Numerical Solution of Optimal Control Problems Using New Third Kind Chebyshev Wavelets Operational Matrix of Integration

Asmaa Abdalelah Abdurrahman
Applied Siences, University of Technology/Baghdad
Email:abdalah-asmaa@yahoo.com

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ABSTRACT

In this paper, we first construct third kind Chebyshev wavelets on the interval [0,1). Then, a $2^k M \times 2^k M$ matrix P, named as almost third kind Chebyshev wavelets operational matrix of integration is constracted and used to reduce the optimal control problem to a system of algebric equation with the aid of spectral method, which can be solved easily. The uniform convergence of third kind Chebyshev wavelets is also discussed in this paper. The method is then tested on numerical example.

Keywords: Third Kind Chebyshev Polynomials, Wavelets, Optimal Control Problems, Spectral Methed.

الحل العددي لمسائل السيطرة المثلى باستخدام مصفوفة العمليات الجديدة لشيبيشيف الموجية الثالثة للتكاملات .

الخلاصة

في هذا البحث تم اولا بناء شيبيشيف الموجية الثالثة للفترة (0,1] ثم تكوين مصفوفة العمليات للتكاملات $2^k M \times 2^k M$ واستخدامها لحل مسائل السيطرة المثلى وتحويلها الى مجموعة من المعادلات الجبرية الخطية وحلها بطريقة الطيف التي تجعل الحل بسيطا. الاقتراب بانتظام لشيبيشيف الموجية الثالثة نوقش في هذا البحث وهذه الطريقة تم تطبيقها في حل المثال العددي.

INTRODUCTION

Wavelets are localized function which are a useful tool in many different applications: signal analysis, data compression, operator analysis, PDE solving, Vibration analysis and solid mechanics [1-4].

One of the popular families of wavelets is Haar wavelets [5], harmonic wavelets [6], Shnnon wavelets [7], Legendre wavelets [8], and Chebyshev wavelets of the first and second kinds [9,10].

The aim of the present paper is to constract new wavelets named third kind Chebyshev wavelets on the interval [0,1]. The related operational matrix of

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2412-0758/University of Technology-Iraq, Baghdad, Iraq

integration is derived which is suitable for approximate solution of optimal control problems.

THIRD KIND CHEBYSHEV POLYNOMIALS

Third kind Chebyshev polynomials are encountered in several areas of numerical analysis, and they hold particular impotance in varions subjects [11,12].

Defenetion(1) [13]: The third kind Chebyshev polynomial in [-1,1] of degree n is denoted by V_n and is defined by

$$V_n(x)\frac{\cos\left(n+\frac{1}{2}\right)\theta}{\cos\left(\frac{\theta}{2}\right)}, \text{ where} x=\cos\theta,\quad \frac{\theta}{2}\neq\frac{\pi}{2}+n\pi \qquad \dots (1)$$

This class of Chebyshev polynomials is satisfied in the following relations:

$$V_0(x)=1,\quad V_1(x)=2x-1,\quad V_n(x)=2xV_{n-1}(x)-V_{n-2}(x) \quad \text{n=2, 3…}$$

Defenetion (2) [13]: The third kind of Chebyshev polynomial in [a, b] of degree n is denoted by V_n^* and is defined by

$$V_n^*(x)=\frac{\cos\left(n+\frac{1}{2}\right)\theta}{\cos\left(\frac{1}{2}\right)\theta}, \theta\neq n\pi \quad \text{where} \quad \cos\theta=\frac{2x-(a+b)}{b-a} \quad \theta\in[0,\pi], b\neq a$$

For $x\in[a,b]$ if we put $=\frac{2x-(a+b)}{b-a}$, then s∈[-1,1] and $V_n^*(x)=V_n(s)$.

Third kind Chebyshev polynomials of degree n are orthogonal with respect to weight function

$$w(x)=\frac{\sqrt{(1+x^2)}}{\sqrt{(1-x^2)}}\ ,\ x\neq\mp1$$

THIRD KIND CHEBYSHEV WAVELETS

Wavelets constitut a family of functions constructed from dilation and translation of a single function called the mother wavelts. When the dilation parameter b vary continuously, we have the following family of continuous wavelets as

$$\Psi_{a,b}(t)=|a|^{-\frac{1}{2}}\left(\frac{t-b}{a}\right) \qquad ,a,b\in R, a\neq 0$$

Now Third kind Chebyshev wavelets

$$\Psi_{nm}^3=\Psi^3(k,m,n,t)$$

have four arguments; k=1,2.3…..,n=1,2,…,2^k ,m is the order for third kind Chebyshev polynomials and t is the normalized time.

They are defined on the interval [0,1) by:

$$\Psi^3_{n,m}(t)=\begin{cases}2^{\frac{k}{2}} & \check{V}_m(2^{k+1}t-2n+1)\ \frac{n-1}{2^k}\le t<\frac{n}{2^k}\\ 0 & otherwise\end{cases} \qquad \dots (3)$$

Where $\check{V}_m=\frac{1}{\sqrt{\pi}}V_m$, m=0, 1, 2,…, M-1 n=1,2,…,2^k

THIRD KIND CHEBYSHEV WAVELETS
Operational Matrix of Integration.

A function f(t) defined over [0,1) may be expanded as:

$$f(t)=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}f_{n,m}\,\Psi^3_{n,m}(t) \qquad \dots (4)$$

where

$$f_{n,m}=\left(f(t),\,\Psi^3_{n,m}(t)\right)$$

If the infinite series in equation(6) is trancated , then equation(6) can be written as:

$$f(t)\cong f_{2^k,M-1}=\sum_{n=1}^{2^k}\sum_{m=0}^{M-1}f_{n,m}\,\Psi^3_{n,m}(t)$$
$$=F^T\,\Psi^3(t) \qquad \dots (5)$$

where F and $\Psi^3(t)$ are $2^kM\times 1$ matrices given by:

$$F=\left[f_{1,0},f_{1,1},\dots,f_{1,M-1},f_{2,0},\dots,f_{2,M-1},\dots,f_{2^k,0},\dots,f_{2^k,M-1}\right] \qquad \dots (6)$$

$$\Psi^3(t)=$$
$$\left[\Psi^3_{1,0}(t)\,\Psi^3_{1,1}(t),\dots,\Psi^3_{1,M-1}(t),\Psi^3_{2,0}(t),\dots,\Psi^3_{2,M-1}(t),\dots,\Psi^3_{2^k,0}(t),\dots,\Psi^3_{2^k,M-1}(t)\right]^T \quad \dots (7)$$

For third kind Chebyshev wavelets the integration of the vector $\Psi^3(t)$ definds in equation (7) can be obtained as:

$$\int_0^t\Psi^3(s)ds\cong P\,\Psi^3(t)$$

Now, we will derive the operational matrix P of integration which plays a great role in dealing with the problem of optimal control . First we constract the 8×8 matrix P for k=1 ,M=3 .

By itegrating (3) from 0 to t and representing it to the matrix from, we obtain

$$\int_0^t \Psi^3_{1,0}(t)dt = \frac{3}{8}\Psi^3_{1,0} + \frac{1}{8}\Psi^3_{1,1} + \frac{1}{2}\Psi^3_{2,0}$$
$$= \begin{bmatrix}\frac{3}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0\end{bmatrix}\Psi^3_8$$
$$\int_0^t \Psi^3_{1,1}(t)dt = -\frac{1}{2}\Psi^3_{1,0} - \frac{1}{16}\Psi^3_{1,1} + \frac{1}{16}\Psi^3_{1,2} - \frac{1}{2}\Psi^3_{2,0}$$
$$= \begin{bmatrix}-\frac{1}{2} & -\frac{1}{16} & \frac{1}{16} & 0 & -\frac{1}{2} & 0 & 0 & 0\end{bmatrix}\Psi^3_8$$
$$\int_0^t \Psi^3_{1,2}(t)dt = \frac{5}{24}\Psi^3_{1,0} - \frac{1}{16}\Psi^3_{1,1} - \frac{1}{48}\Psi^3_{1,2} + \frac{1}{6}\Psi^3_{2,0}$$
$$= \begin{bmatrix}\frac{5}{24} & -\frac{1}{16} & -\frac{1}{48} & 0 & \frac{1}{6} & 0 & 0 & 0\end{bmatrix}\Psi^3_8$$
$$\int_0^t \Psi^3_{1,3}(t)dt = \frac{-657}{32}\Psi^3_{1,0} - \frac{1}{32}\Psi^3_{1,1} + \frac{33}{8}\Psi^3_{1,2} + \frac{9}{32}\Psi^3_{1,3} - \frac{1}{6}\Psi^3_{2,0}$$
$$= \begin{bmatrix}\frac{-657}{32} & -\frac{1}{32} & \frac{33}{8} & \frac{9}{32} & -\frac{1}{6} & 0 & 0 & 0\end{bmatrix}\Psi^3_8$$

$$\int_0^t \Psi^3_{2,0}(t)dt = \frac{3}{8}\Psi^3_{2,0} + \frac{1}{8}\Psi^3_{2,1}$$
$$= \begin{bmatrix}0 & 0 & 0 & 0 & \frac{3}{8} & \frac{1}{8} & 0 & 0\end{bmatrix}\Psi^3_8$$
$$\int_0^t \Psi^3_{2,1}(t)dt = -\frac{1}{2}\Psi^3_{2,0} - \frac{1}{16}\Psi^3_{2,1} + \frac{1}{16}\Psi^3_{2,2}$$
$$= \begin{bmatrix}0 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{1}{16} & \frac{1}{16} & 0\end{bmatrix}\Psi^3_8$$
$$\int_0^t \Psi^3_{2,2}(t)dt = \frac{5}{24}\Psi^3_{2,0} - \frac{1}{16}\Psi^3_{2,1} - \frac{1}{48}\Psi^3_{2,2}$$
$$= \begin{bmatrix}0 & 0 & 0 & 0 & \frac{5}{24} & -\frac{1}{16} & -\frac{1}{48} & 0\end{bmatrix}\Psi^3_8$$
$$\int_0^t \Psi^3_{2,3}(t)dt = -\frac{657}{32}\Psi^3_{2,0} - \frac{1}{32}\Psi^3_{2,1} + \frac{33}{8}\Psi^3_{2,2} + \frac{9}{32}\Psi^3_{2,3}$$
$$= \begin{bmatrix}0 & 0 & 0 & 0 & \frac{-657}{32} & -\frac{1}{32} & \frac{33}{8} & \frac{9}{32}\end{bmatrix}\Psi^3_8$$

Thus $\int_0^t \Psi^3_8(t')dt' = P_{8\times 8}\Psi^3_8(t)$

Where

$$\Psi^3_8(t) = [\Psi^3_{10}(t) \quad \Psi^3_{11}(t) \quad \Psi^3_{12}(t) \quad \Psi^3_{13}(t) \quad \Psi^3_{20}(t) \quad \Psi^3_{21}(t) \quad \Psi^3_{22}(t) \quad \Psi^3_{23}(t)]^T$$

For general case, we have

$$\int_0^t \Psi^3(t')dt' = P\,\Psi^3(t)$$

where P is a $2^k M \times 2^k M$ matrix for integration and given by

$$P = \begin{bmatrix} L & F & F & \dots & F & F \\ 0 & L & F & \dots & F & F \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & L & F \\ 0 & 0 & 0 & \dots & 0 & L \end{bmatrix}$$

Where F and L are $M \times M$ and given by

$$F=\frac{1}{2^k}\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ -1 & 0 & 0 & \dots & 0 \\ \frac{1}{3} & 0 & 0 & \dots & 0 \\ \frac{-1}{3} & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \frac{(-1)^{M+1}}{M} & 0 & 0 & \ddots & 0 \\ \frac{(-1)^{M+1}}{M-1} & 0 & 0 & \dots & 0 \end{bmatrix}$$

$$M \neq 1$$

$$L_{M\times M}=\frac{1}{4^k}\begin{bmatrix} \frac{3}{2} & \frac{1}{2} & 0 & 0 & \dots & 0 \\ -\frac{8}{4} & -\frac{1}{4} & \frac{1}{4} & 0 & \dots & 0 \\ \frac{5}{6} & -\frac{2}{8} & -\frac{2}{24} & 0 & \dots & 0 \\ \frac{-678}{8} & -\frac{2}{16} & -\frac{66}{8} & \frac{18}{16} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{2M} & -\frac{1}{2^M} & -\frac{1}{2^M} & -\frac{1}{M(2^M)} & \dots & \frac{1}{2^M} \\ \frac{-1}{2M} & -\frac{1}{2^M} & -\frac{1}{2^M} & -\frac{1}{2^M} & \dots & \frac{1}{2^M} \end{bmatrix}$$

CONVERGENCE OF THE THIRD KIND CHEBYSHEVE WAVELETS BASES

We shall prove that the third kind Chebyshev wavelets expansion of a function f(x), with bounded second derivative, converge uniformly to f(x).

Theorem(1): Afunction f(x) defined on [0,1), with bounded second derivative, say $|f''(x)| \leq B$, can be expanded as an infinite sum of third kind Chebyshev wavelets,

and the series f(x) $=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty} C_{nm}\, \Psi^{3}_{nm}(x)$ converges … (8)

where $C_{nm} = \int_0^1 f(t)\,\Psi^3_{nm}(t) w_k(t) dt$ … (9)

Proof:

Using eq.(9), yields

$$C_{nm} = \int_{(n-1)/2^k}^{n/2^k} 2^{\frac{k}{2}} f(t) \tilde{V}_m(2^{k+1}t - 2n + 1) w(2^{k+1}t - 2n + 1) dt \quad \dots (10)$$

Substitute $\cos\theta = 2^{k+1}t - 2n + 1$ when m>1in eq.(10) to get

$$C_{nm} = \frac{1}{2^{\frac{k}{2}}} \int_0^\pi f\left(\frac{\cos\theta + 2n - 1}{2^{k+1}}\right) \frac{1}{\sqrt{\pi}} \cos\left(m + \frac{1}{2}\theta\right) \cos\left(\frac{1}{2}\theta\right)$$

By using the integration by part,wehave,

$$= \frac{1}{2^{\frac{k}{2}} 2^{k+1} \sqrt{\pi}} \int_0^\pi f'\left(\frac{\cos\theta+2n-1}{2^{k+1}}\right) \sin\theta \left[\frac{\sin(m\theta)}{m} - \frac{\sin(m+1)\theta}{(m+1)}\right]$$

$$= \frac{1}{2^{\frac{k}{2}} 2^{2k+2} m\sqrt{\pi}} \int_0^\pi f'\left(\frac{\cos\theta+2n-1}{2^{k+1}}\right) \sin\theta \sin m\theta \, d\theta$$

$$- \frac{1}{2^{\frac{k}{2}} 2^{2k+2} (m+1)\sqrt{\pi}} \int_0^\pi f'\left(\frac{\cos\theta+2n-1}{2^{k+1}}\right) \sin\theta \sin(m+1)\theta \, d\theta$$

$$= \frac{1}{2^{\frac{k}{2}} 2^{4k+4} m\sqrt{\pi}} \int_0^\pi f''\left(\frac{\cos\theta+2n-1}{2^{k+1}}\right) \sin\theta \left[\frac{\sin(m-1)}{(m-1)} - \frac{\sin(m+1)\theta}{(m+1)}\right] d\theta$$

$$- \frac{1}{2^{\frac{k}{2}} 2^{4k+4} (m+1)\sqrt{\pi}} \int_0^\pi f''\left(\frac{\cos\theta+2n-1}{2^{k+1}}\right) \sin\theta \left[\frac{\sin m\theta}{(m)} - \frac{\sin(m+2)\theta}{(m+2)}\right] d\theta$$

$$= \frac{1}{2^{\frac{9k}{2}} 2^4 m(m+1)\sqrt{\pi}} \int_0^\pi f''\left(\frac{\cos\theta+2n-1}{2^{k+1}}\right) [(m+1)L_m(\theta) - mH_m(\theta)] \, d\theta$$

where $L_m(\theta) = \sin\theta \left[\frac{\sin(m-1)\theta}{m-1} - \frac{\sin(m+1)\theta}{m+1}\right]$

and $H_m(\theta) = \sin\theta \left[\frac{\sin(m)\theta}{m} - \frac{\sin(m+2)\theta}{m+2}\right]$

thus we get

$$|C_{nm}| = \left| \frac{1}{2^{9\frac{k}{2}} 2^4 m(m+1)\sqrt{\pi}} \int_0^\pi f''\left(\frac{\cos\theta + 2n - 1}{2^{k+1}}\right) [(m+1)L_m - mH_m] \, d\theta \right| \leq$$

$$\frac{1}{2^{9\frac{k}{2}} 2^4 m(m+1)\sqrt{\pi}} \int_0^\pi \left| f''\left(\frac{\cos\theta + 2n - 1}{2^{k+1}}\right) [(m+1)L_m - mH_m] \right| d\theta \leq$$

$$\frac{N}{2^{9\frac{k}{2}}2^4 m(m+1)\sqrt{\pi}}\int_0^{\pi}|[(m+1)L_m - mH_m]|\,d\theta$$

However

$\int_0^{\pi}|[(m+1)L_m - mH_m]|\,d\theta$

$\le (m+1)\int_0^{\pi}\left|\sin\theta\left[\frac{\sin(m-1)\theta}{(m-1)} - \frac{\sin(m+1)\theta}{(m+1)}\right]\right|d\theta - m\int_0^{\pi}\left|\sin\theta\left[\frac{\sin m\theta}{m} - \frac{\sin(m+2)\theta}{m+2}\right]\right|d\theta$

1) $(m+1)\int_0^{\pi}|L_m(\theta)|\,d\theta = (m+1)\int_0^{\pi}\sin\theta\left|\frac{\sin(m-1)\theta}{m-1} - \frac{\sin(m+1)\theta}{m+1}\right|d\theta$

$\le (m+1)\int_0^{\pi}\left|\frac{\sin\theta\sin(m-1)\theta}{m-1}\right| + \left|\frac{\sin\theta\sin(m+1)\theta}{m+1}\right|d\theta \le \frac{2m\pi}{(m^2-1)}$

2) $m\int_0^{\pi}|H_m(\theta)|\,d\theta = m\int_0^{\pi}\sin\theta\left|\frac{\sin(m)\theta}{m} - \frac{\sin(m+2)\theta}{m+2}\right|d\theta$

$\le (m)\int_0^{\pi}\left|\frac{\sin\theta\sin(m)\theta}{m}\right| + \left|\frac{\sin\theta\sin(m+2)\theta}{m+2}\right|d\theta \le \frac{2\pi}{(m+2)}$

Then since n$\le 2^{k-1}$ we obtained

$$|C_{nm}| < \frac{N\sqrt{2\pi}}{(2n)^{\frac{9}{2}}(m^2-1)(m+2)} \qquad \dots (11)$$

Now, if m=1, by using (11), we have

$$|C_{1n}| < \frac{\sqrt{2\pi}}{(2n)^{9/2}}\max_{0\le x\le 1}|f'(x)|$$

$\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}C_{nm}$ Absolutly convergent for m=0

$\{\Psi_{n0}\}_{n=1}^{\infty}$ Orthogonal system

$\left|\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}C_{nm}\,\Psi^3_{nm}(x)\right| \le \left|\sum_{n=1}^{\infty}C_{n0}\,\Psi^3_{n0}(x)\right| + \sum_{n=1}^{\infty}\sum_{m=0}^{\infty}|C_{nm}|\left|\Psi^3_{nm}(x)\right|$

$\le \left|\sum_{n=1}^{\infty}C_{n0}\,\Psi^3_{n0}(x)\right| + \sum_{n=1}^{\infty}\sum_{m=0}^{\infty}|C_{nm}| < \infty$

Then $\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}C_{nm}\,\Psi^3_{nm}(x)$ converges to f(x) uniformly.

APPLICATION OF THE THIRD CHEBYSHEV WAVELETS OPERATIONAL MATRIX OF INTEGRATION.

In this section, we demonstrate the application of the operational matrix of which derived in this work to solve optimal Control problem.

The optimal Cntrol problem

$$J = \int_0^1 (x^T Q x + u^T R u)\,dt \qquad \dots (12)$$

Subject to

$$\dot{x} = Ax + Bu \qquad \dots (13)$$

$$x(0)=x_0$$

We must solve this problem by find solution Hamiltonian Equation [].

The initial condition vector x_0 can be expressed via third Chebyshev wavelets as

$$x_0 = \frac{\sqrt{\pi}}{2^{k/2}}\left(I_s \otimes \Psi^3_{nm}\right)G_0$$

$G_0 = [c_0^1 \quad c_0^2 \dots \dots c_0^s]$ I_s is identity matrix s×s $\Psi^3(t)$ is s×1

$c_0^i = [x_i(0)\ \ 0\ \ 0 \dots \dots 0\ \ x_i(0)\ \ 0\ \ 0 \dots \dots 0]$ From Equation (3) we can obtained

$$\Psi^3_{n0} = \frac{2^{k/2}}{\sqrt{\pi}}$$

then $x_0 = \frac{\sqrt{\pi}}{2^{k/2}} \Psi^3_{n0}(t)$.

First we assume:

$x^r(t) = C^T \Psi^3_{nm}(t)$ r is higher derivitiv in Hameltonian equation. … (14)

$$x(t) = C^T P^r \Psi^3_{nm} + x_0^{(r-1)} t + x_0^{(r-2)} \frac{t^2}{2} + \dots + \dot{x}_0 \frac{t^r}{r} + x_0 \ \frac{n-1}{2^k} \le t < \frac{n}{2^k} \quad \dots (15)$$

Example 1

Cosider the Optemal Control problem

$$\min J = \int_0^1 (x^2 + u^2)\, dt \quad \dots (16)$$

subject to $\dot{x} = u, x(0) = 1$ using Hamiltonian method

H=$f_0 + \lambda f$ $\quad f_0 = x^2 + u^2 \quad f = u$

$\frac{-\partial H}{\partial x} = \dot{\lambda} \qquad \frac{\partial H}{\partial U} = 0$

The Hamiltonian is

$$H = x^2 + u^2 + \lambda u$$

By solving this equation we obtain

$$\ddot{x} - x = 0 \quad \dots (17)$$

exact solution $x(t) = \frac{cosh(1-t)}{cosh\, 1}$, $u(t) = \frac{-sinh(1-t)}{cosh\, 1}$

Let $\ddot{x} \simeq C^T \Psi^3_{nm}$ … (18)

$$\dot{x}(t) = C^T P\, \Psi^3_{nm}(t) + \dot{x}(0)$$

$$x(t) = C^T P^2\, \Psi^3_{nm} + \dot{x}(0)t + x(0) \quad \dots (19)$$

$x(0) = \frac{\sqrt{\pi}}{2^{k/2}} G_0$, $\quad G_0 = [1\ 0\ 0 1\ 0\ 0]\, \Psi^3_{nm}$ then $x(0) = \frac{\sqrt{\pi}}{\sqrt{2}} \left[\Psi^3_{10}\ 0\ 0\ \Psi^3_{20}\ 0\ 0 \right]$ … (20)

$$t = \frac{\sqrt{\pi}}{\sqrt{2}} \left[\frac{3}{8}\ \ \frac{1}{8}\ \ 0\ \ \frac{7}{8}\ \ \frac{1}{8}\ \ 0 \right] \Psi^3_{nm} \qquad \dots (21)$$

Substitute (20),(21) in (19) yeilds

$$x(t) = C^T P^2\, \Psi^3_{nm} + d^T\, \Psi^3_{nm}$$

$$d^T \Psi^3_{nm} = [0.895370\ \ -0.11931459\ \ 0\ \ 0.41811204\ \ -0.11931459\ \ \ 0\,]^T\ \Psi^3_{nm}$$

From abov we obtaned

$$C^T P^2\, \Psi^3_{nm} + d^T \Psi^3_{nm} - C^T\, \Psi^3_{nm} = 0 \qquad (33)$$

By solving (33) by spectral method we obtaied the following eqs.

$$\frac{5}{64} C_{10} - \frac{55}{384} C_{11} + \frac{121}{1152} C_{12} - C_{10} = -0.89537037$$

$$\frac{5}{128} C_{10} - \frac{1}{16} C_{11} + \frac{1}{32} C_{12} - C_{11} = 0.11931452$$

$$\frac{1}{128} C_{10} - \frac{1}{192} C_{11} - \frac{1}{288} C_{12} - C_{12} = 0$$

$$\frac{5}{16} C_{10} - \frac{38}{96} C_{11} + \frac{56}{288} C_{12} + \frac{5}{64} C_{20} - \frac{55}{384} C_{21} + \frac{121}{1152} C_{22} - C_{20} = -0.41811204$$

$$\frac{1}{16} C_{10} - \frac{1}{16} C_{11} + \frac{1}{48} C_{12} + \frac{5}{128} C_{20} - \frac{1}{16} C_{21} + \frac{1}{32} C_{22} - C_{21} = 0.11931452$$

$$\frac{1}{128} C_{20} - \frac{1}{192} C_{21} - \frac{1}{288} C_{22} - C_{22} = 0$$

Solving the above system to get C^T

$$C^T = [0.98395684\ -0.07588431\ \ 0.00805443\ \ 0.82510875\ \ -0.01926789\ \ \ 0.00652386\ \,]^T$$

Following table shows numerical reselts of example above

Table (1) shows reselts of third Chebyshev wavelets.

t	ex x(t)	Rd Ch.w x(	error	2ed Ch.w x(t	ex u(t)	Rd Ch.w u(t)	error
0	1	0.99885723	0.00114277	0.99930248	-0.76159416	-0.75971976	0.00187440
0.1	0.92871776	0.92882665	0.00000889	0.92886474	-0.66523855	-0.66534063	0.00000208
0.2	0.86673043	0.86702199	0.00002916	0.86686167	-0.57554088	-0.57597663	0.00004357
0.3	0.81341764	0.81344326	0.00000256	0.81329249	-0.49160341	-0.49162776	0.00000244
0.4	0.76824580	0.76809044	0.00001554	0.76815747	-0.41258607	-0.41229402	0.00002921
0.5	0.73076282	0.73096356	0.00002007	0.73145661	-0.33769804	-0.33779754	0.00002774
0.6	0.70059357	0.70069986	0.00001063	0.70125513	-0.26618980	-0.26625406	0.00000643
0.7	0.6774360	0.6775740	0.00001380	0.67794531	-0.19734568	-0.19768738	.00003417
0.8	0.6610586	0.6611110	0.00000524	0.66139415	-0.13047666	-0.13048939	.00000127
0.9	0.6512972	0.6513108	0.00000136	0.65160163	-0.06491340	-0.06466008	.00002533
	0.6480542	0.6481733	0.00001191	0.64856776	0	0	0
Max Error=0.00114277					Max Error=0.00187440		

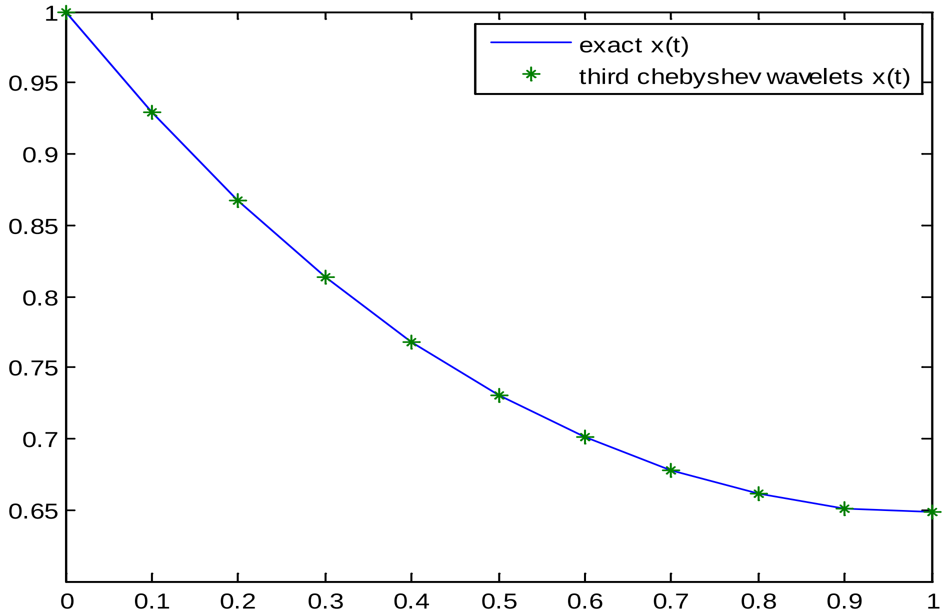

Figure (1) shows third Chebyshev wavelets x(t) with exact x(t).

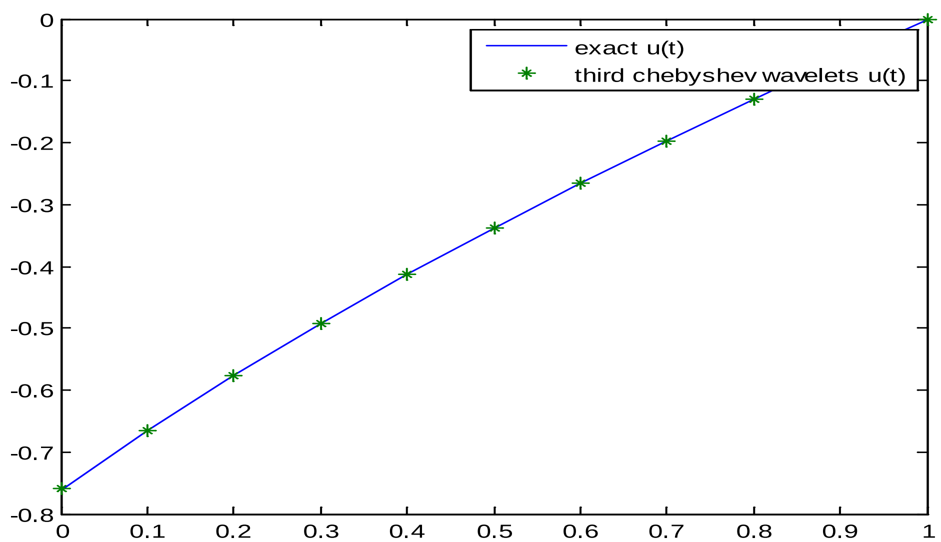

Figure (2) shows third Chebyshev wavelets u(t) with exact u(t).

Example 2

Consider the finite time quadratic problem

Min $[J] = \int_0^1 u^2 dt$

The exact solution to this problem is given by

$x_1(t) = t^3 - 3t^2 + t + 1$ and $u = 6t - 6$.

In order to apply the proposed method , one first finds
The Hamiltonian equation

$$H = u^2 + \lambda_1 x_2 + \lambda_2 u$$

and the adjoint equations

$$-\frac{\partial H}{\partial x_1} = \dot{\lambda}_1 \quad , -\frac{\partial H}{\partial x_2} = \dot{\lambda}_2, \quad \frac{\partial H}{\partial u} = 0$$

From the above equations , we obtain the following

$$\dot{x} + \ddot{x} = 3t^2 - 5$$

Similerly example(1)

$d = [-6.9324564 \ \ 0.29374550 \ \ 0.058749100 \ \ -4.5824298 \ \ 0.76373830 \ \ 0.058749100 \ ]^T$

and

$C^T = [0.98395684 \ -0.07588431 \ \ 0.00805443 \ \ 0.82510875 \ -0.01926789 \ \ 0.00652386 \ ]^T$

Therefore the following approximate solution with be achieved

$$x_1(t) = C^T P^2 \Psi^3_{nm} + \dot{x}_1(0)t + x_1(0)$$

which is the exact solution

CONCLUSIONS

In this paper a general formulation for the third kind Chebyshev wavelets Ψ^3 was presented then the convergence theorem of Ψ^3. Its operational matrix of itegration has been derived. Then an approximated method based on third kind Chebyshev wavelets expansions together with the operational matrix of integrationare proposed to obtaind an approximate solution of optimal control problems using spectral method.

The numerical reselt show the method is very efficient for the numerical solution of optemal control problems and only few number of Ψ^3 expansion terms are needed to obtain a good approximate solution for these problems.we can modify this method for the numerical solution of other problems such as nonlinear optimal control problems and integral equations.

Using third Chebyshev wavelets give high accuracy approximation of solution Optimal problems in Example (1) show accuracy of this method compared with second Chebyshev wavelets.

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