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A Comparative Study between Analytical and Numerical Solution of Unsteady State One-Dimensional Heat Transfer for Different Materials

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ABSTRACT

The present investigation includes a comparative study of numerical solution methods with analytical solution for one dimensional transient conduction problems. For this purpose, six different materials were selected and distributed in two groups (metallic and nonmetallic) materials. Group of metallic materials were such as pure copper, pure Aluminum and Iron, whereas a group of nonmetallic materials such as sandstone, concrete and building brick. Four numerical solution methods were selected as (finite difference, thermal capacitance, and finite element) and also Ansys program to compare them with analytical solution. The simulation results showed that the error ratio between numerical solutions and analytical solution would be higher at the transient period in all materials, while this ratio decreased when the case of the materials would reaches steady state, and the results indicated that the materials having high thermal conductivity would give less error ratio in the beginning of simulation than other materials which had low thermal conductivity.

Keywords: *one dimensional heat transfer, transient heat transfer, numerical solution.*

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دراسة مقارنة بين الحلول التحليلية والعددية لانتقال الحرارة عند الحالة الغير مستقرة و أحادي البعد في المواد المختلفة

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الملخص

الدراسة الحالية تشمل مقارنة الحل التحليلي لمعادلة انتقال الحرارة بالتوصيل أحادي البعد عند الحالة المتغيرة مع الزمن بالطرق الحلول العددية لها، ولهذا الغرض فقد تم اختيار ست مواد مختلفة وزعت على مجموعتين (مواد معدنية و مواد غير معدنية). حيث ان مجموعة المواد المعدنية تضم النحاس النقي، والألمنيوم النقي و الحديد، بينما مجموعة المواد غير معدنية تضم الحجر، الخرسانة و طابوق البناء، أما اختيار الطرق العددية الأربعة كانت (الفروق المحددة ،السعة الحرارية ، العناصر المحددة والبرنامج الهندسي Ansys) لأجل لمقارنتها بالحل التحليلي. أظهرت نتائج المحاكاة بان نسبة خطاء بين الحلول العددية والحل التحليلي تكون عالية خلال فترة عدم وصول المواد الحالة الاستقرارية ، بينما تقل هذه النسبة عند وصولها إلى الحالة الاستقرارية، كذلك تشير النتائج إلى إن نسبة خطاء عند البدء بالمحاكاة في المواد التي لها موصلية حرارية عالية اقل من الأخرى التي لها موصلية حرارية اقل.

الكلمات الدالة: انتقال الحرارة أحادي البعد، انتقال الحرارة الغير مستقرة، الحل العددي.

1.INTRODUCTION

Heat transfer is the study of thermal energy transport within a medium or among neighboring media by molecular interaction, fluid motion, and electro-magnetic waves, resulting from a spatial variation in temperature. This variation in temperature is governed by the principle of energy conservation, which when applied to a control volume or a control mass, states that the sum of the flow of energy and heat across the system, the work done on the system, and the energy stored and converted within the system, is zero. Heat transfer finds application in many important areas, namely design of thermal and nuclear power plants including heat engines, steam generators, condensers and other heat exchange equipment's, catalytic convertors, heat shields for space vehicles, furnaces, electronic equipment's etc., internal combustion engines, refrigeration and air conditioning units, design of cooling systems for electric motors generators and transformers, heating and cooling of fluids etc. in chemical operations, construction of dams and structures, minimization of building heat losses using improved insulation techniques, thermal control of space vehicles, heat treatment of metals, dispersion of atmospheric pollutants. A thermal system contains matter or substance and this substance may change by transformation or by exchange of mass with the surroundings. To perform a thermal analysis of a system, we need to use thermodynamics, which allows for quantitative description of the substance. This is done by defining the boundaries of the system, applying the conservation principles, and examining how the system participates in thermal energy exchange and conversion [1].Conduction is one of the three principle heat transfer modes, the others being convection and radiation, it is customarily distinguished as being an energy diffusion process in materials which do not contain molecular convection. Kinetic energy is exchanged molecules resulting in a net transfer between regions of different energy levels, these energy levels are commonly called temperature. The process of heat transfer in materials has been studied for many centuries, even early Greek philosophers, such as Lucretius(C 98-55 B. C.), meditated on the subject and recorded their conclusions, much later, the famous mathematical physicist, Josef B. j. Fourier (1768-1830),developed a mathematical expression that become the basis of practically all heat conduction solution. He postulated that the local heat flux rate in a material is proportional to the local temperature gradient in the direction of heat flow [2]:

$$q_x \propto \frac{dT}{dx} \qquad ..(1)$$

where q_x is the heat flow in the *x*-direction per unit area ($W.m^{-2}$) and T is temperature (^oC). Material properties are accounted for by including a proportionality constant,

$$q_x = -k\frac{\partial T}{\partial x} \qquad ..(2)$$

where the constant k is called thermal conductivity ($W.m^{-1}.^oC^{-1}$) and the minus sign must be included to satisfy the second law of thermodynamic. Equation (2) is called Fourier's law of heat conduction for homogeneous isotropic continua.

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(273-291)
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There were more studies developed to show the accuracy of the numerical methods solution with comparison to the analytical solution. A brief summary provided below. An extensive analytic studies has been carried out by Liet al.[3] to solve rapid transient heat conduction in multilayer material under pulsed heating numerically based on a hyperbolic heat conduction equation. Mydin M.A.O.[4] developed one dimensional finite deference model which can be used to solve transient heat conduction problems in multilayer panels. Blomberge T. [5] have utilized numerical modeling techniques to solve two dimensional transient heat conduction equation for steel ingot process and compared the results with result of analytical solution. Dhawan and Kumar [6] are studied temperature decay in an aluminum plate is observed using Galerkin finite element method for 2-Dimensional transient heat conduction equation. Temperature variation is studied using unconditionally stable first order and second order accurate schemes - backward Euler and modified Crank-Nicholson respectively.

2.MATHEMATICAL DESCRIPTION

Many heat conduction problems encountered in engineering applications involve time as an independent variable. The goal of analysis is to determine the variation of the temperature as a function of time and position. The general form of conductive heat transfer, in case of constant physical properties as ρ , *k* ..etc. may be written as [7]:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau} \qquad ..(3)$$

$$\alpha=\frac{k}{\rho c} \qquad ..(4)$$

where *x*, *y* ,*z* are space coordinates in Cartesian system, *c* is specific heat J.kg^{-1}. ^{o}C, ρ is density (kg.m^{-3}), τ is time (sec) and $\dot{q}$ is heat generated per unit volume (W.m^{-3}).

Examining equation (3), one can distinguish between three possible situations for the case of no heat generated inside the body:

a) Temperature variation throughout the body can be neglected, i.e. the temperature distribution is uniform everywhere, but is varied with time, then:

$T=f(\tau)$

In such case Biot number $(Bi)<0.1$ and it is known as lumped system, where $Bi=\frac{h\,\Delta x}{k}$

b) Temperature is function of time and heat is transferred in one-direction, in such case $Bi > 0.1$, and satisfied the condition of large wall, long cylinder, sphere and semi-infinite solid. Then transient conduction can be simplified in x-direction according to equation (3) to the following form as:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau} \quad ..(5)$$

Equation (5) is one-dimensional transient heat conduction equation. Temperature in this case is function of both *time* and *x*-direction:

$$T = f(x, \tau)$$

c) Temperature is function of time and heat is transferred in more than one direction. In such case, one speaks about Transient heat conduction in multidimensional system. Temperature, in such case, can be express as:

$$T = T(x, y, \tau) \ \text{or}\ T = (x, y, z, \tau)$$

For the solution of equation (5), we need two boundary conditions in x-direction and one initial condition. The analytical solution of equation (5) is useful for case of simple geometry. Due to complexity of many geometric shapes and boundary conditions of practical interest, analytical solutions to equation (5) are not always possible. In this study, analytical solution was used for solution of one-dimensional transient heat conduction equation on a sample as shown in Figure (1) for different materials and comparing the results with those of several kinds of the numerical solutions which were used to the solution of afore mentioned equation through the time needed to change the case of the sample from unsteady condition to steady state.

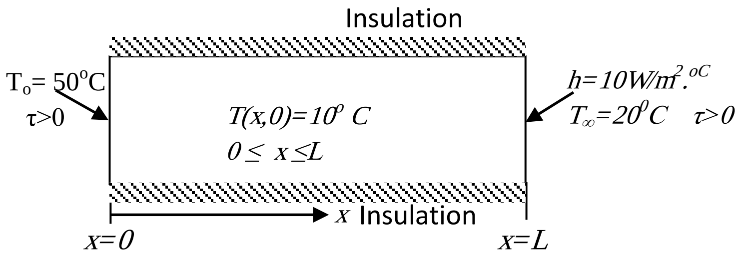

Figure (1): Heat conduction in a sample

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(273-291)
ISSN 1992 – 0849

2.1. Analytical solution

Figure (1) shows the shape and details of the boundary conditions on the selected sample. To calculate the value of the temperature at any x-position for any time, we introduce the method of separated variables to solve equation (5), under homogeneous boundary conditions. The sample is insulated from upper and lower surfaces and the sample initially at a temperature $T = F(x)$, and times $\tau > 0$ the boundary surface at $x = 0$ is kept at (T_o) while the boundary at $x = L$ dissipates heat by convection with heat transfer coefficient (h) and ambient temperature (T_∞). The mathematical formulation of this problem as follow is [8]:

$$\frac{\partial^2 T(x,\tau)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T(x,\tau)}{\partial \tau} \qquad in \quad 0 < x < L,\ \tau > 0 \qquad ..(6\text{-}a)$$

Boundary conditions:

1) $$\frac{\partial T}{\partial x} = 0 \qquad at \quad x = 0, \quad \tau > 0 \qquad ..(6\text{-}b)$$

2) $$k\frac{\partial T}{\partial x} + hT = 0 \qquad at \quad x = L, \quad \tau > 0 \qquad ..(6\text{-}c)$$

Initial condition:

3) $$T = F(x) \qquad for \quad \tau = 0, \quad 0 \le x \le L \qquad ..(6\text{-}d)$$

So that the solution to eq. (6-a) yields the solution for temperature distribution equation in the medium eq. (7) as a function of position and time.

$$T(x,\tau) = \sum_{m=1}^{\infty} e^{-\alpha\beta_m^2\tau}\frac{1}{N(\beta_m)}X(\beta_m,x)\int_0^L X(\beta_m,x)F(x)dx$$

..(7)

where β_m is the separating constant, and

$$\frac{1}{X(x)}\frac{d^2X(x)}{dx^2} = \mp\beta_m^2$$

..(8)

2.2. Numerical solution methods:

a)Finite Difference Method (FDM).

Finite difference method is a relatively simple technique which can provide approximate numerical solutions to many practical cases. FDM replaces derivative expressions in Eq. (5) with approximately equivalent partial difference quotients. Two approaches can be taken to transform the partial differential equation into finite difference equation as (*Explicit formulation and implicit formulation*).The two techniques often result in the same finite

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(273-291)
ISSN 1992 – 0849

difference equations [9]. Consider a solid body divided into m increments in x-direction as shown in Figure (2).by using explicit techniques to determine nodal equation as follows:

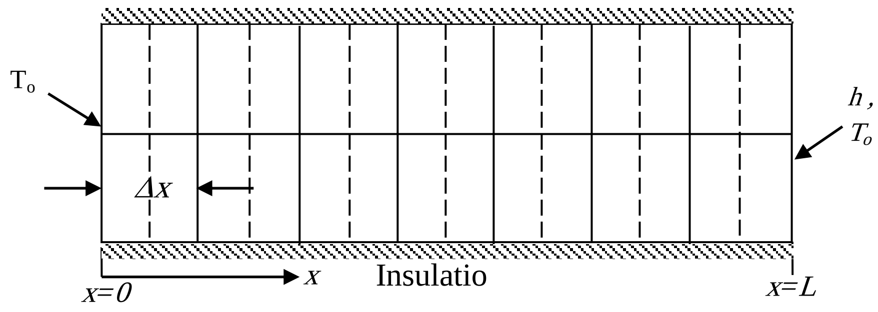

Figure (2): Schematic of mathematic model

$$\Delta x = \frac{L}{m} \qquad ..(9)$$

Nodal relation for node on left surface as shown in Figure (3) is:

$$T_m^{p+1} = Fo\left[2Bi \cdot T_\infty + 2T_{m-1}^p + \left(\frac{1}{Fo} - 2Bi - 2\right)T_m^p\right] \qquad ..(10)$$

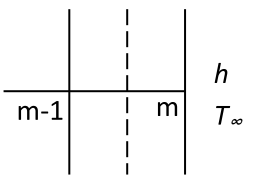

Figure (3): Nomenclature for nodal equation with convection boundary

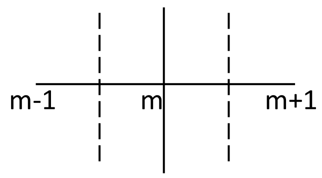

Figure (4): Nomenclature for numerical solution for interior

and for interior nodes as shown in Figure (4).

$$T_m^{p+1} = Fo\left(T_{m+1}^p + T_{m-1}^p\right) + [1 - 2Fo]T_m^p \qquad ..(11)$$

where Δx is thickness of the node (m), L is length of the sample (m), Fo is Fourier number and equal to $\frac{\alpha\,\Delta\tau}{(\Delta x)^2}$.

The aforementioned method is implicit and simultaneous equations for all node points need to be solved at each time step. For stability and to ensure convergence of numerical solution, all selections of the parameter (Fo) must be restricted according to

$$Fo \le \frac{1}{2(Bi+1)} \quad \text{for equation (10)} \qquad ..(12)$$

and

$Fo \le \frac{1}{2}$ for equation (11) ..(13)

The valve of Fo must be choose to achieve the result of equations (12 and 13).

b)Thermal resistance and capacity formulation.

In this method resistance concept used for writing the heat transfer between nodes. In transient condition, the net heat transfer into the node must be evidenced as an increase in internal energy of the element. Each volume element behaves like small lumped capacity, and the interaction of all elements determines the behavior of the solid during a transient process [10]. So the internal energy of node (m) as shown in Figure (4) can be expressed in terms of specific heat and temperature, the rate of change with time is approximated by

$$\frac{\Delta E}{\Delta \tau} = C_m \frac{T_m^{p+1} - T_m^p}{\Delta \tau} \quad ..(14)$$

Where ΔE is change in internal energy (J), C is thermal capacity (J.^{o}C^{-1}) and equal to($\rho c \Delta V$), and $\Delta V$is the volume of the element, then the general resistance capacity formulation for energy balance on the node (m) in the explicit formulation is

$$\frac{T_{m+1}^p - T_m^p}{R_{m+1}} + \frac{T_{m-1}^p - T_m^p}{R_{m-1}} = C_m \frac{T_m^{p+1} - T_m^p}{\Delta \tau} \quad ..(15)$$

$$T_m^{p+1} = \left[\frac{T_{m+1}^p}{R_{m+1}} + \frac{T_{m-1}^p}{R_{m-1}}\right]\frac{\Delta \tau}{C_m} + \left\{1 - \frac{\Delta \tau}{C_m}\left(\frac{1}{R_{m+1}} + \frac{1}{R_{m-1}}\right)\right\} T_m^p \quad ..(16)$$

Where:

$$R_{m+1} = R_{m-1} = \frac{\Delta x}{kA} \quad ..(17)$$

and resistance capacity formulation for convection boundary condition as shown in Figure (3) is:

$$\frac{T_{m-1}^p - T_m^p}{R_{m-1}} + \frac{T_\infty - T_m^p}{R_\infty} = \frac{1}{2} C_m \frac{T_m^{p+1} - T_m^p}{\Delta \tau} \quad ..(18)$$

$$T_m^{p+1} = 2\left[\frac{T_{m-1}^p}{R_{m-1}} + \frac{T_\infty}{R_\infty}\right]\frac{\Delta \tau}{C_m} + \left\{1 - \frac{2\,\Delta \tau}{C_m}\left(\frac{1}{R_{m-1}} + \frac{1}{R_\infty}\right)\right\} T_m^p \quad ..(19)$$

where

$$R_\infty = \frac{1}{hA} \quad ..(20)$$

The stability requirement in an explicit formulation, the value of ($\Delta \tau$) must be chosen the minimum value from

$$\Delta \tau \le \frac{C_m}{\sum \frac{1}{R_{th}}} \quad ..(21a)$$

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(273-291)
ISSN 1992 – 0849

or $\Delta\tau \leq \frac{2\,C_m}{\sum\frac{1}{R_{th}}}$ ..(21b)

c)Finite Element Method (FEM)

Application of the finite element method for solution of equation (5) proceeds by dividing the problem domain into finite-length, one - dimensional elements and discrediting the temperature distribution within each element as [11]

$$T(x,\tau) = N_1(x)T_1(\tau) + N_2(x)T_2(\tau) = [N(x)]\{T(\tau)\} \quad ..(22)$$

By using Garelrkins finite element method for the equation (5) to obtain the residual equations

$$\int_{x1}^{x2}\left(k\frac{\partial^2 T}{\partial x^2} - \rho c\frac{\partial T}{\partial \tau}\right)N_i(x)A dx i{=}1{,}2 \quad ..(23)$$

Noting that

$$N_i\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(N_i\frac{\partial T}{\partial x}\right) - \frac{\partial N_i}{\partial x}\frac{\partial T}{\partial x} \quad ..(24)$$

the residual equations can be rearranged and expressed as

$$\int_{x1}^{x2} k\frac{\partial N_1}{\partial x}\frac{\partial T}{\partial x}Adz + \int_{x1}^{x2}\rho c\frac{\partial T}{\partial \tau}N_i A dx = \int_{x1}^{x2}\frac{\partial}{\partial x}\left(N_i\frac{\partial T}{\partial x}\right)A dx i{=}1{,}2 \quad ..(25)$$

And equation (21) can be written in detailed matrix form as

$$\rho c A\int_{x1}^{x2}\begin{bmatrix}N_1\\N_2\end{bmatrix}\begin{bmatrix}N_1 & N_2\end{bmatrix}dx\begin{Bmatrix}\dot{T}_1\\\dot{T}_2\end{Bmatrix} +$$

$$kA\int_{x1}^{x2}\begin{bmatrix}\frac{dN_1}{dx}\\\frac{dN_2}{dx}\end{bmatrix}\begin{bmatrix}\frac{dN_1}{dx} & \frac{dN_2}{dx}\end{bmatrix}dx\begin{Bmatrix}T_1(\tau)\\T_2(\tau)\end{Bmatrix} = \{f_g\} \quad ..(26)$$

Equation (26) is most often expressed as

$$\left[C^{(e)}\right]\left\{\dot{T}^{(e)}\right\} + \left[k^{(e)}\right]\left\{T^{(e)}\right\} = \left\{f_g^{(e)}\right\} \quad ..(27)$$

where $\left[C^{(e)}\right]$ is the element capacitance matrix, and the dot denotes differentiation with respect to time, $T^{(e)}$ is the column matrix of element nodal temperature, $k^{(e)}$ is element conductance matrix, $f_g^{(e)}$is nodal forcing function, N_i is interpolation function, the dot denotes differentiation with respect to time.

d)Engineering Program (ANSYS V.14.0)

ANSYS is a general purpose finite element modeling tools for numerically solving a wide variety of physics such as structural, vibration, fluid dynamics, heat transfer and electromagnetic for engineers, and commonly used in academia, it's self-contained analysis tools incorporating processing (geometry certain, meshing), solver and post-processing modules in a unified general graphical user interface [12].

3.RESULTS AND DISCUSSIONS

By applying one-dimensional transient heat conduction without internal heat generation for body which its dimensions, boundary and initial conditions on the sketch as shown in Figure (1), this problem can be solved by methods as (analytical solution, finite difference, thermal capacitance, finite element, and ANSYS) by using physical as shown in Table (1), and these material can be divided into two groups (metals and nonmetals). The simulation processes which were executed by the language of technical computing (Math lab R2010a) for all methods except ANSYS programming method. In a fact, the rate of heat transfer through materials which dependent on a an important physical property, which is a thermal diffusivity (α) and its value that will limit a time of that the sample reached to steady state case. Table (2) shows the amount of time which was required to reach a steady state for each material. The results indicated that the progress of the temperatures inside the sample from left side to right side changed quickly in metals and slow in nonmetals, so the temperature progress in materials which were selected in this study as shown in Figures (5 to 10) for pure Copper, pure Aluminum, pure Iron, Sandstone, Concrete and Brick respectively and these were represented in analytical solution results and Figures (11 and 12) shows results of temperature distribution in Pure Copper and Sandstone samples respectively which were obtained by Ansys method.

Table (1): Physical properties of materials

Type of Material	Density (ρ) kg/m^3	Specific Heat (*c*) kJ/kg.℃	Thermal Conductivity (*k*)W/m.℃	Thermal Diffusivity (α) m^2/s
Pure Copper	8954	0.3831	386	11.234×10^{-5}
Pure Aluminum	2707	0.986	204	8.418×10^{-5}
Pure Iron	2897	0.452	73	2.034×10^{-5}
Sandstone	2160	0.71	1.83	11.2×10^{-7}
Concrete	1900	0.88	1.37	8.2×10^{-7}
Building Brick	1600	0.84	0.69	5.2×10^{-7}

Table (2): the periods which were requiring for samples even reaches a steady state.

Type of material	Time (sec)
Pure Copper	600
Pure Aluminum	1000
Pure Iron	1800
Sandstone	17500
Concrete	20000
Building Brick	25000

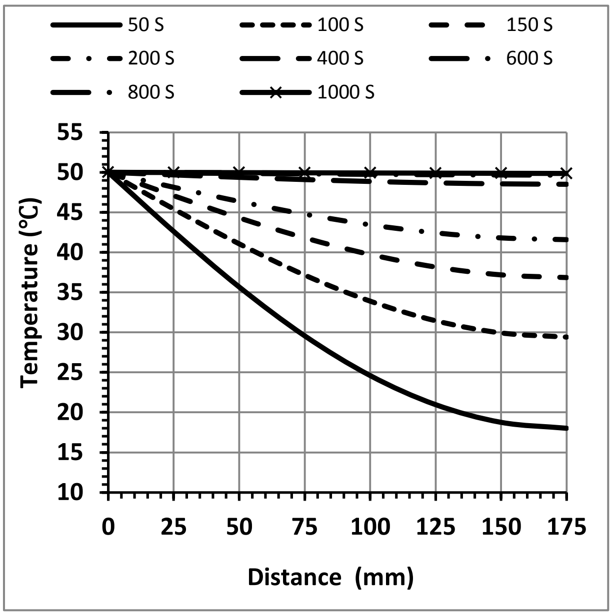

Figure (5): Temperature distribution in piece of pure Copper for several time periods through heating process.

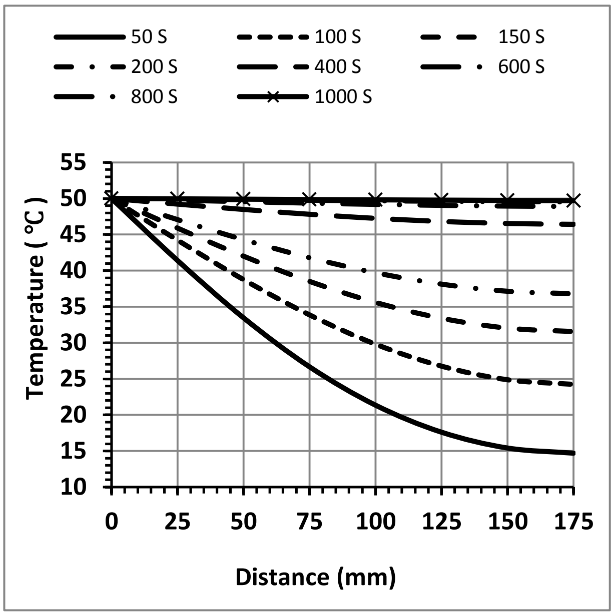

Figure (6): Temperature distribution in piece of pure Aluminum for several time periods through heating process.

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(273-291)
ISSN 1992 – 0849

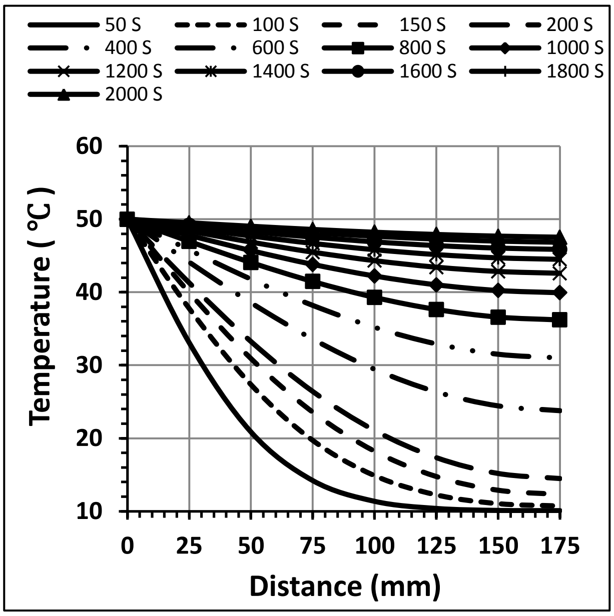

Figure (7): Temperature distribution in piece of pure Iron for several time periods through heating process.

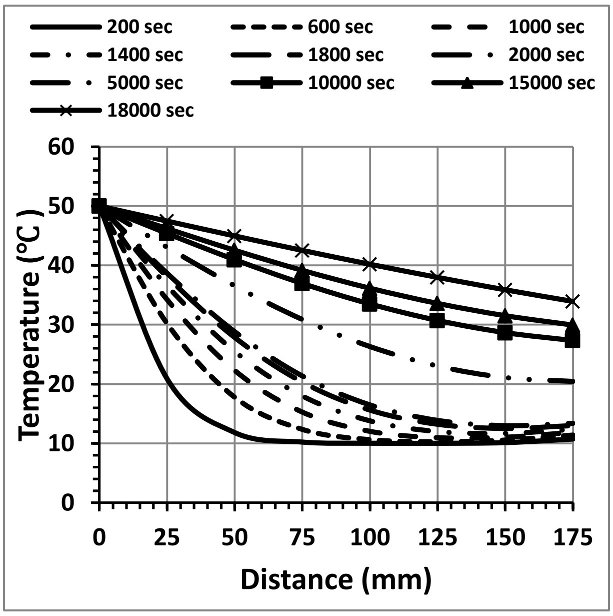

Figure (8): Temperature distribution in piece of for Sand stone several time periods through heating process.

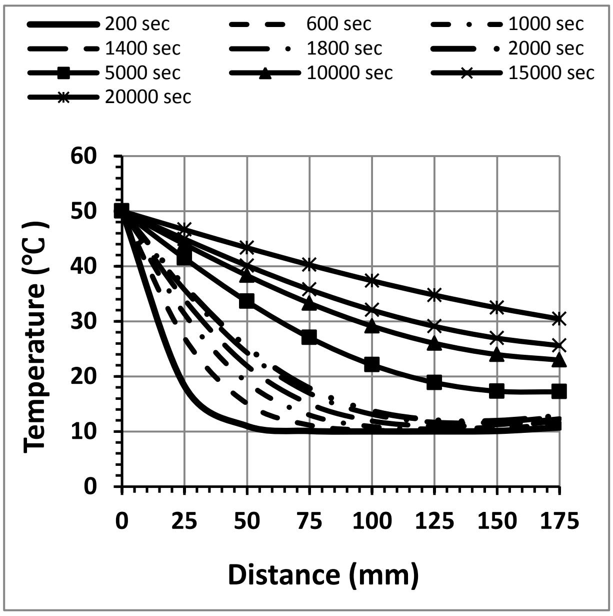

Figure (9): Temperature distribution in piece of Concrete for several periods through heating process.

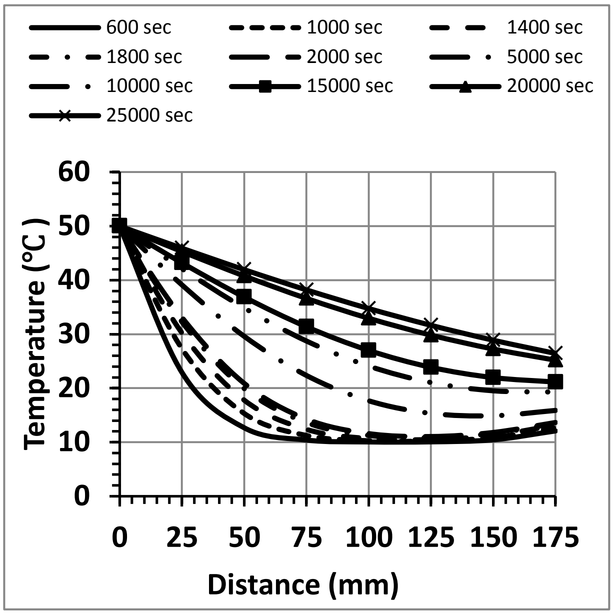

Figure (10): Temperature distribution in piece of Brick for several periods through heating process.

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(273-291)
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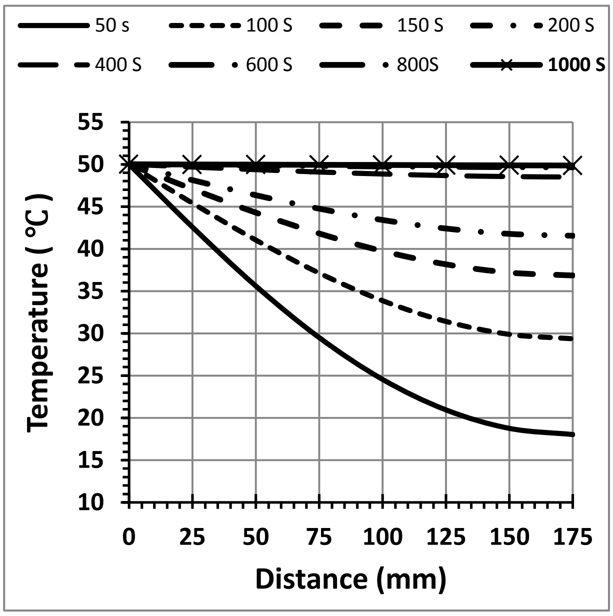

Figure (11): Temperature distribution in Copper sample piece which was calculated by Ansys Method

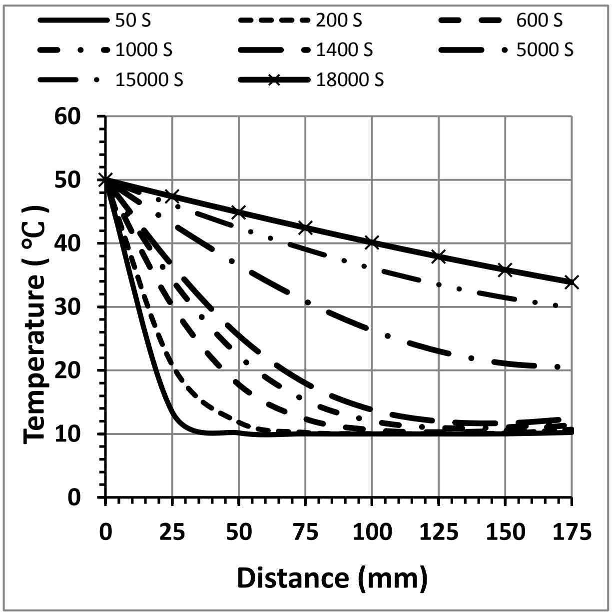

Figure (12): Temperature distribution in Sand Stone sample piece which was calculated by Ansys Method

Results of analytical solution which was taken as a reference result in comparison to all other four methods to obtain the result of which type numerical methods are near to the reference method, where Table (3) shows the error percentages in results of all numerical solution methods which were compared to the analytical solution, from the dataset shown that at the beginning of the simulation process in each material gives high error percentages, but this ratio decreases to zero, when the state of the samples changes to steady state condition. While we indicate beginning error ratio dependent on the physical properties of material, this ratio will be small in materials which has high thermal conductivity and vice versa is true. In addition to errors ratio between result of analytical solution and other numerical methods, we obtained that an identical results in both methods finite element and heat capacitance since the beginning to the end of the simulation for all materials.

4.CONCLUSION

Simulation outcome indicate that using numerical solutions application in transient heat transfer problems instead of analytical solutions led to some errors in the results as shown below:

1. The error ratio in materials which have high thermal conductivity at the outset of simulation was less than other materials of lower thermal conductivity for instance in time of 50 sec. the error ratio in Copper sample was about 0.53 to 0.5% , while in Brick sample was about 15.74 to 15.55% as shown in Table (3).
2. But upon reaching the samples to the steady-state the application of these numerical methods was an appropriate for all material types.

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 10, Issue 3, September 2015 , p.p(273-291)
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Table (3): Error ratio between average temperature of analytical solution and average temperature of numerical solution methods.

Time	Copper			Aluminum			Iron			Sand stone			Concrete			Brick		
(sec)	e1	e2	e3	e1	e2	e3	e1	e2	e3	e1	e2	e3	e1	e2	e3	e1	e2	e3
50	0.53	0.59	0.5	1.41	1.49	1.36	2.96	2.9	2.99	12.25	12.35	12.2	14.06	14.26	13.92	15.74	16.53	15.55
100	0.36	0.42	0.33	1.36	1.46	1.32	2.53	2.57	2.5	11.47	11.5	11.44	12.4	12.54	12.28	14.23	15.13	13.89
150	0.25	0.26	0.25	1.32	1.41	1.27	2.42	2.44	2.43	10.73	10.77	10.7	11.36	11.46	11.31	13.34	14.11	12.92
200	0.08	0.11	0.06	1.08	1.15	1.04	2.13	2.18	2.17	9.63	9.67	9.63	10.43	10.47	10.38	12.21	13.09	11.92
400	0.04	0.05	0.03	0.93	0.95	0.91	1.54	1.55	1.55	8.41	8.45	8.39	9.92	9.99	9.88	11.32	12.76	11.03
600	0.02	0.03	0.01	0.37	0.38	0.36	1.21	1.23	1.21	7.43	7.43	7.43	9.06	9.26	8.97	11.02	11.56	10.98
800	0.0	0.005	0.0	0.11	0.12	0.11	1.02	1.03	1.02	6.51	6.55	6.47	8.16	8.26	8.11	10.87	10.87	10.87
1000	0.0	0.0	0.0	0.07	0.07	0.07	0.92	0.95	0.91	5.71	5.75	5.68	7.08	7.28	7.0	10.00	11.08	9.76
1200				0.05	0.05	0.05	0.53	0.55	0.52	4.96	4.99	4.92	6.04	6.24	6.01	9.14	10.23	9.07
1400				0.02	0.02	0.02	0.29	0.31	0.28	4.22	4.26	4.19	5.01	5.18	4.96	7.99	9.39	7.69
1600				0.01	0.01	0.01	0.27	0.28	0.26	3.88	3.91	3.85	4.21	4.36	4.12	7.10	8.10	6.46
1800				0.0	0.0	0.0	0.23	0.24	0.22	3.30	3.35	3.28	3.53	3.73	3.33	6.39	7.24	6.13
2000							0.1	0.11	0.1	2.86	2.89	2.82	3.01	3.13	2.95	5.36	6.31	5.18
2500							0.09	0.09	0.08	2.48	2.52	2.45	2.73	2.93	2.62	4.81	5.24	4.34
3000							0.04	0.05	0.04	2.1	2.26	1.95	2.45	2.55	2.33	4.31	4.61	4.03
3500							0.02	0.02	0.02	1.89	1.94	1.85	2.19	2.30	2.06	3.61	3.92	3.18
4000							0.0	0.0	0.0	1.72	1.75	1.70	1.93	2.03	1.81	2.96	3.26	2.76
4500										1.57	1.60	1.55	1.71	1.86	1.56	2.59	2.85	2.39
5000										1.22	1.25	1.20	1.46	1.65	1.33	1.99	2.52	1.78
7500										0.93	0.96	0.90	1.21	1.45	1.18	1.60	1.92	1.43
10000										0.70	0.73	0.68	1.07	1.27	1.02	1.27	1.73	1.15
12500										0.41	0.43	0.39	0.89	1.09	0.80	1.07	1.37	1.00
15000										0.20	0.22	0.19	0.63	0.83	0.59	0.81	1.11	0.78
17500										0.12	0.14	0.11	0.29	0.49	0.25	0.58	0.77	0.51
20000										0.06	0.07	0.05	0.14	0.24	0.11	0.32	0.52	0.26
22500										0.02	0.03	0.01	0.08	0.18	0.06	0.25	0.33	0.22
25000										0.0	0.0	0.0	0.04	0.08	0.03	0.16	0.27	0.14
27500													0.0	0.3	0.0	0.10	0.20	0.09
30000													0.0	0.0	0.0	0.06	0.11	0.03
35000																0.02	0.07	0.01
40000																0.0	0.4	0.0
42500																0.0	0.0	0.0

$e1 = \frac{\bar{T}_1 - \bar{T}_2}{\bar{T}_1} \times 100\%$: $e2 = \frac{\bar{T}_1 - \bar{T}_3}{\bar{T}_1} \times 100\%$: $e3 = \frac{\bar{T}_1 - \bar{T}_4}{\bar{T}_1} \times 100\%$

$\bar{T}_1$: Average Temperature of analytical solution, $\bar{T}_2$: Average temperature of finite difference method, $\bar{T}_3$: Average temperature of finite element method and $\bar{T}_4$: Average temperature of ANSYS Method.

Steady state region

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