Solution of Fuzzy Initial Value Problem Using Variational Iteration Method

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ABSTRACT

Variation iteration method is powerful analytical method for solving linear and nonlinear equations.

In this paper, the variational iteration method has been applied to solve fuzzy linear and nonlinear differential equations with fuzzy initial conditions,

We introduce two examples to solve the problem by using this method ,and applied in in MathCAD 7 computer software .

Keywords: Fuzzy Differential Equations, Variation Iteration Method.

حد المعادلات التفاضلية الضبابية باستخدام الطريقة التكرارية التغايرية

الخلاصة

الطريقة التكرارية التغايرية هي الأسلوب والطريقة التحليلية القوية من أجل حل المعادلات الخطية وغير الخطية في هذا البحث، قد تم تطبيق طريقة تكرار التغاير لحل المعادلات الخطية الضبابية والمعادلات التفاضلية غير الخطية مع الشروط الابتدائية الضبابية، .MathCAD 7 ونحن نقدم مثالين على حل المسالة باستخدام هذا الأسلوب، وتطبيقها في برامج الكمبيوتر في البرنامج.

INTRODUCTION

Many authors have been worked about Variational Iteration Method (VIM) that for more information sees [3] ,[4] .

In some case, the analytical solution may be so difficult to be evaluated, therefore numerical and approximate methods seems to be necessary to be used which cover the problem under consideration. The method that will be consider in this work is the Variational Iteration Method (VIM) for finding the solution of a linear and nonlinear problems.

This method is a modification of the general Lagrange multiplier method into an iteration method, which is called the correction functional [6]. Heuristic interpretation of these concepts leads to a new comers in the field to start working immediately without the long search and preparation of advanced calculus and calculus of variations. At the same time those concepts already familiar with variational iteration method, which will find the most recent new result.

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In this paper, the VIM is extent to solve fuzzy differential equations and obtain approximate fuzzy solution; we explain some basic concepts of VIM

Our results show this important and efficient method for solving fuzzy initial value problem.

Basic Concepts of VIM [11]

To illustrate the basic idea of the VIM, we consider the following general non-linear system:

$$L(y(t)) + N(y(t)) = g(t),\ t \in [a, b] \subset R \qquad ...(1)$$

Where L is a linear operator, N is a nonlinear operator and g(t) is any given function. Construct a correction functional for the system (1), which reads as follows:

$$y_{n+1}(t) = y_n(t) + \int_0^t \lambda\{Ly_n(s) - N\tilde{y}_n(s) - g(s)\}\, ds, \forall\ n = 0, 1 \qquad \dots (2)$$

Where λ is the general Lagrange multiplier which can be identified optimally via the variation theory [4, 5, 6], y_0 is an initial approximation and $\tilde{y}_n$ denotes a restricted variation, i.e., $\delta\ \tilde{y}_n = 0$.

To illustrate this method, consider the first order initial value problem:

$$y'(t) = f(t, y(t)),\ y(t_0) = y_0 \qquad \dots(3)$$

supstution equation (3) into equation (2), yields :

$$y_{n+1}(t) = y_n(t) + \int_0^t \lambda\left\{\frac{dy_n(\tau)}{d\tau} - f(\tilde{y}_n(\tau), \tau)\right\} d\tau, \forall\ n = 0, 1 \qquad \dots (4)$$

Making the above correction functional stationary, and notice that the restricted variation $\delta y(0) = 0$,

$$\delta y_{n+1}(t) = \delta y_n(t) + \delta\int_0^t \lambda\{y'_n(\tau) - f(\tilde{y}_n(\tau), \tau)\}\, d\tau$$

Hence, using the method of integration by parts on the above equation will give the following formula:

$$= \delta y_n(t) + \lambda(\tau)\delta y_n(\tau)\Big|_{\tau=t} + \int_0^t \lambda'(\tau)\delta y_n(\tau)\, d\tau = 0 \qquad \dots (5)$$

Thus, we obtain the following stationary conditions:

$$1 + \lambda(\tau)\Big|_{\tau=t} = 0 \quad \text{and} \quad \lambda'(\tau)\Big|_{\tau=t} = 0 \qquad \dots (6)$$

The Lagrange multiplier, therefore, can be readily identified to be $\lambda = -1$ and the following iteration formula can obtained:

$$Y_{n+1}(t) = y_n(t) - \int_0^t \{y'_n(\tau) - f(\tilde{y}_n(\tau), \tau)\}\, d\tau, \quad y_0(0) = c \qquad \ldots(7)$$

Notice that, it is not necessary to take $y_n(\tau)$ as restricted in $f(y_n(\tau), \tau)$ and for obtaining better value for λ in initial value problems, one can use methods introduced in [5, 9, 10].

Fuzzy Number

An arbitrary fuzzy number is represented by an order pair of functions $(\underline{y}(\alpha), \overline{y}(\alpha))$, for all $\alpha \in [0, 1]$; which satisfy the following requirements [2]; $\underline{y}(\alpha)$ and $\overline{y}(\alpha)$ are bounded, where $\underline{y}(\alpha) \le \overline{y}(\alpha)$, $0 \le \alpha \le 1$, $\underline{y}(\alpha)$ and $\overline{y}(\alpha)$ refers to the left and right bounds of the fuzzy number.

SPECIAL CASES

Case (1)[1]:

Let $\tilde{y}(\alpha) = (\underline{y}(\alpha), \overline{y}(\alpha))$, $\alpha \in [0, 1]$ be a fuzzy number, we take:

$$y^c(\alpha) = \frac{\overline{y}(\alpha) + \underline{y}(\alpha)}{2}, \quad y^d(\alpha) = \frac{\overline{y}(\alpha) - \underline{y}(\alpha)}{2}$$

it is clear that $y^d(\alpha) \ge 0$ and $\underline{y}(\alpha) = y^c(\alpha) - y^d(\alpha)$ and $\overline{y}(\alpha) = y^c(\alpha) + y^d(\alpha)$ also a fuzzy number $\tilde{y} \in E$ is said to be symmetric if $\tilde{y}(\alpha)$ is independent of α, for all $\alpha \in [0, 1]$.

Case (2), [1]:

Let $\tilde{y}(\alpha) = (\underline{y}(\alpha), \overline{y}(\alpha))$, $\tilde{v}(\alpha) = (\underline{v}(\alpha), \overline{v}(\alpha))$, $\alpha \in [0, 1]$ and also k, s are arbitrary real numbers. If $\tilde{w} = k\tilde{y} + s\tilde{v}$, then:

$w^c(\alpha) = ky^c(\alpha) + sv^c(\alpha)$

$w^d(\alpha) = |k|y^d(\alpha) + |s|v^d(\alpha)$

Definition (1), [2]:

Let $\tilde{y}(\alpha) = (\underline{y}(\alpha), \overline{y}(\alpha))$, $\tilde{v}(\alpha) = (\underline{v}(\alpha), \overline{v}(\alpha))$, $\alpha \in [0, 1]$ be fuzzy numbers then the Hausdorff distance between $\tilde{y}$, $\tilde{v}$ is given by:

$$D(\tilde{y}, \tilde{v}) = \sup_{\alpha \in [0,1]} \max\{|\underline{y}(\alpha) - \underline{v}(\alpha)|, |\overline{y}(\alpha) - \overline{v}(\alpha)|\}$$

case (3), [1]:

Clearly from case (2), we have:

$|\overline{y}(\alpha) - \overline{v}(\alpha)| \le |y^c(\alpha) - v^c(\alpha)| + |y^d(\alpha) - v^d(\alpha)|$

$|\underline{y}(\alpha) - \underline{v}(\alpha)| \le |y^c(\alpha) - v^c(\alpha)| + |y^d(\alpha) - v^d(\alpha)|$

Hence, for all $\alpha \in [0, 1]$:

$\max\{|\underline{y}(\alpha) - \underline{v}(\alpha)|, |\overline{y}(\alpha) - \overline{v}(\alpha)|\} \leq |y^c(\alpha) - v^c(\alpha)| + |y^d(\alpha) - v^d(\alpha)|$

and then:

$$D(\tilde{y}, \tilde{v}) = \sup_{\alpha\in[0,1]} \{|y^c(\alpha) - v^c(\alpha)| + |y^d(\alpha) - v^d(\alpha)|\}$$

Therefore, if $|y^c(\alpha) - v^c(\alpha)|$ and $|y^d(\alpha) - v^d(\alpha)|$ tends to zero hence $D(\tilde{y}, \tilde{v})$ tends to zero.

Let E be the set of all upper semi-continuous normal convex fuzzy numbers with bounded α-level intervals. This means that if $\tilde{v} \in E$, then the α-level set:

$$[\tilde{v}]_\alpha = \{s : \tilde{v}(s) \geq \alpha\}$$

is closed bounded interval which is described by:

$$[\tilde{v}]_\alpha = [\underline{v}(\alpha), \overline{v}(\alpha)], \text{ for } \alpha \in [0, 1]$$

and

$$[\tilde{v}]_0 = \overline{\bigcup_{\alpha\in[0,1]}[\tilde{v}]_\alpha}$$

Two fuzzy numbers $\tilde{y}$ and $\tilde{v}$ are called equal, $\tilde{y} = \tilde{v}$, if $\tilde{y}(s) = \tilde{v}(s)$, for all
$s \in R$ or $[\tilde{y}]_\alpha = [\tilde{v}]_\alpha$, for all $\alpha \in [0, 1]$.

Lemma (1), [7]:

If $\tilde{y}, \tilde{v} \in E$, then for $\alpha \in (0, 1]$:

$$[\tilde{y} + \tilde{v}]_\alpha = [\underline{y}(\alpha) + \underline{v}(\alpha), \overline{y}(\alpha) + \overline{v}(\alpha)]$$

$$[\tilde{y} . \tilde{v}]_\alpha = [\min k, \max k]$$

where $k = \{\underline{y}(\alpha)\,\underline{v}(\alpha), \underline{y}(\alpha)\,\overline{v}(\alpha), \overline{y}(\alpha)\,\underline{v}(\alpha), \overline{y}(\alpha)\,\overline{v}(\alpha)\}$.

Lemma (2), [7]:

Let $\tilde{v}(\alpha) = [\underline{v}(\alpha), \overline{v}(\alpha)]$, $\alpha \in (0, 1]$ be a given family of nonempty intervals. If:

(i) $[\underline{v}(\alpha), \overline{v}(\alpha)] \supset [\underline{v}(s), \overline{v}(s)]$, for $0 < \alpha \leq s$.

(ii) $[\lim_{k\to\infty} \underline{v}(\alpha, k), \lim_{k\to\infty} \overline{v}(\alpha, k)] = [\underline{v}(\alpha), \overline{v}(\alpha)]$.

whenever (α, k) is a non-decreasing sequence converging to $\alpha \in (0, 1]$, then the family $[\underline{v}(\alpha), \overline{v}(\alpha)]$, $\alpha \in (0, 1]$, represent the α-level sets of a fuzzy number v in E. Conversely, if $[\underline{v}(\alpha), \overline{v}(\alpha)]$, $\alpha \in (0, 1]$ are the α-level sets of a fuzzy number $\tilde{v} \in E$, then the conditions (i) and (ii) hold true.

Definition (2), [8]

Let I be a real interval. A mapping $\tilde{v} : I \longrightarrow E$ is called a fuzzy process and we denote the α-level set by $[\tilde{v}(t)]_\alpha = [\underline{v}(t, \alpha), \overline{v}(t, \alpha)]$.

The Seikkala derivative $\tilde{v}'(t)$ of $\tilde{v}$ is defined by:

$$[\tilde{v}'(t)]_\alpha = [\underline{v}'(t,\alpha), \overline{v}'(t,\alpha)]$$

provided that this equation defines a fuzzy number $\tilde{v}(t) \in E$.

Definition (3), [8]:

The fuzzy integral of a fuzzy process $\tilde{v}$, denoted by $\int_a^b \tilde{v}(t)\,dt$, for a, b $\in$ I, is defined by:

$$\left[\int_a^x \tilde{v}(t)\,dt\right]_\alpha = \left[\int_a^x \underline{v}(t,\alpha)\,dt, \int_a^x \overline{v}(t,\alpha)\,dt\right]$$

provided that the Lebsegue integrals exist.

VARIATIONAL ITERATION METHOD FOR SOLVING FUZZY DIFFERENTIAL EQUATIONS

In this section, the theory of VIM for solving initial value problems of fuzzy ordinary differential equations will be considered, which consists of the general form of the vriational iteration formula and its convergence to the exact solution. These two results are presented in the following main two theorems that may be labeled as the main results.

Theorem (1)[12]

Consider the fuzzy initial value problem:

$$\tilde{y}'(x) = f(x, \tilde{y}(x)),\ \tilde{y}(a) = \tilde{\alpha},\ a \le x \le b \qquad \dots(8)$$

where α is a fuzzy constant number and a(x) is continuous on [a, b], $\tilde{y}(x)$ is the solution which have to be determined. Then the variational iteration formula for solving (8) is given for all j = 1, 2, …, n; by:

$$\begin{aligned} \underline{y}_j^{k+1}(x,t) &= \underline{y}_j^{k}(x,t) - \int_a^x [\underline{y}_j'^{k}(t,\alpha) - \underline{y}_{j+1}^{k}(t,\alpha)]\,dt \\ \overline{y}_j^{k+1}(x,t) &= \overline{y}_j^{k}(x,t) - \int_a^x [\overline{y}_j'^{k}(t,\alpha) - \overline{y}_{j+1}^{k}(t,\alpha)]\,dt \end{aligned} \qquad \dots(9)$$

Proof:

Let ($\underline{y}(x,\alpha), \overline{y}(x,\alpha)$), $0 \le \alpha \le 1$, $a \le x \le b$ is a parametric form of $\tilde{y}(x)$. Then:

$$\underline{y}_j'(x,t) = \underline{y}_{j+1}(x,\alpha),\ \overline{y}_j'(x,t) = \overline{y}_{j+1}(x,\alpha);\ j = 1,2,\dots,n-1 \qquad \dots(10)$$

To solve eq.(4) by the VIM, the following formulae are obtained:

$$\begin{aligned} \underline{y}_j^{k+1}(x,t) &= \underline{y}_j^{k}(x,\alpha) + \int_a^x \underline{\lambda}_j(t,\alpha)[\underline{y}_j'^{k}(t,\alpha) - \underline{y}_{j+1}^{k}(t,\alpha)]\,dt \\ \overline{y}_j^{k+1}(x,t) &= \overline{y}_j^{k}(x,\alpha) + \int_a^x \overline{\lambda}_j(t,\alpha)[\overline{y}_j'^{k}(t,\alpha) - \overline{y}_{j+1}^{k}(t,\alpha)]\,dt \end{aligned} \qquad \dots(11)$$

where $\lambda(x, \alpha)$ is a general Lagrange multiplier which can be identified optimally via variational theory, $\underline{y}^k$ and $\overline{y}^k$ denotes the restricted variation, i.e.,

$$\delta \underline{y}^k = 0,\ \delta \overline{y}^k = 0$$

Where k is the number of iteration step. Then the iteration is calculated with respect to $\underline{y}_j^k$ and $\overline{y}_j^k$, j = 1, 2, …, n; respectively. Then we have:

$$\delta \underline{y}_j^{k+1}(x,t) = \delta \underline{y}_j^{k}(x,\alpha) + \delta \int_a^x \underline{\lambda}_j(t,\alpha)[\underline{y}_j'^{k}(t,\alpha) - \underline{y}_{j+1}^{k}(t,\alpha)]\, dt$$

$$= \delta \underline{y}_j^{k}(x,\alpha) + \underline{\lambda}_j(t,\alpha)\delta \underline{y}_j^{k}(t,\alpha)\Big|_{t=x} - \int_a^x \frac{\partial \underline{\lambda}_j(t,\alpha)}{dt}\delta \underline{y}_j^{k}(t,\alpha)\, dt$$

$$= (1+\underline{\lambda}_j(t,\alpha))\delta \underline{y}_j^{k}(t,\alpha) + \int_a^x -\frac{\partial \underline{\lambda}_j(t,\alpha)}{dt}\delta \underline{y}_j^{k}(t,\alpha)\, dt = 0$$

for arbitrary $\delta \underline{y}_j^{k}$, j = 1, 2, …, n; the following stationary condition is obtained:

$$-\frac{\partial \underline{\lambda}_j(t,\alpha)}{dt} = 0$$

with the natural boundary condition:

$$1+\underline{\lambda}_j(x,t) = 0,\ j = 1, 2, \ldots, n$$

The lower bound Lagrange multiplier can be defined as:

$$\underline{\lambda}_j(x,t) = -1,\ j = 1, 2, \ldots, n$$

Similarly, the upper bound Lagrange multiplier is given by:

$$\overline{\lambda}_j(x,t) = -1,\ j = 1, 2, \ldots, n$$

Hence, the following iteration formula can be obtained as:

$$\left.\begin{aligned} \underline{y}_j^{k+1}(x,t) &= \underline{y}_j^{k}(x,t) - \int_a^x [\underline{y}_j'^{k}(t,\alpha) - \underline{y}_{j+1}^{k}(t,\alpha)]\, dt \\ \overline{y}_j^{k+1}(x,t) &= \overline{y}_j^{k}(x,t) - \int_a^x [\overline{y}_j'^{k}(t,\alpha) - \overline{y}_{j+1}^{k}(t,\alpha)]\, dt \end{aligned}\right\} \quad \ldots (12)$$

So, beginning with:

$$\underline{y}_j^{0}(a) = \underline{\alpha}_j \text{ and } \overline{y}_j^{0}(a) = \overline{\alpha}_j \quad \ldots (13)$$

and hence the sequence of numerical solutions approximating the exact solution of eq.(8) may be obtained.

Theorem(2):

Consider:

$$y^c(\alpha) = \frac{\overline{y}(\alpha)+\underline{y}(\alpha)}{2},\ y^d(\alpha) = \frac{\overline{y}(\alpha)-\underline{y}(\alpha)}{2}$$

Then the fuzzy version of eq.(3) can be written as:

$$y_j'^{c}(x,\alpha) = y_{j+1}^{c}(x,\alpha),\ y_j'^{d}(x,\alpha) = y_{j+1}^{d}(x,\alpha),\ j = 1,2,\ldots,n-1 \cdots (14)$$

and for all j = 1, 2, …, n

$$y_j^c(a,\alpha) = \frac{\overline{y}_j(a)+\underline{y}_j(a)}{2} = \frac{\overline{\alpha}_j+\underline{\alpha}_j}{2}$$

$$y_j^d(a,\alpha) = \frac{\overline{y}_j(a)-\underline{y}_j(a)}{2} = \frac{\overline{\alpha}_j-\underline{\alpha}_j}{2} \quad \ldots (15)$$

Similarly, from eq.(5), we can obtain the following formula for all j = 1, 2, …, n – 1:

$$\left.\begin{aligned} \underline{y}_j^{k+1^c}(x,t) &= \underline{y}_j^{k^c}(x,t) - \int_a^x [\underline{y}_j'^{k^c}(t,\alpha) - \underline{y}_{j+1}^{k^c}(t,\alpha)]\,dt \\ \overline{y}_j^{k+1^c}(x,t) &= \overline{y}_j^{k^c}(x,t) - \int_a^x [\overline{y}_j'^{k^c}(t,\alpha) - \overline{y}_{j+1}^{k^c}(t,\alpha)]\,dt \end{aligned}\right\} \quad \ldots (16)$$

If $E_j^{k^c}(x,\alpha) = y_j^{k^c}(x,\alpha) - y_j^c(x,\alpha)$, then we must prove that $E_j^{k^c}(x,\alpha) \longrightarrow 0$ as $k \longrightarrow \infty$.

Proof:

Let $\underline{E}_j^{k^c}(x,\alpha) = \underline{y}_j^{k^c}(x,\alpha) - \underline{y}_j^c(x,\alpha)$, then we may obtain:

$\underline{E}_j^{k+1^c}(x,t) = \underline{E}_j^{k^c}(x,t) - \int_a^x [\underline{E}_j'^{k^c}(t,\alpha) - \underline{E}_{j+1}^{k^c}(t,\alpha)]\,dt$, j = 1, 2… n – 1

… (17)

Then eq.(17) may be rewritten as follows:

$$\underline{E}_j^{k+1^c}(x,t) = \int_a^x \underline{E}_{j+1}^{k^c}(t,\alpha)\,dt \quad \ldots (18)$$

Suppose $|\underline{E}_j^{k^c}| = \max_{a\le t\le b} |\underline{E}_j^{k^c}(t,\alpha)| = \max_j |\underline{E}_j^{k^c}(t,\alpha)|$ and $\underline{A}_j = \max_{a\le t\le b} |\underline{a}_j(t)| = \max_j |\underline{a}_j(t)|$, j = 1, 2, …, n; k = 0, 1, …. Then:

$$|\underline{E}_j^{1^c}(x,t)| \le \int_a^x |\underline{E}_{j+1}^{0^c}(t,\alpha)|\,dt \le x|\underline{E}^{0^c}|,\ j=1,2\ldots n-1 \quad \ldots (19)$$

$$|\underline{E}_j^{2^c}(x,t)| \le \int_a^x |\underline{E}_{j+1}^{1^c}(t,\alpha)|\,dt \le \frac{x^2}{2!}|\underline{E}^{0^c}|,\ j = 1, 2\ldots n-1 \quad \ldots (20)$$

and so on we may obtain in general for any k = 1, 2, …:

$$|\underline{E}_j^{k^c}(x,t)| \le \frac{x^k}{k!}|\underline{E}^{0^c}|,\ j = 1, 2\ldots n-1 \quad \ldots (21)$$

Thus, as $k \longrightarrow \infty$, we have $\underline{E}_j^{k^c} \longrightarrow 0$.

In a similar manner, we can prove that $\overline{E}_j^{k^c} \longrightarrow 0$ as $k \longrightarrow \infty$. Hence:

$$E_j^{k^c}(x,\alpha) \longrightarrow 0 \text{ as } k \longrightarrow \infty$$

Which imply the convergence of the sequence of approximations using VIM to the exact solution of the fuzzy initial value problem?

APPLICATIONS

In this section, two examples are solved to illustrate the applicability of the VIM derived in theorem (1) for solving fuzzy initial value problem. The results and the graphs are obtained using the computer software Mathcad 7.

Example (1):

Consider the nonlinear fuzzy initial value problem:

$$\tilde{y}' = x^2y + xy^2,\ \tilde{y}(0) = \tilde{1} \quad \dots (22)$$

where the initial conditions may be characterized in terms of α-level sets as:

$$[\tilde{y}_0]_\alpha = [\underline{\tilde{y}}_{0\alpha}(x),\ \overline{\tilde{y}}_{0\alpha}(x)]$$

$$= [1 - \sqrt{1-\alpha},\ 1 + \sqrt{1-\alpha}\,] \quad \dots (23)$$

The approximate solution by VIM

$$y_1(x) = \frac{x^2(2x+3)}{6} + 1$$

$y_2(x)=$

$$\frac{x^2(2x+3)}{6} + \frac{x^4(35x^4+120x^3+245x^2+588x+630)}{2520} + 1$$

$$y_3(x) = \frac{x^2(2x+3)}{6} + \frac{x^4(35x^4+120x^3+245x^2+588x+630)}{2520} + \frac{x^6}{12} - \frac{43x^7}{420} + \frac{61x^8}{720} + \frac{1493x^9}{22680} + \frac{1969x^{10}}{50400} + \frac{167x^{11}}{7560} + \frac{1249x^{12}}{100800} + \frac{593x^{13}}{98280} + \frac{143x^{14}}{51840} + \frac{17x^{15}}{16200} + \frac{631x^{16}}{2032128} + \frac{x^{17}}{12852} + \frac{x^{18}}{93312} + 1$$

and the fuzzy solution is:

$$\underline{\tilde{y}}_1(x,\alpha) = 1 - \frac{x^2\left(\sqrt{1-\alpha}-1\right)\left(2x-3\sqrt{1-\alpha}+3\right)}{6} - \sqrt{1-\alpha}$$

$$\overline{\tilde{y}}_1(x,\alpha) = \sqrt{1-\alpha} + \frac{x^2\left(\sqrt{1-\alpha}+1\right)\left(2x+3\sqrt{1-\alpha}+3\right)}{6} + 1$$

and when α = 1, we have the crisp solution:

$$\underline{\tilde{y}}_1(x) = \overline{\tilde{y}}_1(x) = \frac{x^2(2x+3)}{6} + 1$$

Similarly:

$$\underline{\tilde{y}}_2(x) = \overline{\tilde{y}}_2(x) \text{ and } \underline{\tilde{y}}_3(x) = \overline{\tilde{y}}_3(x), \text{ when } \alpha = 1$$

The results obtained for this example have been sketched in Fig.(1). And Table

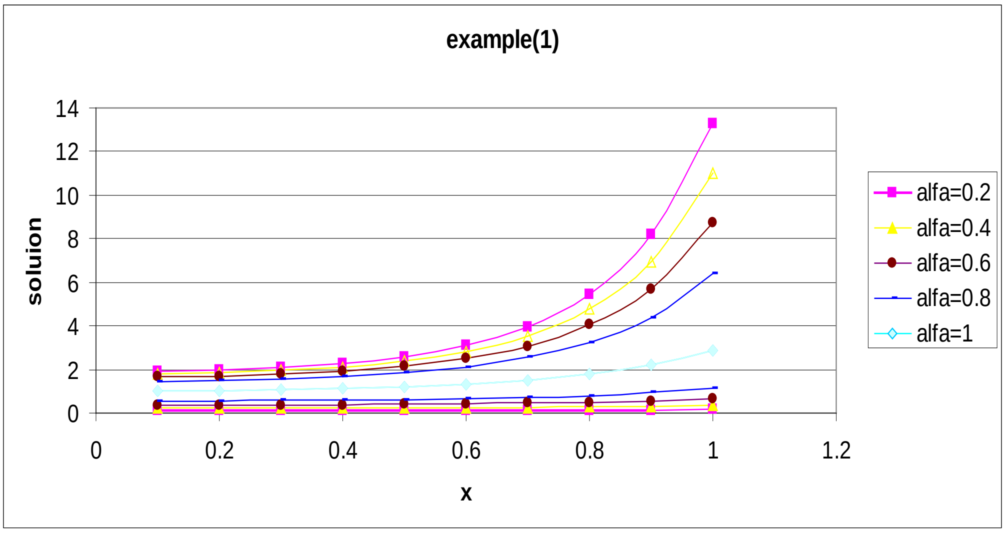

Figure (1) approximate solutions using VIM of example (1).

Table (1).

X	$\underline{\tilde{y}}_{0.2}$	$\overline{\tilde{y}}_{0.2}$	$\underline{\tilde{y}}_{0.4}$	$\overline{\tilde{y}}_{0.4}$	$\underline{\tilde{y}}_{0.6}$	$\overline{\tilde{y}}_{0.6}$	$\underline{\tilde{y}}_{0.8}$	$\overline{\tilde{y}}_{0.8}$	$\underline{\tilde{y}}_{1}$	$\overline{\tilde{y}}_{1}$
0	0.106	1.894	0.225	1.775	0.368	1.632	0.553	1.447	1	1
0.1	0.106	1.913	0.226	1.791	0.368	1.646	0.555	1.458	1.005	1.005
0.2	0.106	1.974	0.227	1.845	0.371	1.692	0.56	1.494	1.023	1.023
0.3	0.107	2.09	0.23	1.947	0.377	1.778	0.572	1.562	1.057	1.057
0.4	0.109	2.284	0.235	2.115	0.387	1.92	0.591	1.674	1.111	1.111
0.5	0.112	2.596	0.242	2.385	0.402	2.145	0.62	1.848	1.194	1.194
0.6	0.116	3.106	0.253	2.819	0.424	2.501	0.662	2.118	1.318	1.318
0.7	0.122	3.961	0.268	3.536	0.455	3.077	0.722	2.543	1.502	1.502
0.8	0.13	5.455	0.29	4.763	0.498	4.039	0.807	3.234	1.781	1.781
0.9	0.141	8.173	0.319	6.947	0.56	5.709	0.93	4.392	2.212	2.212
1	0.157	13.3	0.36	10.979	0.646	8.713	1.109	6.401	2.895	2.895

Example (2):

Consider the linear fuzzy initial value problem:

$$\tilde{y}' = 2y + e^x, \ \tilde{y}(0) = \tilde{1} \qquad \ldots (24)$$

where the initial conditions may be characterized in terms of α-level sets as:

$$[\tilde{y}_0]_\alpha = [\underline{\tilde{y}}_{0\alpha}(x), \ \overline{\tilde{y}}_{0\alpha}(x)]$$

$$= [1 - \sqrt{1-\alpha}, 1 + \sqrt{1-\alpha}] \qquad \ldots (25)$$

The approximate solution by VIM

$y_1(x) = 2x + e^x$

$y_2(x) = 3e^x + 2x^2 - 2$

$y_3(x) = 7e^x - 4x + \dfrac{4x^3}{3} - 6$

and the fuzzy solutions in lower case are given by:

$$\underline{\tilde{y}}_1(x,\alpha) = 2x + e^x - 2x\sqrt{1-\alpha} - \sqrt{1-\alpha}$$

$$\underline{\tilde{y}}_2(x,\alpha) = 3e^x - 2x\sqrt{1-\alpha} - \sqrt{1-\alpha} + 2x^2 - 2x^2\sqrt{1-\alpha} - 2$$

$$\underline{\tilde{y}}_3(x,\alpha) = 7e^x - 4x - 2x\sqrt{1-\alpha} - \sqrt{1-\alpha} + \frac{4x^3}{3} - 2x^2\sqrt{1-\alpha} - \frac{4x^3\sqrt{1-\alpha}}{3} - 6$$

and similarly for the upper case:

$$\overline{\tilde{y}}_1(x,\alpha) = 2x + e^x + 2x\sqrt{1-\alpha} + \sqrt{1-\alpha}$$

$$\overline{\tilde{y}}_2(x,\alpha) = 3e^x + 2x\sqrt{1-\alpha} + \sqrt{1-\alpha} + 2x^2 + 2x^2\sqrt{1-\alpha} - 2$$

$$\overline{\tilde{y}}_3(x,\alpha) = 7e^x - 4x + 2x\sqrt{1-\alpha} + \sqrt{1-\alpha} + \frac{4x^3}{3} + 2x^2\sqrt{1-\alpha} + \frac{4x^3\sqrt{1-\alpha}}{3} - 6$$

So, when α = 1, we have:

$$\underline{\tilde{y}}_1(x) = \overline{\tilde{y}}_1(x); \ \underline{\tilde{y}}_2(x) = \overline{\tilde{y}}_2(x) \text{ and } \underline{\tilde{y}}_3(x) = \overline{\tilde{y}}_3(x)$$

Which is also the same as the crisp solution?

The results obtained for this example have been sketched in Figure (2). And Table (2)

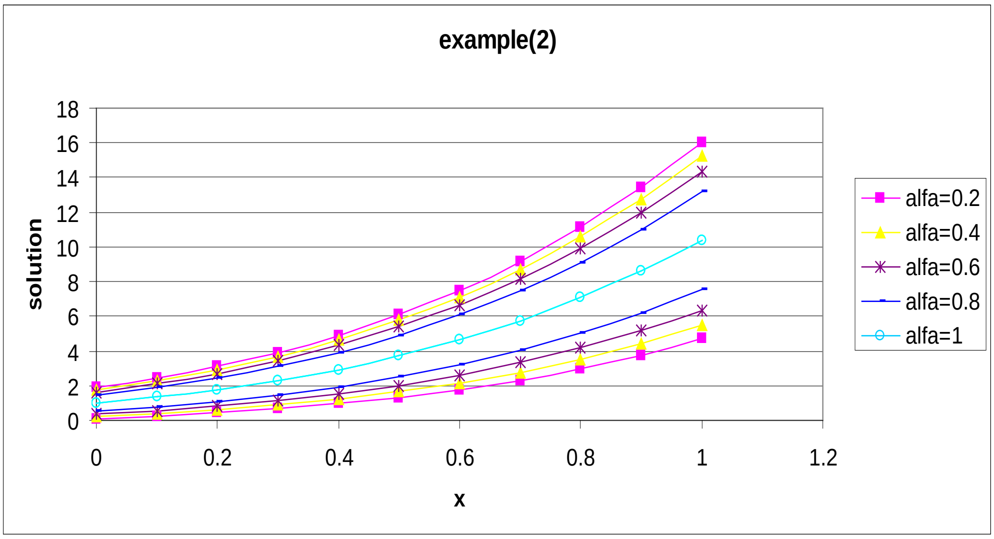

Figure (2) approximate solutions using VIM of example (2).

Table (2)

X	$\underline{\tilde{y}}_{0.2}$	$\overline{\tilde{y}}_{0.2}$	$\underline{\tilde{y}}_{0.4}$	$\overline{\tilde{y}}_{0.4}$	$\underline{\tilde{y}}_{0.6}$	$\overline{\tilde{y}}_{0.6}$	$\underline{\tilde{y}}_{0.8}$	$\overline{\tilde{y}}_{0.8}$	$\underline{\tilde{y}}_{1}$	$\overline{\tilde{y}}_{1}$
0	0.106	1.894	0.225	1.775	0.368	1.632	0.553	1.447	1	1
0.1	0.245	2.43	0.391	2.284	0.565	2.11	0.791	1.884	1.338	1.338
0.2	0.427	3.094	0.606	2.915	0.818	2.703	1.094	2.427	1.76	1.76
0.3	0.661	3.909	0.878	3.692	1.136	3.434	1.473	3.097	2.285	2.285
0.4	0.956	4.901	1.22	4.636	1.533	4.323	1.942	3.914	2.928	2.928
0.5	1.323	6.093	1.642	5.773	2.021	5.394	2.515	4.9	3.708	3.708
0.6	1.774	7.512	2.158	7.128	2.614	6.672	3.208	6.077	4.643	4.643
0.7	2.321	9.186	2.781	8.726	3.327	8.181	4.037	7.47	5.754	5.754
0.8	2.98	11.142	3.527	10.596	4.176	9.947	5.021	9.102	7.061	7.061
0.9	3.766	13.412	4.413	12.766	5.179	11.999	6.178	11.001	8.589	8.589
1	4.697	16.026	5.456	15.267	6.356	14.367	7.529	13.194	10.361	10.361

CONCLUSIONS

In this paper, we have proposed and used the He's variational iteration method to find the approximate analytical solution for fuzzy initial value problems in which the effective of the method had been proved and shown using two examples.

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