the propellants in unit of second and $\Delta m/m$ (propellant mass ratio). As seen from figure at constant Δυ the $\Delta m/m$ (propellant mass ratio) decreases with increasing the propellant Specific Impulse therefore this lead to minimize the propellant mass carried all aloft in favor of payload.

Tables (5,6) list position vectors in two different orbits and the computed minimum delta-υ using series solution with the suitable equations (4) to (8) for Hohmann and (14) to (16) for Bi- elliptic transfer.

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(155-170)
ISSN 1992 - 0849

Table (5): Hohmann transfer

position vector components (km)	(delta-υ) (km/s)	Flight time (sec)	*R*
$\mathbf{r}_{initial} = 8000\,\hat{\mathbf{i}} + 0.0\,\hat{\mathbf{j}} + 0.0\hat{\mathbf{k}}$	2.438	42840	5.2
$\mathbf{r}_{final} = -42166\,\hat{\mathbf{i}} - 0.0\,\hat{\mathbf{j}} + 0.0\,\hat{\mathbf{k}}$			

Table (6): Bi-elliptic transfer

position vector components (km)	(delta-υ) (km / s)	Flight time (sec)	R^*
$\mathbf{r}_{initial} = 8000\,\hat{\mathbf{i}} + 0.0\,\hat{\mathbf{j}} + 0.0\hat{\mathbf{k}}$	6.8	86400	7.06
$\mathbf{r}_b = 56500\,\hat{\mathbf{i}} - 0.0\,\hat{\mathbf{j}} + 0.0$			
$\mathbf{r}_{final} = -42166\,\hat{\mathbf{i}} - 0.0\,\hat{\mathbf{j}} + 0.0\,\hat{\mathbf{k}}$			

The Hohmann transfer is the minimum-energy transfer between most but not all coplanar orbits. In some cases, the Bi-elliptic transfer may use less energy [2]. The computed change in velocity show that the less change in velocity required in Hohmann than that in Bi-elliptic transfer.

In Tables (7,8,9) list position vector sin two different or bits with *R* ratio much than15.58, the change in velocity required by Bi-elliptic transfer is smaller than that computed in Hohmann transfer also, therefore from the above concludes that for *R* ratio less than 11.94 the Hohmann transfer is Appropriate, while for *R* greater than15.58 the Bi-elliptic saves Δυ with an extreme in time.

Table (7): Hohmann transfer

position vector components (km)	(delta-υ) (km/s)	Flight time (sec)	Semi-major axis (km)	*R*
$\mathbf{r}_{initial} = 6569.3\,\hat{\mathbf{i}} + 0.0\,\hat{\mathbf{j}} + 0.0\hat{\mathbf{k}}$	3.17	1.17×10^6	282718	58.2
$\mathbf{r}_{final} = 282688\,\hat{\mathbf{i}} - 0.0\,\hat{\mathbf{j}} + 0.0\,\hat{\mathbf{k}}$				

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(155-170)
ISSN 1992 - 0849

Table (8): Bi-elliptic transfer

position vector components (km)	(delta-υ) (km/s)	Flight time (sec)	Semi-major axis (km)	R^*
$\mathbf{r}_{initial} = 6569.3\,\hat{\mathbf{i}} + 0.0\,\hat{\mathbf{j}} + 0.0\hat{\mathbf{k}}$	0.987	2.43×10^6	391344	60.8
$\mathbf{r}_b = 400000\,\hat{\mathbf{i}} - 0.0\,\hat{\mathbf{j}} + 0.0$				
$\mathbf{r}_{final} = 282688\,\hat{\mathbf{i}} - 0.0\,\hat{\mathbf{j}} + 0.0\,\hat{\mathbf{k}}$				

Table (9): Bi-elliptic transfer

position vector components (km)	(delta-υ) (km/s)	Flight time (sec)	Semi-major axis (km)	R^*
$\mathbf{r}_{initial} = 6569.3\,\hat{\mathbf{i}} + 0.0\,\hat{\mathbf{j}} + 0.0\hat{\mathbf{k}}$	0.794	3.06×10^6	456344	80.6
$\mathbf{r}_b = 529485.58\,\hat{\mathbf{i}} - 0.0\,\hat{\mathbf{j}} + 0.0$				
$\mathbf{r}_{final} = 282688\,\hat{\mathbf{i}} - 0.0\,\hat{\mathbf{j}} + 0.0\,\hat{\mathbf{k}}$				

The Bi-elliptic transfer requires much longer transfer time computed to the Hohmann transfer. However Bi-elliptic is more efficient for long distance the change in velocity for long distance orbit transfer at different R^* (the ratio of apogee radius of transfer orbit to initial orbit) in Bi-elliptic at the same value of the ration R.

The results in Table (7, 8, and 9) show that the Bi-elliptic transfer requires much longer transfer time computed to the Hohmann transfer. However Bi-elliptic is more efficient for long distance the change in velocity for long distance orbit transfer at different R^* (the ratio of apogee radius of transfer orbit to initial orbit) in Bi-elliptic at the same value of the ration R. The results agree with concept that when R^* is increase the $\Delta\upsilon$ decrease as given in the reference [1].

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(155-170)
ISSN 1992 - 0849

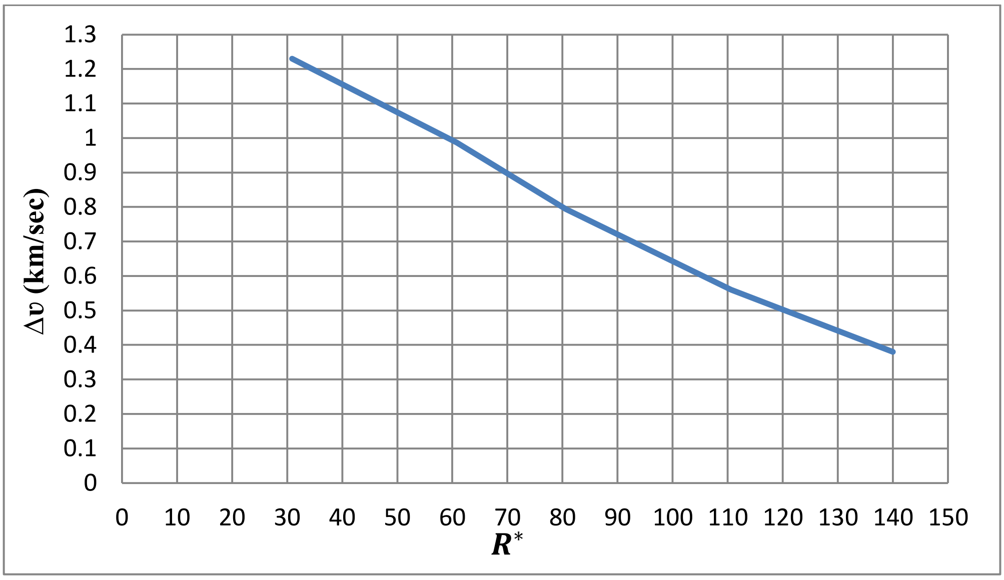

Figure (6): the change in velocity for Bi-elliptic transfer orbit at different R^*

Figure (6) represent the relationship between the minimum changes in velocity Δυ with respect to R^* show that the minimum changes in velocity Δυ decreases with the increasing R^* which is agree with the concept that the Bi-elliptic transfer perform better as increasing R^* ratio.

5. CONCLUSION

With a complete set of series solutions available for every case of Lambert's Theorem, it is possible to apply it for coplanar orbit transfer and gives good results in magnitude of change of velocity. Using analytical methods for multiple-impulse missions to minimize total fuel requirements. Computed change in velocity for different types of maneuvers (Hohmann and Bi-elliptic). The results show that the Hohmann transfer is saves fuel by reduce the change in velocity and extreme in time.

REFERENCES

[1] D. A. Vallado Fundamentals of Astrodynamics and Applications California 2007, 3rd edn. Microcosm Press.

[2] D. C. Howard Orbital Mechanics for Engineering Students 2005, 2nd edn.

[3] P. R. Escobal 66 Methods of Astrodynamics 1979. New York: John Wiley & Sons. Reprint: Malabar FL: Krieger Publishing Co.

[4] L. G. Taff Celestial Mechanics a Computational Guide for the Practitioner 1985, Johon Wilyey and Sons.

[5] R. H. Battin an Introduction to the Mathematics and Methods of Astrodynamics 1987, AIAA Education series, New York.

[6] B. R. Roger., D.D. Mueller, and J.E. White, Fundamentals of Astrodynamics, 1971, New Dover Publications, New York.

[7] R.H. Battin A New Solution for Lambert's Problem, 1968, Proceedings of the XIX International Astronautical Congress, Oxford, Vol. 2, pp. 131-150.

[8] W. Hohmann, Die Erreichbarkeit der Himmelskörper 1960, (The Attainability of Heavenly Bodies). NASA Technical Translation TTF44. Washington, DC: National Aeronautics and Space Administration November

[9] J.D. Thorne, Orbital maneuver optimization using time-explicit power series 2011 Journal of Celest Mech Dyn Astr 110:31–42

[10] W. P. Michael A numerical investigation of the series reversion/inversion and series reversion of Lambert's Time Function Thesis, December, 1990. School of Engineering of the Air Force Institute of Technology (AU), Captain, USA F.

AUTHOR

Mayada Jasim Hamwdi Abdullah: is lecturer in the digital control laboratory in the Control and Electronic Eng. Department in Technical College \ Kirkuk. She received her B. Sc. in Applied Physics in 1996 from University of Technology \ Iraq. She is a M. Sc. Student in the College of Science, University of Kirkuk \ Iraq.