

Optical Design Optimization for Indoor Solar Illumination Using Truncated Tetrahedral Pyramid Concentrator

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ABSTRACT

A concept of indoor solar illumination is described and designed. The solar illumination system is composed of a none tracking primary reflector and controlling mirrors. The primary reflector is in form of a truncated tetrahedral pyramid covered by a thin flat glass sheet on the wide opening of the pyramid to prevent dust to accumulate on the reflecting sides of the pyramid besides preventing dust from coming inside the room. The controlling mirrors are plane and rectangular. Each part of the solar illumination system is optically suited and compatible with other parts to realize high efficiency. The optical design is conducted for Baghdad city at interior building in two solstice days over the year. Research results showed that the design of the solar system is achieved on the base of minimum and maximum solar illuminance level in 21 June. Tetrahedral pyramid's dimensions in a question as a solar concentrator for this paper are: concentration ratio is 3 for 90 cm entrance opening, 25° for half angle and 5° for controlling mirrors.

Keywords: Pyramid, Half Angle, Concentration Ratio, Illumination, Solstice Day

تحقيق امثلية التصميم البصري للارانة الشمسية الداخلية بأستخدام المركز الهرمي الرباعي الناقص

الخلاصة

فكرة الانارة الشمسية الداخلية قد تم وصفها وتصميمها. تتألف منظومة الانارة الشمسية من عاكس اولي غير متتبع , مرايا للسيطرة على الانارة. يكون العاكس الاولي على شكل هرم رباعي ناقص مغطى بشريحة زجاجية رقيقة من جهة قاعدة الهرم و ذلك لمنع الغبار من التجمع على السطوح العاكسة للهرم اضافة الى منع دخول الغبار الى داخل الغرفة. كل جزء من اجزاء منظومة الانارة الشمسية يكون متناسبا و متوافقا بصريا مع الاجزاء الاخرى و ذلك لتحقيق اعلى كفاءة لمنظومة الانارة. تم تنفيذ التصميم البصري للمنظومة الشمسية لمدينة بغداد و ليومي الانقلاب الشمسي. بينت نتائج البحث بأن التصميم تم انجازه على اساس مستوى الانارة الشمسية الدنيا و القصوى ليوم 21 من شهر حزيران. ابعاد الهرم الرباعي الناقص قيد الدرس كمركز شمسي في هذا

البحث كانت كالاتي: نسبة التركيز الشمسي هي ثلاثة مرات عندما يكون قياس فتحة الدخول 90 سم و فتحة الخروج 30 سم و نصف زاوية ميلان الهرم 25 درجة و زاوية ميلان مرآة السيطرة هي 5 درجات.

INTRODUCTION

Natural light is very important element in the quality of vision. Skylights can provide satisfactory lighting for activities that can tolerate large variations in illumination level. Getting good performance from skylights is not as simple as it may appear. To satisfy a number of requirements, some of which may not be easily compatible with each other. In single-floor buildings, skylights may provide a large fraction of illumination requirements. Sunlight is so intense that skylights can provide virtually any illumination level that is required. Artificial lighting is still needed at night.

Outdoor sunlight from a clear sky produces an illumination of about 60,000 lux, most of which comes directly from the sun. Using this fact, you can easily calculate the fraction of the ceiling area that needs to be converted to skylight. Skylights must be located where the sun can shine on them directly. A skylight does not produce a useful amount of daylight if it is shaded by adjacent structures or foliage. Similarly, skylights are not worthwhile in areas that have heavy cloud cover for a large fraction of the time, unless the climate is mild and the structure can accommodate a large area of skylights. Clouds typically reduce solar illumination by a factor of five to ten [1].

NON- IMAGING OPTICS

Non-imaging optics is a field that deals with the optimal transfer of light between a source distribution and a target distribution. The most common use of nonimaging optics is achieving maximal concentration of light. In the simplest terms, nonimaging optics strives to gather light rays that are incident on an opening or aperture of a given area, A_{in} , and to ensure that these rays manage to make their way to the exit aperture, A_{out} , which has a smaller area. As long as the entering rays become exiting rays, the definition of the concentration ratio, C_r , is given as the ratio of the two areas in equation (1) [2].

$$C_r = \frac{A_{in}}{A_{out}} \quad \dots (1)$$

The development of nonimaging concentrators has reached a very high stage of development. The standard design for these applications is to use truncated pyramids or hexagons, which are easier to fabricate reliably. The following analytic results will be specific to the truncated pyramid.

If one considers two sides of the pyramid, as shown in Figure (1), and imagines a light ray incident on the extreme left edge of the entrance aperture, then the concentration factor, C_r , can be shown from simple geometrical optics to be [2]:

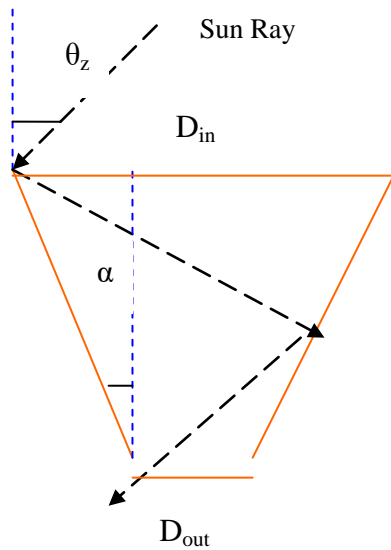


Figure (1) Side view of truncated pyramid which shows angular relationships involved in estimating concentration ratio.

$$C_r = \frac{D_{in}}{D_{out}} = \frac{\text{Sin}[(2n + 1)\alpha + \theta_z]}{\text{Sin}(\alpha + \theta_z)} \quad \dots(2)$$

In Equation (2) [2], the quantities are as follows: D_{in} is the width of the entry aperture, D_{out} is the width of the exit aperture, n is number of reflections that the light rays can make with the interior walls of the concentrator, α is the half – angle opening of the pyramid and θ_z is the angle between the vertical (normal incidence) and the incident light ray.

There is a restriction on the angles and number of reflections. That restriction is that after n reflections, the light ray will have acquired no more than a 90° angle with respect to the vertical axis of the concentrator [3].

$$2(n + 1)\alpha + \theta_z \leq 90^\circ \quad \dots(3)$$

In order to achieve a certain concentration ratio, these three quantities, n , α and θ_z , must be optimized. Once that is done, there will be a necessary relationship between the quantities α , θ_z , n and the size (including the slope height) of the concentrator [3]:

$$\frac{L}{D_{out}} = \frac{\text{Sin}[(2n + 1)\alpha + \theta_z] - \text{Sin}(\alpha + \theta_z)}{2\text{Sin}\alpha \cdot \text{Sin}(\alpha + \theta_z)} \quad \dots(4)$$

It should be noted that repeated reflections from the inner surfaces of the concentrator entails some loss of light power. This is so because no surface is perfectly reflecting, and thus some light energy is absorbed at each reflection. This is may expressed as a reflection coefficient less than unity ($r < 1$). As a light ray

strikes the surface, the power contained in the reflected light ray will be reduced from its initial value, P_{in} , to a new value, given by ρP_{in} , where ρ is reflectivity of the interior wall of the pyramid. This new light ray will suffer yet another reflection on its downward path into the concentrator, with a consequent reduction of its power to $\rho^2 P_{in}$, and so on. It is clear that too many repeated reflections reduce the power available at the exit aperture of the concentrator.

TRUNCATED PYRAMID'S GEOMETRY AND GATHERING SOLAR POWER CAPABILITY

A – Concentrator Geometry

To calculate the surface area of reflecting sides of truncated pyramid. One has to consider Figure (2) which shows geometry of truncated pyramid [3].

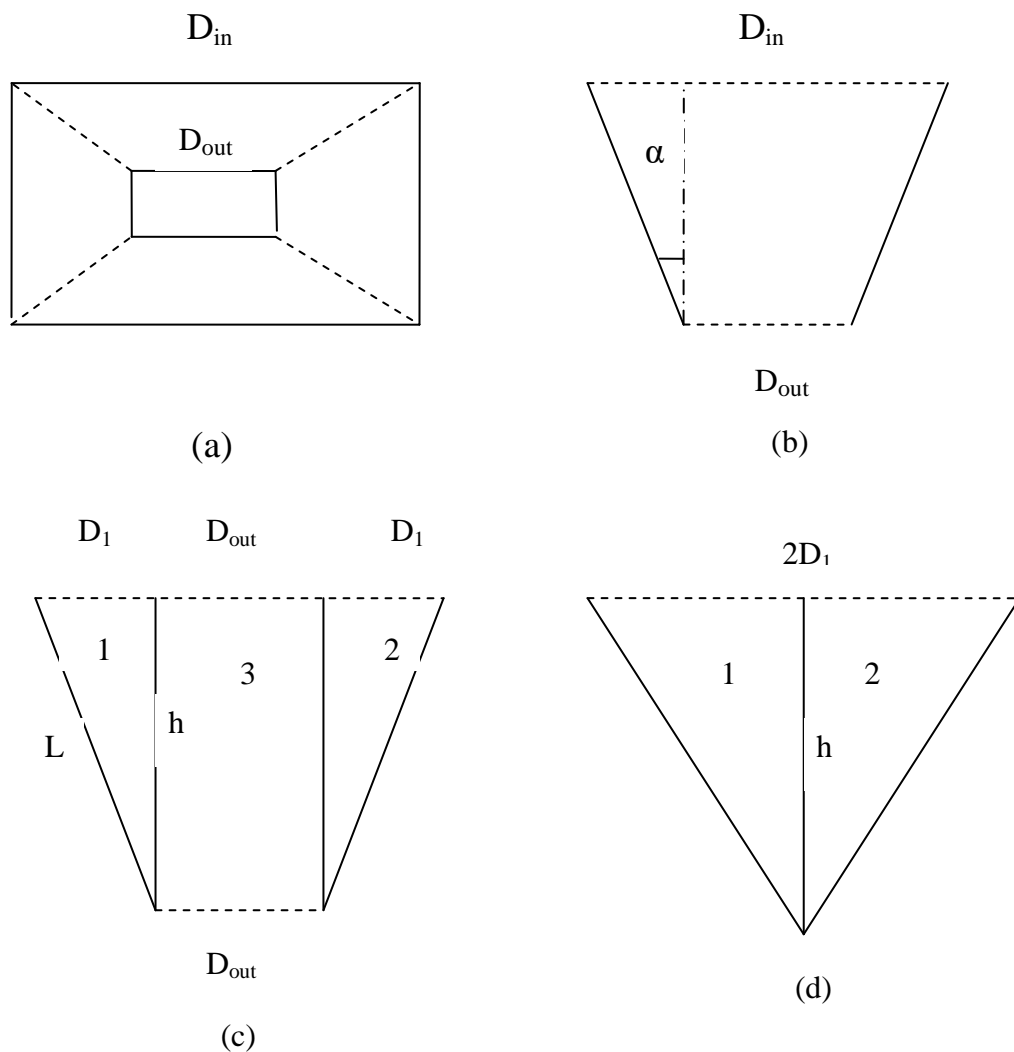


Figure (2) Pyramid's (a) Top view (b) Side view (c) One side area distribution (d) areas of triangle (1) and (2) together.

Surface area of one side, A_{Surface} , of truncated tetrahedral pyramid can be calculated geometrically from Figure (2 - C) given as:

$$A_{\text{Surface}} = \text{Area (1)} + \text{Area (2)} + \text{Area (3)} \quad \dots (5)$$

$$D_1 = \frac{D_{in} - D_{out}}{2} \quad \dots (6)$$

Also, height of the triangle, h , which is the height of the pyramid, is given as:

$$h = \frac{D_1}{\tan(\alpha)} \quad \dots (7)$$

And, slant length, L , of the pyramid is given as:

$$L = \frac{D_1}{\sin(\alpha)} \quad \dots (8)$$

Figure (1) shows α , θ_z and $\frac{\pi}{2}$ are angles of right angle triangle, sum of its angles are π which is given in the following equation:

$$\alpha = \pi - (\theta_z + \frac{\pi}{2}) \quad \dots (9)$$

Area (1) & (2) form a triangle see Figure (2 - D).

$$A_{\text{Triangle}} = \frac{1}{2} * 2D_1 * \frac{D_1}{\tan(\alpha)} \quad \dots (10)$$

$$A_{\text{Oblong}} = D_{out} * \frac{D_1}{\tan(\alpha)} \quad \dots (11)$$

So, total surface areas, A_{TS} , for four reflecting sides will be:

$$A_{\text{TS}} = (\frac{D_{in} - D_{out}}{\tan(\alpha)}) * (\frac{D_{in} - D_{out}}{\tan(\alpha)} + 2D_{out}) \quad \dots (12)$$

B – Solar Power Reaching the Interior Building

The visible solar power reaching the interior building i.e; the living room is given as [4]:

$$P_{in} = I_{dir} * A_{TS} * \rho^n * T_{Glass} * V_{Spectrum} \dots (13)$$

Where, I_{dir} is the direct (beam) solar intensity reaching certain place in certain time.

T_{Glass} is transmittance of cover glass sheet on the truncated tetrahedral Pyramid, which is given as:

$$T_{Glass} = 1 - \left(\frac{n_g - n_a}{n_g + n_a} \right)^2 \dots (14)$$

Where n_g and n_a are refractive index of glass and air, respectively. $V_{Spectrum}$ is the percentage of visible solar light which is 45 % [5].

RESEARCH METHODOLOGY

The aim of this paper is to design a reflecting concentrator as smaller as possible, besides it achieves as less as possible reflection times on the sides of the concentrator to minimize losing much solar power as a function of ρ^n . To calculate solar power reaching the indoor building, living room, the following data should be known:

1. Interior total surface area of the truncated tetrahedral pyramid, the concentrator.
2. Reflectivity of the interior surface, ρ .
3. Half angle of the truncated tetrahedral pyramid.
4. Solar incidence angle θ_z .
5. Concentration ratio of the truncated tetrahedral pyramid C_r .
6. Controlling angle, ϕ , should be known.
7. The city at which the designer has to conduct his design, namely the latitude.

RESULT & DISCUSSION

For choosing proper surface area for the truncated pyramid, knowing the *solar irradiance* over a year for Baghdad city (Latitude is $\phi = 33^\circ 20' N$ [6]) will make the choice easy. Maximum and minimum levels for solar irradiance are the base for electing the corresponding days over the year. June 21th is the maximum level will Dec. 21th is the minimum level see Figure (4).

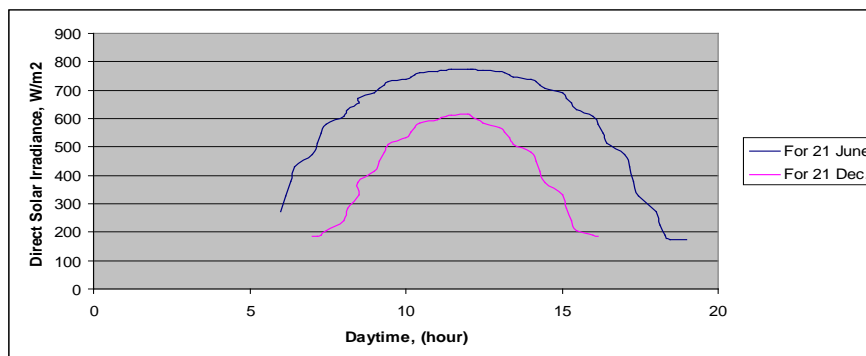


Figure (4) distribution of direct solar irradiance for chosen days [7].

Chosen days for solar solstice are detailed on Table (1).

Table (1) Direct solar irradiance fallen on Baghdad city for chosen days.

| Day | Time | Direct Solar Irradiance, I_{dir} , W/m ² |
|---------|----------|---|
| 21 June | 8:00 AM | 608.1776 |
| | 12:00 PM | 773.7154 |
| 21 Dec. | 8: AM | 241.5614 |
| | 12:00 PM | 614.1756 |

Solar angle incidence over Baghdad city is given in Figure (5).

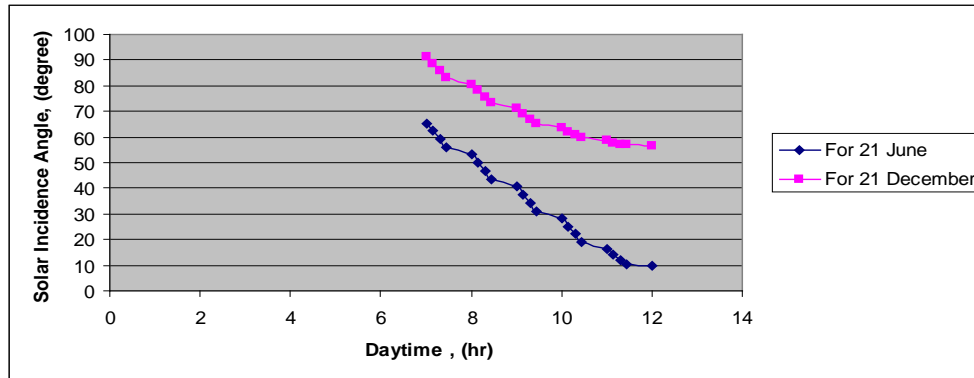


Figure (5) Solar incidence angle for Baghdad city in 21 June and 21 December.

It is clear solar incidence angles decreases as daytime passes, and this decrease is greater in 21 June rather than 21 December.

The optical design of the pyramid is chosen to achieve concentration ratio equal to three. So, the input opening will be 0.9 m for 0.3 m output opening facing the roof of the interior building, i.e; the living room.

For truncated tetrahedral pyramid design, first of all, one has to check different values of half angle of the pyramid to find height (h), slope height (L) and total surface area ($A_{Surface}$), see Figures (6 and 7).

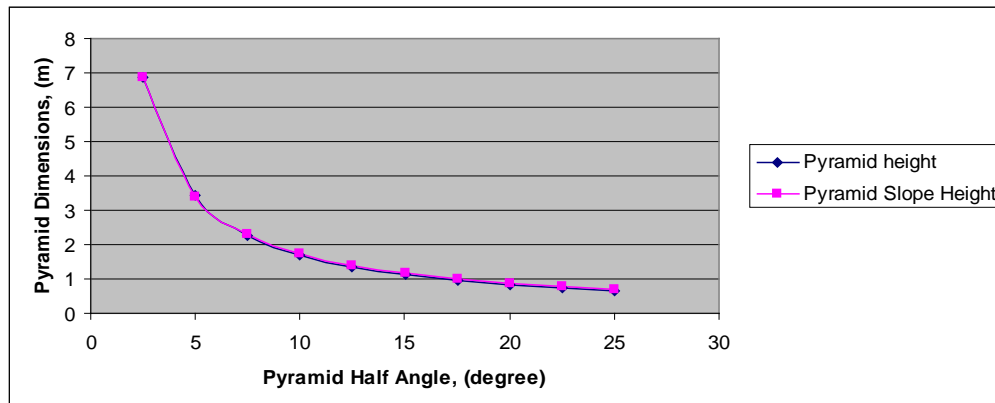


Figure (6) Pyramid's dimension, height and slope height as a function to pyramid's half angle α for $D_{in} = 0.9$ m, $D_{out} = 0.3$ m and $C_r = 3$.

It is clear as pyramid's half angle α increases, pyramid's dimensions decrease. So, $\alpha = 25^\circ$ is the proper half angle.

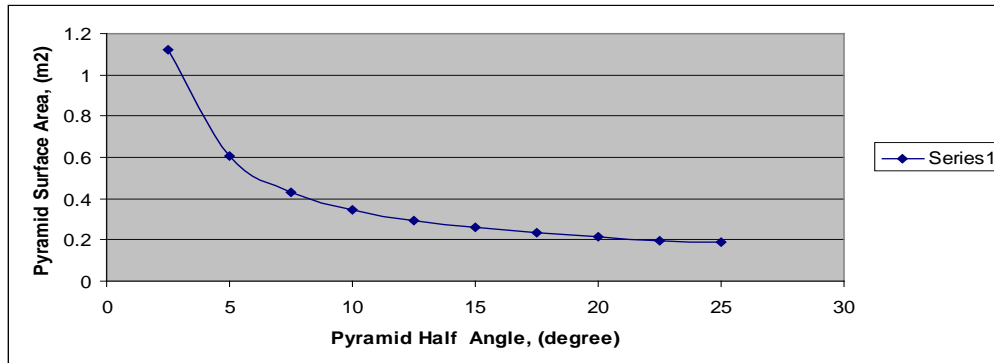


Figure (7) Pyramid's surface area as a function to pyramid's half angle α for $D_{in} = 0.9$ m, $D_{out} = 0.3$ m and $C_r = 3$.

Figure (6) and (7) are detailed in Table (1).

Table (1) Pyramid's height, slope height and surface area as a function to half angle.

| Pyramid Half Angle α , (degree) | Height (m) | Slope Height (m) | Surface Area (m ²) |
|--|------------|------------------|--------------------------------|
| 2.5 | 6.868 | 6.874 | 1.120 |
| 5.0 | 3.427 | 3.407 | 0.6041 |
| 7.5 | 2.277 | 2.297 | 0.4316 |
| 10.0 | 1.70 | 1.72 | 0.3451 |
| 12.5 | 1.352 | 1.385 | 0.2929 |
| 15.0 | 1.119 | 1.158 | 0.2578 |
| 17.5 | 0.951 | 0.997 | 0.2326 |
| 20.0 | 0.823 | 0.876 | 0.2135 |
| 22.5 | 0.724 | 0.783 | 0.1985 |
| 25.0 | 0.643 | 0.709 | 0.186 |

Also choosing different values of α versus number of reflection on the sides of reflecting interior pyramid, n , for 21 June and 21 December is given in Figure (8 and 9).

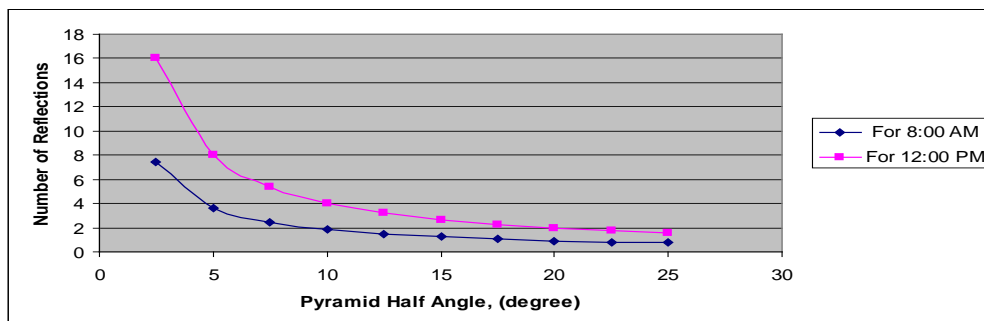


Figure (8) number of reflections on the interior sides of the pyramid as a function to pyramid's half angle α for 21 June.

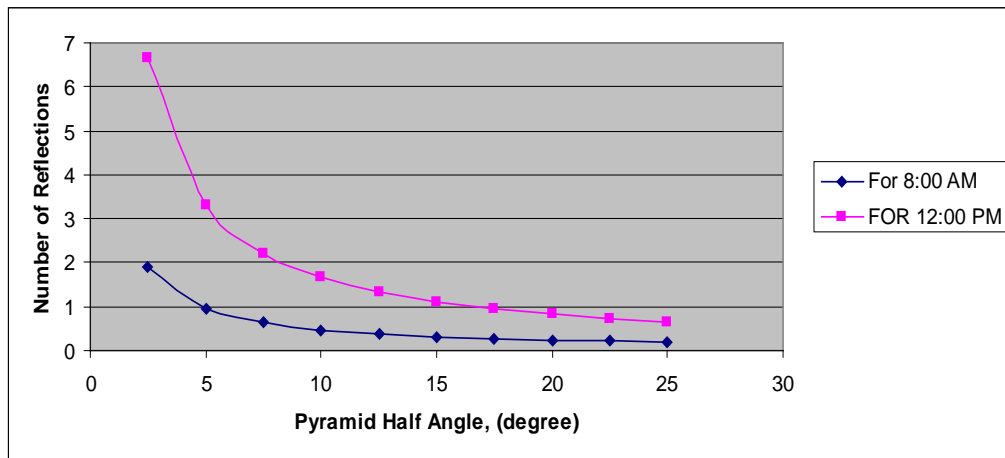


Figure (9) Number of reflections on the interior sides of the pyramid as a function to pyramid's half angle α for 21 December.

It is clear that, as pyramid's half angle increases, number of reflections decrease, hence decreasing solar power loss. Decreasing number of reflections is lower at 21 December rather than 21 June. Also it is lower at 8:00 AM rather than 12:00 PM for 21 December.

Solar spot rib length on the ground of the interior building which is shown in Figure (10).

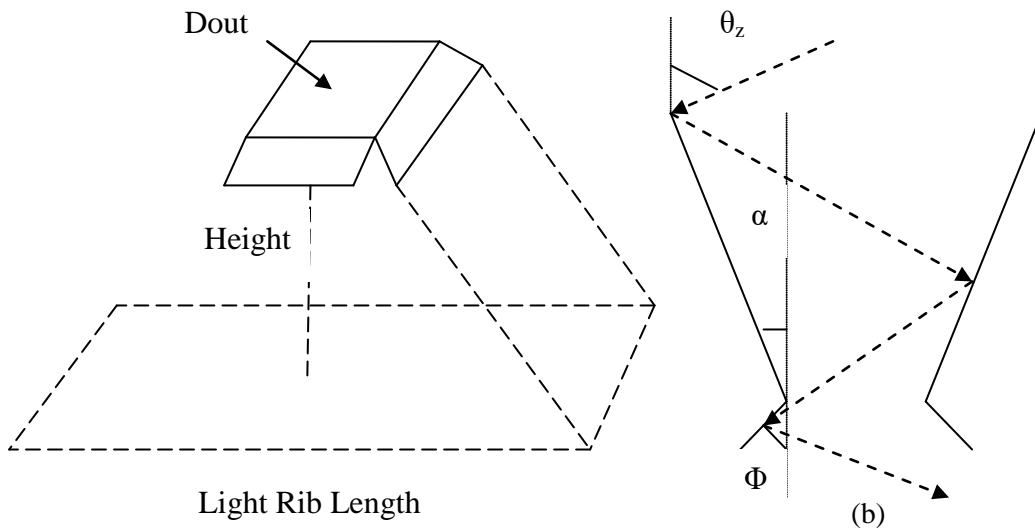


Figure (10) Shows (a) solar spot rib length on the ground of the interior building (b) controlling mirror angle.

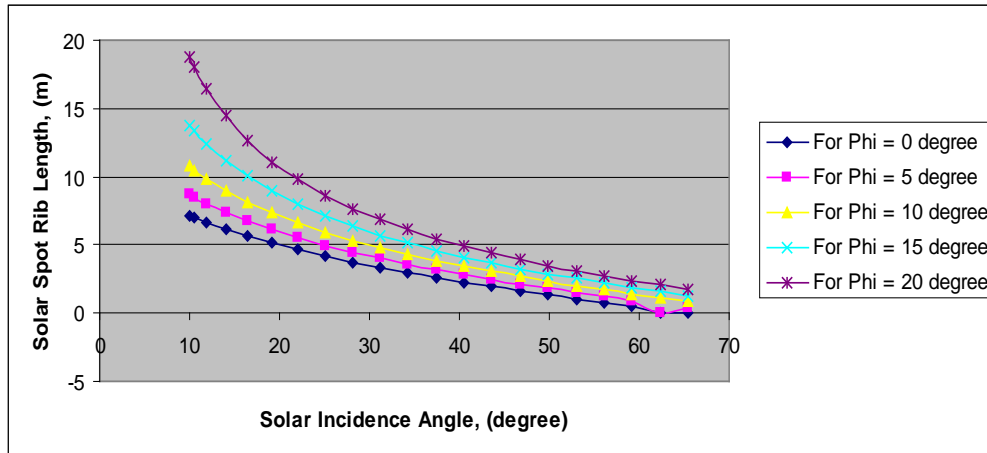


Figure (11) Solar spot rib length versus solar incidence angle for different angles of controlling mirror.

It is clear that as solar incidence angle decreases, solar spot rib length increases. The increase of rib length is greater as controlling mirror angle, ϕ , increases. Solar illuminance is given in the following Figure.

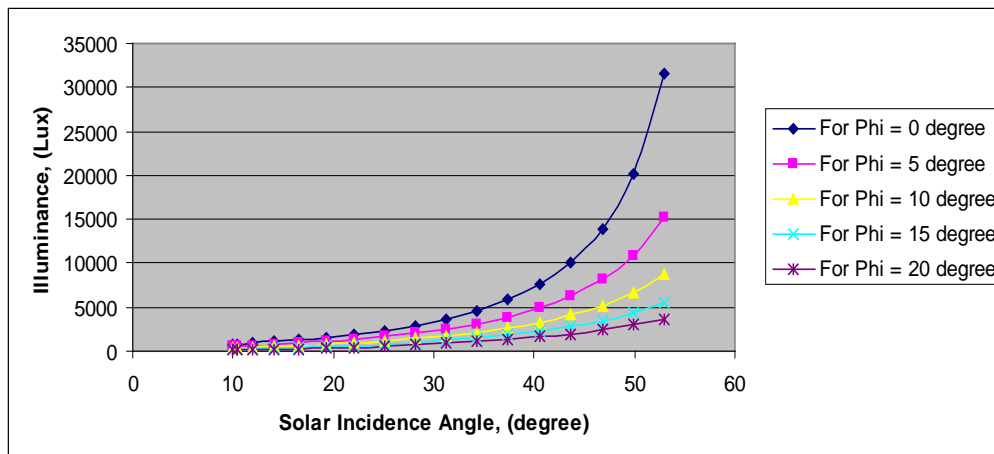


Figure (12) Solar illuminance versus different solar incidence Angles @ 21 June.

Figure (12) is detailed in Table (2).

Table (3) solar angle incidence at 8:00 AM and 12:00 PM for different mirror controlling angles versus illumination.

| Daytime (hr) | Angle of Incidence (deg) | Illumination for different mirror controlling angles (Lux) | | | | |
|--------------|--------------------------|--|-------|------|------|------|
| | | 0° | 5° | 10° | 15° | 20° |
| 8:00 | 53°.026 | 31494 | 15177 | 8677 | 5451 | 3622 |
| 12:00 | 9°.929 | 822 | 558 | 364 | 221 | 119 |

To find the proper luminance for human eye, see Table (3) which shows the level of each case as follows [9]:

Table (4) Illuminance level for different cases.

| Illuminance, (lux) | Example |
|---------------------------|---|
| 50 | Family living room. |
| 80 | Hallway / toilet. |
| 100 | Very dark overcast day. |
| 400 | Sunrise or sunset on a clear day. |
| 500 | Office lighting. |
| 1000 | Overcast day, typical TV studio lighting. |

Geometrical data of the truncated tetrahedral pyramid are given as following: 90 Cm for input opening, 30 Cm for output opening, 3 for concentration ratio, 94% for reflecting sides reflectivity, 25° for half angle, 5° for angle of controlling mirror and 0,1864 m² for surface area.

Illumination data for the upper geometrical, for 2.5 m living room height, data are:

For 21 June: 15177 Lux @ 8:00 AM and 558 Lux @ 12:00 PM.

It is clear the values of illumination over a day are going with Table (3).

CONCLUSIONS

It is clear that, one can use truncated tetrahedral pyramid concentrator as a cheap, practical and easy to be handled as a free, green illumination source. A concentrator of three times concentration ratio is achieved by designing a reflecting pyramid of silver sides of 25° half angle, 5° for controlling mirror and 0.1864 m² as a surface area. The lower illumination level is achieved by these dimensions for Baghdad city which is 558 Lux.

REFERENCES

- [1]. Rannels, J. (2000). The DOE office of solar energy technologies' vision for Advancing solar technologies in the new millennium. Solar Energy. Vol. 69, No. 5, p. 363-370.
- [2]. Burkhard, D.G., Strobel, G.L., and Burkhard, D.R. (1978). Flat-sided rectilinear trough as a solar concentrator: an analytical study. Applied Optics. Vol. 17, No. 12, p.1870-1881.
- [3]. Burkhard, D.G., Strobel, G.L., and Shealy, D.L. (1978). Solar concentrating Properties of truncated hexagonal, pyramidal circular cones, Applied Optics. Vol. 17, No. 15, p. 2431-2440.
- [4]. Snyder, Allan W. and Love, John D. 1983). Optical Waveguide Theory. New York: Chapman and Hall. Welford, W. and Winston, R. (1978). The Optics of Nonimaging Concentrators. New York: Academic Press.
- [5]. Geyer, Micheal, Stine B., William, "Power from the Sun", J. T. Lyle Center 2001.
- [6]. Radiometry and photometry in astronomy FAQ, section 7, Paul Schlyter. (1980).
- [7]. Baghdad City, Wikipedia.

- [8]. Hisham A. Maliek, Simulation of Indoor Solar Illumination for a Library Hall for Baghdad City, Joint Physics National Conference between Baghdad's College of Science and Babylon's College of Education for Pure Science, 7-8 / 11 / 2012.
- [9]. Australian Greenhouse Office (May, 2005). "Chapter 5: Assessing lighting savings". Working Energy Resource and Training Kit: Lighting. Archived from the original on April 15, 2007. Retrieved March 13, 2007.