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Some common random fixed point theorems In S- metric Space with new type of rational contractive conditions.

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ABSTRACT

In this paper , we prove some common random fixed point theorems for new type of rational contraction mappings in generalized metric space which is called S – metric space .

Keywords : Rondom fixed point , common rondom fixed point ,rational contraction mapping , S-metric space .

بعض مبرهنات النقطة الصامدة العشوائيةالمشتركةلصنف جديد من التطبيقات الانكماشية النسبية في فضاء-S- المتري

المستخلص

في هذا البحث ,برهنا بعض مبرهنا ت النقطة الصامدة العشوائية المشتركة لصنف جديد من التطبيقات الانكماشية النسبية في فضاءات مترية موسعة والتي عرفت بأسم فضاء -S- المتري. نتائجنا هي تعميم وتوسيع لبعض النتائج المعروفة.

الكلمات المفتاحية

النقطة الصامدة العشوائية, النقطة الصامدة العشوائية المشتركة, التطبيق الانكماشي النسبي ,فضاء –S- المتري .

INTRODUCTION

Probalistic functional analysis has emerged as one of the important mathematical disciplines in view of its role in analyzing probabilistic models in the applied sciences. The study of fixed points of rondom operaters form a central topic in the area .The prague school of probabilistic initiated its study in the 1950 .However , the research in thise area flourished after the publication of the survey article of Bharucha –Reid [1] .Since then many instructing random fixed point results and several applications have appeared in the literature for example the work of Beg and shahazad [2,3], Lin[4],Oregan [5],papag eargiou [6],Xu[7]. In recent years, the study of random fixed points has attracted much attention . In particular random iteration

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schemes leading to random point of random operators have been discussed in [8,9,10,11,12,13,14].Recently , shaban and Nabi [15,16] introduced the concept of a generalized metric space which is more generalized of D – metric space and D^*-metric space that is S-metric space and they proved some basic properties and some fixed point theorems in S-metric space . In this paper , we prove some random fixed point and common random fixed point for random operaters satisfying new type of rational conditions which are more modifying than the results in [15] in the setting of S-metric space .

Preliminaries

Definitions

Let X be nonempty set .A generalized metric (or S- metric) on X is a function S : : $X\times X \times X \to [0,\infty]$ that satisfies the following conditions for each x,y,z,a $\in X$

1. $S(x,y,z) \geq 0$
2. $S(x,y,z) = 0$ if and only if x=y=z
3. $S(x,y,z) \leq S(x,x,a)+ S(y,y,a) +S(z,z,a)$

The pair (X,S) is called a generalized metric space or (S – metric) space .

Example

1. Let $X=R^n$ and ||.|| a norm on X, then S(x, y, z)= ||x-z||+||y-z|| is an S-metric space.
2. Let X be non empty set, d is ordinary metric on X, then S(x, y, z)=d(x,z)+d(y,z) is an S-metric space.

As a special case of previous example (2) is the following :

Example

If X=R ,define $S(x,y,z)=|a^{y+z} - a^{2x}| +|y-z|$ for every x,y,z,a $\in R$, $a > 0$, $a \neq 1$
Then (R,S) is S- metric space
For more examples of S-metric space we refer to [15] .

Remark

In a S- metric space , proved that S(x,y,y) = S(y,x,x) , for proof can see [15] .

Remark

Let (X,S) be a S-metric space . If define f : $X\times X \to [0,\infty)$ as f(x,y) =S(x,y,y) for all x,y$\in X$ then f is an ordinary metric on X. For proof can see [15] .

Let (X,S) be S-metric space .For $r > 0$ define $B_s(x,r)=\{y\in X: S(x,y,y) < r\}$[15].

Example

Let X=R,denote $S(x,y,z)=|3^{y+z} - 3^{2x}|+|y-z|$ for all x,y,z $\in R$. Thus $B_s(1,2)=\{y\in R: S(1,y,y) < 2\} = \{y \in R: |3^{2y} - 3^2| < 2\}$

$$=\{y\in R: \frac{\log_3^7}{2} < y < \frac{\log_3^{11}}{2}\}$$

$$= (\frac{\log_3^7}{2}, \frac{\log_3^{11}}{2}).$$

Definition

(Let (X,S) be S-metric space and x$\in X$:

1-If for every x$\in A$ *there exists* $r > 0$ *such that* $B_s(x,r) \subset$ A, then subset A is called open subset of X.

2- A sequence $\{x_n\}$ in X converges to x if and only if $S(x_n,x,x) = S(x, x_n, x_n) \to 0$ as $n \to \infty$ .That is for each $\epsilon > 0$ there exists $n_0 \in N$ such that $\forall n \geq n_0 \to S(x, x_n, x_n) < \epsilon$.

3-A sequence $\{x_n\}$ in X is called a Cauchy sequence if for each $\epsilon > 0$, there exists $n_0 \in N$ such that $S(x_n, x_m, x_m) < \epsilon$ for each $n,m \geq n_0$.

4-.The S-metric space (X,S) is said to be complete if every cauchy sequence is convergent

Let τ be the set of all $A \subset X$ if and only if there exists $r > 0$ such that $B_s(x,r) \subset$ A.Then τ is a topology on X (induced by the S-metric).

Lemma

Let (X,S) be S-metric space .If $r > 0$,then ball $B_s(x,r)$ with centre x$\in$X and Redius r is open ball . For proof can see [15].

Lemma

Let (X,S) be a S-metric space .If sequence $\{x_n\}$ in X Converges to x, then x is unique .For proof can see [15].

Lemma

Let (X,S) be a S-metric space .The sequence $\{x_n\}$ in X is Convergent to x,then sequence $\{x_n\}$ is a Cauchy sequence .For proof can see [15]

For more basic concepts in S- metric space , can see [15] . Now we will recall the following concepts which are necessary in this paper .(can see [4],[12],[14]).

Let (X,d) be metric space and (Ω, ε) be measurable space . Let 2^x be a family Of all subsets of X. A mapping T: $\Omega \to 2^x$ is called measurable if for any open Subset C of X , $T^{-1}(c)=\{w\in \Omega: T(w) \cap c \neq \emptyset\} \in \varepsilon$A mapping $\varepsilon: \Omega \to X$ is said to be measurable selector of a measurable

Mapping T:D$\to 2^x$ if ε is measurable and for any w$\in \Omega$, $\varepsilon(w) \in$ T(w).

A mapping f: $\Omega \times X \to X$ is called random mapping , if for any x$\in X$, $f(\cdot, x)$ is Measurable .A measurable mapping $\varepsilon: \Omega \to X$ is called random fixed point of A random mapping F : $\Omega\times X \to$ X , if for every w$\in \Omega$,f(w, $\varepsilon(w)$) = $\varepsilon(w)$.

In this paper we will use the previous concepts in the setting of S-metric space .

In the following section we take Ω =R , where R is the set of real numbers.

Main results

Theorem(3.1)

Let (X,S) be complete S-metric space and let F,T :R$\times X \to X$ be two random mappings satisfying the following condition :

S(f(w,x),T(w,y),T(w,y)) $\leq$

$$\alpha(w)\frac{[S(y,T(w,y),T(w,y))+S(x,F(w,x),F(w,x))]S(x,y,y)}{S(x,T(w,y),T(w,y))+S(y,F(w,x),F(w,x))}$$

$$+\beta(w)\frac{[S(x,T(w,y),T(w,y))+S(x,F(w,x),F(w,x))]S(x,y,y)}{S(y,T(w,y),T(w,y))+S(y,F(w,x),F(w,x))} \quad \ldots(3.1.1)$$

Where

$S(x,T(w,y),T(w,y)) + S(y,F(w,x),F(w,x)) \geq$

$S(y,T(w,y),T(w,y)) + S(x,F(w,x),F(w,x))$ ……………(3.1.2)

And $S(y,T(w,y),T(w,y)) + S(y,F(w,x),F(w,x)) \geq$

$S(x,T(w,y),T(w,y)) + S(x,F(w,x),F(w,x))$ ……………….(3.1.3)

For each x,y $\in X$, w $\in R$ and α,β are measurable mappings defined by $\alpha,\beta: R \rightarrow (0,1)$ with $0 \leq \propto(w) + \beta(w) < 1$. Then there exists a unique common random fixed point of F and T .

Proof

Let g_0 :R→ X be an arbitrary measurable mapping and choose a measurable mapping g_1 :R→ X such that $g_1(w) = F(w,g_0(w))$ for each w∈ R . Then by(3.1.1) we have:

$$S(F(w,g_0(w)),T(w,g_1(w)),T(w,g_1(w))) \leq$$

$$\alpha(w)\frac{[S(g_1(w),T(w,g_1(w)),T(w,g_1(w)))+S(g_0(w),F(w,g_0(w)),F(w,g_0(w)))]S(g_0(w),g_1(w),g_1(w))}{S(g_0(w),T(w,g_1(w)),T(w,g_1(w))+S(g_1(w),F(w,g_0(w),F(w,g_0(w)))}$$

$$+\beta(w)\frac{[S(g_0(w),T(w,g_1(w)),T(w,g_1(w))+S(g_0(w),F(w,g_0(w)),F(w,g_0(w)))]S(g_0(w),g_1(w),g_1(w))}{S(g_1(w),T(w,g_1(w)),T(w,g_1(w)))+S(g_1(w),F(w,g_0(w)),F(w,g_0(w)))}$$

Further there exists g_2 :R→ X such that for all w∈ R , $g_2(w) = T(w,g_1(w))$ so,

$$S(g_1(w),g_2(w),g_2(w)) = S(F(w,g_0(w)),T(w,g_1(w)),T(w,g_1(w))) \leq$$

$$\alpha(w)\frac{[S(g_1(w),g_2(w)),g_2(w))+S(g_0(w),g_1(w),g_1(w))]S(g_0(w),g_1(w),g_1(w))}{S(g_0(w),g_2(w),g_2(w))+S(g_1(w),g_1(w),g_1(w))}$$

$$+\beta(w)\frac{[S(g_0(w),g_2(w),g_2(w))+S(g_0(w),g_1(w),g_1(w))]S(g_0(w),g_1(w),g_1(w))}{S(g_1(w),g_2(w),g_2(w))+S(g_1(w),g_1(w),g_1(w))}$$

$$\leq\left(\begin{array}{l}\frac{\alpha(w)[S(g_1(w),g_2(w)),g_2(w))+S(g_0(w),g_1(w),g_1(w))]}{S(g_0(w),g_2(w),g_2(w))}+\\ \frac{\beta(w)[S(g_0(w),g_2(w),g_2(w))+S(g_0(w),g_1(w),g_1(w))]}{[S(g_1(w),g_2(w)),g_2(w))}\end{array}\right)S(g_0(w),g_1(w),g_1(w))$$

By (3.1.2) and (3.1.3) we have :

$$S(g_1(w),g_2(w)),g_2(w)) \leq \left(\alpha(w)+\beta(w)\right)S(g_0(w),g_1(w),g_1(w))$$

Let K $=\left(\alpha(w)+\beta(w)\right)$

So, $S(g_1(w),g_2(w),g_2(w)) \leq$ K $S(g_0(w),g_1(w),g_1(w))$

By Beg and Shah zad [2,lemma 2.3] we obtain g_3 :R→ X such that for all w∈ R , $g_3(w) = F(w,g_2(w))$ and

$$S(g_2(w),g_3(w),g_3(w)) = S(T(w,g_1(w)),F(w,g_2(w)),F(w,g_2(w)))$$

$$= S(F(w,g_2(w)), T(w,g_1(w)), T(w,g_1(w)))$$

$$\le \alpha(w)\left(\frac{S(g_1(w),T(w,g_1(w)),T(w,g_1(w)))+S(g_2(w),F(w,g_2(w),F(w,g_2(w)))}{S(g_2(w),T(w,g_1(w)),T(w,g_1(w)))+S(g_1(w),F(w,g_2(w),F(w,g_2(w)))}\right)\cdot S(g_2(w),g_1(w),g_1(w))$$

$$+\beta(w)\left(\frac{S(g_2(w),T(w,g_1(w)),T(w,g_1(w)))+S(g_2(w),F(w,g_2(w),F(w,g_2(w)))}{S(g_1(w),T(w,g_1(w)),T(w,g_1(w)))+S(g_1(w),F(w,g_2(w),F(w,g_2(w)))}\right) S(g_2(w),g_1(w),g_1(w))$$

$$=\alpha(w)\left(\frac{S(g_1(w),g_2(w)),g_2(w))+S(g_2(w),g_3(w)),g_3(w))}{S(g_2(w),g_2(w)),g_2(w))+S(g_1(w),g_3(w)),g_3(w))}\right)S(g_2(w),g_1(w)),g_1(w))$$

$$+\beta(w)\left(\frac{S(g_2(w),g_2(w)),g_2(w))+S(g_2(w),g_3(w)),g_3(w))}{S(g_1(w),g_2(w)),g_2(w))+S(g_1(w),g_3(w)),g_3(w))}\right)S(g_2(w),g_1(w)),g_1(w))$$

$$=\left(\begin{array}{c}\dfrac{\alpha(w)[S(g_1(w),g_2(w)),g_2(w))+S(g_2(w),g_3(w)),g_3(w))]}{S(g_1(w),g_3(w)),g_3(w))}+\\ \left(\dfrac{\beta(w)[S(g_2(w),g_3(w)),g_3(w))}{S(g_1(w),g_2(w)),g_2(w))+S(g_1(w),g_3(w)),g_3(w))}\right)\end{array}\right)S(g_2(w),g_1(w)),g_1(w))$$

By (3.1.2) and (3.1.3) we have :

$S(g_2(w),g_3(w)),g_3(w)) \le (\alpha(w)+\beta(w))\, S(g_2(w),g_1(w)),g_1(w))$

$\le K\ S(g_2(w),g_1(w)),g_1(w))$

Also by remark (2.5) we have :

$S(g_2(w),g_3(w)),g_3(w)) \le K\ S(g_1(w),g_2(w)),g_2(w))$

$\le K^2 S(g_0(w),g_1(w)),g_1(w))$

Similarly ,proceeding in the same way by induction we produce a sequence of mappings g_n:R→X , such that for n>0 and any w∈ R ,

$g_{2n+1}(w)=F(w,g_{2n}(w))$, $g_{2n+2}(w)=T(w,g_{2n+1}(w))$ and hence ,

$S(g_n(w),g_{n+1}(w),g_{n+1}(w)) \le K^n S(g_0(w),g_1(w)),g_1(w))$

Furthermore ,for m > n,

$S(g_n(w),g_m(w),g_m(w)) \le S(g_{n+1}(w),g_m(w),g_m(w))+ S(g_{n+1}(w),g_n(w),g_n(w))$
$\le S(g_{n+2}(w),g_m(w),g_m(w))+ S(g_{n+2}(w),g_{n+1}(w),g_{n+1}(w))$
$+ S(g_{n+1}(w),g_m(w),g_m(w))$
$\vdots$
$\le S(g_{m-1}(w),g_m(w),g_m(w))+ \cdots +S(g_{n+2}(w),g_{n+1}w),g_{n+1}(w))$
$+ S(g_{n+1}(w),g_n(w),g_n(w))$

By remark (2.5) we have :

$S(g_n(w),g_m(w),g_m(w)) \le S(g_{m-1}(w),g_m(w),g_m(w))+ \cdots +S(g_{n+1}(w),g_{n+2}w),g_{n+2}(w))$ +
$S(g_n(w),g_{n+1}(w),g_{n+1}(w))$

$\leq$ $K^{m-1} S(g_0(w), g_1(w)), g_1(w)) +\ldots..+K^{n+1} S(g_0(w), g_1(w)), g_1(w)) +$
$K^n\ S(g_0(w), g_1(w)), g_1(w))$

$\leq \dfrac{K^{m-1}}{1-K} S(g_0(w), g_1(w)), g_1(w)) \to 0$ as $m\to \infty$

Thus , $\{g_n(w)\}$ is a Cauchy sequence in X , and by completeness of X , $\{g_n(w)\}$ converges to g(w)in X .

Now we prove that F(w,g(w))=g(w) take x =g(w), y=g_{2n}(w) in inequality (3.1.1) we have :S(F(w,g(w)),T(w,g_{2n}(w)), T(w,g_{2n}(w)))

$$\leq \alpha(w)\left(\frac{[S(g_{2n}(w),T(w,g_{2n}(w)),T(w,g_{2n}(w)))+S(g(w),F(w,g(w)),F(w,g(w)))]S(g(w),g_{2n}(w),g_{2n}(w))}{S(g(w),T(w,g_{2n}(w)),T(w,g_{2n}(w))+S(g_{2n}(w),F(w,g(w),F(w,g(w)))}\right)$$

$$+\beta(w)\left(\frac{[S(g(w),T(w,g_{2n}(w)),T(w,g_{2n}(w)))+S(g(w),F(w,g(w)),F(w,g(w)))]S(g(w),g_{2n}(w),g_{2n}(w))}{S(g_{2n}(w),T(w,g_{2n}(w)),T(w,g_{2n}(w))+S(g_{2n}(w),F(w,g(w),F(w,g(w)))}\right)$$

Thus we have : S(F(w,g(w)),g_{2n+1}(w), g_{2n+1}(w))

$$\leq \alpha(w)\left(\frac{[S(g_{2n}(w),g_{2n+1}(w),g_{2n+1}(w))+S(g(w),F(w,g(w)),F(w,g(w)))]S(g(w),g_{2n}(w),g_{2n}(w))}{S(g(w),g_{2n+1}(w),g_{2n+1}(w))+S(g_{2n}(w),F(w,g(w),F(w,g(w)))}\right)$$

$$+\beta(w)\left(\frac{[S(g(w),g_{2n+1}(w),g_{2n+1}(w))+S(g(w),F(w,g(w)),F(w,g(w)))]S(g(w),g_{2n}(w),g_{2n}(w))}{S(g_{2n}(w),g_{2n+1}(w),g_{2n+1}(w))+S(g_{2n}(w),F(w,g(w),F(w,g(w)))}\right)$$

By taking $n\to \infty$ in the above inequality we have :

S(F(w,g(w)),g (w), g (w))

$$\leq \alpha(w)\left(\frac{[S(g(w),g(w),g(w))+S(g(w),F(w,g(w)),F(w,g(w)))]S(g(w),g(w),g(w))}{S(g(w),g(w),g(w))+S(g(w),F(w,g(w),F(w,g(w)))}\right)$$

$$+\beta(w)\left(\frac{[S(g(w),g(w),g(w))+S(g(w),F(w,g(w)),F(w,g(w)))]S(g(w),g(w),g(w))}{S(g(w),g(w),g(w))+S(g(w),F(w,g(w),F(w,g(w)))}\right)$$

So we obtain that S(F(w,g(w)),g (w), g (w)) =0 and then F(w,g(w))=g(w)

Now take x =g(w), y= g(w) in inequality (2.1.1) we have :

S(F(w,g(w)),T(w,g(w)), T(w,g(w)))≤

$$\alpha(w)\left(\frac{[S(g(w),T(w,g(w)),T(w,g(w)))+S(g(w),F(w,g(w)),F(w,g(w)))]S(g(w),g(w),g(w))}{S(g(w),T(w,g(w)),T(w,g(w))+S(g(w),F(w,g(w),F(w,g(w)))}\right)$$

$$+\beta(w)\left(\frac{[S(g(w),T(w,g(w)),T(w,g(w)))+S(g(w),F(w,g(w)),F(w,g(w)))]S(g(w),g(w),g(w))}{S(g(w),T(w,g(w)),T(w,g(w))+S(g(w),F(w,g(w),F(w,g(w)))}\right)$$

So , we obtain that ; S(F(w,g(w)),T(w,g(w)), T(w,g(w))) = 0 and so we get

T(w,g(w))=g(w) .Thus g(w) is a common random fixed point of F and T .
Now to prove the uniqueness of g(w) , let $\gamma(w)$ be another common random fixed point of F and T such that g(w) $\neq \gamma(w)$ .Then by inequality (3.1.1) we have :
S(g(w),$\gamma(w),\gamma(w)$) =S(F(w,g(w)),T(w,$\gamma(W)), T(w,\gamma(w)) \leq$

$$\alpha(w)\left(\frac{[S(\gamma(w),T(w,\gamma(w)),T(w,\gamma(w)))+S(g(w),F(w,g(w)),F(w,g(w)))]S(g(w),\gamma(w),\gamma(w))}{S(g(w),T(w,\gamma(w)),T(w,\gamma(w))+S(\gamma(w),F(w,g(w),F(w,g(w)))}\right)$$

$$+\beta(w)\left(\frac{[S(g(w),T(w,\gamma(w)),T(w,\gamma(w)))+S(g(w),F(w,g(w)),F(w,g(w)))]S(g(w),\gamma(w),\gamma(w))}{S(\gamma(w),T(w,\gamma(w)),T(w,\gamma(w))+S(\gamma(w),F(w,g(w),F(w,g(w)))}\right)$$

$$=\propto(w)\left(\frac{[S(\gamma(w),\gamma(w),\gamma(w))+S(g(w),g(w),g(w))]S(g(w),\gamma(w),\gamma(w))}{S(g(w),\gamma(w),\gamma(w))+S(\gamma(w),g(w),g(w))}\right)$$
$$+\beta(w)\left(\frac{[S(g(w),\gamma(w),\gamma(w))+S(g(w),g(w),g(w))]S(g(w),\gamma(w),\gamma(w))}{S(\gamma(w),\gamma(w),\gamma(w))+S(\gamma(w),g(w),g(w))}\right)$$
$$=\beta(w)\left(\frac{[S(g(w),\gamma(w),\gamma(w))]S(\gamma(w),g(w),g(w))}{S(\gamma(w),g(w),g(w))}\right)$$
So ,(1- $\beta(w)$) $S(g(w),\gamma(w),\gamma(w)) \leq o$,that is , $S(g(w),\gamma(w),\gamma(w))$ = 0
And then g(w)= $\gamma(w)$ .Thus , g(w) is a unique common random fixed point of F and T.

Corollary
Let (X,S) be complete S-metric space and let F :R$\times X \rightarrow X$ be random mappings satisfying the following rational contractive condition :
S(F(w,x),F(w,y),F(w,y)) $\leq$
$$\alpha(w)\frac{[S(y,F(w,y),F(w,y))+S(x,F(w,x),F(w,x))]S(x,y,y)}{S(x,F(w,y),F(w,y))+S(y,F(w,x),F(w,x))}$$
$$+\beta(w)\frac{[S(x,F(w,y),F(w,y))+S(x,F(w,x),F(w,x))]S(x,y,y)}{S(y,F(w,y),F(w,y))+S(y,F(w,x),F(w,x))} \quad \ldots\ldots\ldots(3.2.1)$$
Where
$$S(x,F(w,y),F(w,y))+S(y,F(w,x),F(w,x)) \geq S(y,F(w,y),F(w,y)) + S(x,F(w,x),F(w,x)) \quad \ldots\ldots\ldots\ldots(3.2.2)$$
And $$S(y,F(w,y),F(w,y)) + S(y,F(w,x),F(w,x)) \geq S(x,F(w,y),F(w,y)) + S(x,F(w,x),F(w,x)) \quad \ldots\ldots\ldots\ldots..(3.2.3)$$

For each x,y $\in X$ and w $\in R$ and α,β are two measurable mapping defined by $\alpha,\beta\colon R \rightarrow (0,1)$ with 0$\leq \propto(w) + \beta(w) < 1$ Then there exists a unique random fixed point of F .

Proof
by theorem (3.1)by taking T(w,y) = F(w,y)
Now by remark (2.6) we have the following corollary :

Corollary

Let (X,d) be complete metric space and let F,T :R× $X \to X$ be two random mappings satisfying the following condition :

$$d(F(w,x),T(w,y),) \leq \propto (w)\frac{[d(y,T(w,y))+d(x,F(w,x))]d(x,y)}{d(x,T(w,y))+d(y,F(w,x))}$$

$$+\beta(w)\frac{[d(x,T(w,y))+d(x,F(w,x))]d(x,y)}{d(y,T(w,y))+d(y,F(w,x))} \quad \text{………(3.3.1)}$$

Where

$d(x,T(w,y))+d(y,F(w,x)) \geq d(y,T(w,y))+d(x,F(w,x))$ …(3.3.2)

And $d(y,T(w,y))+d(y,F(w,x)) \geq d(x,T(w,y))+ d(x,F(w,x))$ …..(3.3.3)

For each x,y $\in X$ and w $\in R$ and α, β are two measurable mapping defined by $\alpha, \beta: R \to (0,1)$ with $0 \leq \propto (w) + \beta(w) < 1$ Then there exists a unique random fixed point of F .

Proof

in theorem (3.1), by taking S(x,y,y) =d(x,y)

Theorem(3.4)

Let (X,S) be complete S-metric space and let F,T :R× $X \to X$ be two random mappings satisfying the following rational contraction condition :

S(F(w,x),T(w,y),T(w,y)) ≤

$$\alpha(w)\frac{[S^2(y,T(w,y),T(w,y))+S^2(x,F(w,x),F(w,x))]S(x,y,y)}{S^2(x,T(w,y),T(w,y))+S^2(y,F(w,x),F(w,x))} \quad \text{………(3.4.1)}$$

Where

$$S^2(x,T(w,y),T(w,y)) + S^2(y,F(w,x),F(w,x)) \geq$$
$$S^2(y,T(w,y),T(w,y))+S^2(x,F(w,x),F(w,x)) \quad \text{………(3.4.2)}$$

For each x,y $\in X$ and w $\in R$ and α is a measurable mapping defined by $\alpha: R \to (0,1)$ with $0 < \propto (w) < 1$ Then there exists a unique random fixed point of F .

Proof

Let g_0 :R→ X be an arbitrary measurable mapping and choose a measurable mapping g_1 :R→ X such that $g_1(w) = F(w,g_0(w))$ for each w∈ R . Then by (3.4.1) we have :

$$S(F(w,g_0(w)),T(w,g_1(w)),T(w,g_1(w))) \leq \alpha(w)\frac{[S^2(g_1(w),T(w,g_1(w)),T(w,g_1(w)))+S^2(g_0(w),F(w,g_0(w)),F(w,g_0(w)))]S(g_0(w),g_1(w),g_1(w))}{S^2(g_0(w),T(w,g_1(w)),T(w,g_1(w))+S^2(g_1(w),F(w,g_0(w),F(w,g_0(w)))}$$

Further there exists g_2 :R→ X such that for all w∈ R , $g_2(w) = T(w,g_1(w))$ so

$S(g_1(w), g_2(w), g_2(w)) =$ S(F(w,g_0(w)),T(w,g_1 (w)),T(w,g_1 (w))) $\leq \alpha(w)$

$$\frac{[S^2(g_1(w),g_2(w)),g_2(w))+S^2(g_0(w),g_1(w),g_1(w))]S(g_0(w),g_1(w),g_1(w))}{S^2(g_0(w),g_2(w),g_2(w))+S^2(g_1(w),g_1(w),g_1(w))}$$

$$=\left(\frac{\alpha(w)[S^2(g_1(w),g_2(w)),g_2(w))+S^2(g_0(w),g_1(w),g_1(w))]S(g_0(w),g_1(w),g_1(w))}{S^2(g_0(w),g_2(w),g_2(w))}\right)$$

By (3.4.2) we have :

$$S(g_1(w),g_2(w),g_2(w))\leq \alpha(w)\ S(g_0(w),g_1(w),g_1(w))$$

Let K $=\alpha(w)$, so we obtain that :

$$S(g_1(w),g_2(w),g_2(w))\ \leq\ \mathrm{K}\ S(g_0(w),g_1(w),g_1(w))$$

By Beg and Shah zad [2,lemma 2.3] we obtain $g_3:R\to X$ such that for all $w\in R$, $g_3(w)=F(w,g_2(w))$ and

$S(g_2(w),g_3(w),g_3(w)) = S(T(w,g_1(w)),F(w,g_2(w)),F(w,g_2(w))$

By remark (2.5) we have :

$S(g_2(w),g_3(w),g_3(w)) = S(F(w,g_2(w)), T(w,g_1(w)) T(w,g_1(w)))$

$$\leq\left(\frac{\alpha(w)[S^2(g_1(w),T(w,g_1(w)),T(w,g_1(w)))+S^2(g_2(w),F(w,g_2(w),F(w,g_2(w)))]}{S^2(g_2(w),T(w,g_1(w)),T(w,g_1(w)))+S^2(g_1(w),F(w,g_2(w),F(w,g_2(w)))}\right)$$

$$\cdot S(g_2(w),g_1(w),g_1(w))$$

$$\leq\left(\frac{\alpha(w)[S^2(g_1(w),g_2(w)),g_2(w))+S^2(g_2(w),g_3(w)),g_3(w))]S(g_2(w),g_1(w)),g_1(w))}{S^2(g_2(w),g_2(w)),g_2(w))+S^2(g_1(w),g_3(w)),g_3(w))}\right)$$

By (3.4.2) and remark (2.5) we have :

$$\begin{aligned} S(g_2(w),g_3(w)),g_3(w)) &\leq \alpha(w)\ S(g_1(w),g_2(w)),g_2(w)) \\ &\leq \mathrm{K}\ S(g_1(w),g_2(w)),g_2(w)) \\ &\leq \mathrm{K}^2 S(g_0(w),g_1(w)),g_1(w)) \end{aligned}$$

Similarly ,proceeding in the same way by induction we produce a sequence of mappings $g_n:R\to X$, such that for $n>0$ and any $w\in R$,

$g_{2n+1}(w)=F(w,g_{2n}(w))$, $g_{2n+2}(w)=T(w,g_{2n+1}(w))$ and hence ,

$S(g_n(w),g_{n+1}(w),g_{n+1}(w))\leq K^n S(g_0(w),g_1(w)),g_1(w))$

So by a similar way of theorem (3.1) we obtain that $\{g_n(w)\}$is a Cauchy sequence in X , and by completeness of X , $\{g_n(w)\}$ converges to g(w)in X.

Now we prove that F(w,g(w))=g(w) take $x=g(w)$, $y=g_{2n}(w)$ in inequality (3.4.1) we have :$S(F(w,g(w)),T(w,g_{2n}(w)), T(w,g_{2n}(w)))$

$$\leq\left(\frac{\alpha(w)[S^2(g_{2n}(w),T(w,g_{2n}(w)),T(w,g_{2n}(w)))+S^2(g(w),F(w,g(w)),F(w,g(w)))]S(g(w),g_{2n}(w),g_{2n}(w))}{S^2(g(w),T(w,g_{2n}(w)),T(w,g_{2n}(w))+S^2(g_{2n}(w),F(w,g(w),F(w,g(w)))}\right)$$

Thus we have : S(F(w,g(w)),g_{2n+1}(w), g_{2n+1}(w))

$$\leq\left(\frac{\alpha(w)[S^2(g_{2n}(w),g_{2n+1}(w),g_{2n+1}(w))+S^2(g(w),F(w,g(w)),F(w,g(w)))]S(g(w),g_{2n}(w),g_{2n}(w))}{S^2(g(w),g_{2n+1}(w),g_{2n+1}(w))+S^2(g_{2n}(w),F(w,g(w),F(w,g(w)))}\right)$$

By taking n→ ∞ in the above inequality we have :

S(F(w,g(w)),g (w), g (w))

$$\leq\left(\frac{\alpha(w)[S^2(g(w),g(w),g\ (w))+S^2(g(w),F(w,g(w)),F(w,g(w)))]S(g(w),g(w),g(w))}{S^2(g(w),g(w),g(w))+S^2(g(w),F(w,g(w),F(w,g(w)))}\right)$$

So we obtain that S(F(w,g(w)),g (w), g (w)) =0 and then F(w,g(w))=g(w)
Now take x =g(w), y= g(w) in inequality (3.4.1) we have :
S(F(w,g(w)),T(w,g(w)), T(w,g(w)))≤

$$\left(\frac{\alpha(w)[S^2(g(w),T(w,g(w)),T(w,g(w)))+S^2(g(w),F(w,g(w)),F(w,g(w)))]S(g(w),g(w),g(w))}{S^2(g(w),T(w,g(w)),T(w,g(w))+S^2(g(w),F(w,g(w),F(w,g(w)))}\right)$$

So , we obtain that ; S(F(w,g(w)),T(w,g(w)), T(w,g(w))) = 0 that is S(g(w),T(w,g(w)), T(w,g(w))) =0 and so
T(w,g(w))=g(w) .Thus g(w) is a common random fixed point of F and T .
Now to prove the uniqueness of g(w) , let $\gamma(w)$ be another common random fixed point of F and T such that g(w) $\neq \gamma(w)$ .Then by inequality (3.4.1) we have :
S(g(w),$\gamma(w),\gamma(w)$) =S(F(w,g(w)),T(w,$\gamma(w)),T(w,\gamma(w))$) ≤

$$\left(\frac{\alpha(w)[S^2(\gamma(w),T(w,\gamma(w)),T(w,\gamma(w)))+S^2(g(w),F(w,g(w)),F(w,g(w)))]S(g(w),\gamma(w),\gamma(w))}{S^2(g(w),T(w,g(w)),T(w,g(w))+S^2(\gamma(w),F(w,g(w),F(w,g(w)))}\right)$$

$$=\left(\frac{\alpha(w)[S^2(\gamma(w),\gamma(w),\gamma(w))+S^2(g(w),g(w),g(w))]S(g(w),\gamma(w),\gamma(w))}{S^2(g(w),\gamma(w),\gamma(w))+S^2(\gamma(w),g(w),g(w))}\right)$$

So we obtain that, $S(g(w),\gamma(w),\gamma(w))$ = 0 And then g(w)= $\gamma(w)$ .Thus , g(w) is a unique common random fixed point of F and T.

Corollary

Let (X,S) be complete S-metric space and let F :R× $X\to X$ be random mappings satisfying the following rational contractive condition :
S(F(w,x),F(w,y),F(w,y)) ≤

$$\frac{\alpha(w)[S^2(y,F(w,y),F(w,y))+S^2(x,F(w,x),F(w,x))]S(x,y,y)}{S^2(x,F(w,y),F(w,y))+S^2(y,F(w,x),F(w,x))}\qquad \ldots\ldots\ldots(3.5.1)$$

Where

$$S^2(x,F(w,y),F(w,y))+S^2(y,F(w,x),F(w,x))\geq$$
$$S^2(y,F(w,y),F(w,y))+S^2(x,F(w,x),F(w,x))$$

For each x,y ∈ X and w ∈ R and α is measurable mapping defined by $\alpha: R\to(0,1)$ with 0≤ $\propto(w)<1$. Then there exists a unique random fixed point of F .

Proof

By theorem (3.4)by taking T(w,y) = F(w,y)

Now by remark (2.6) we have the following corollary :

Corollary

Let (X,d) be complete metric space and let F,T :R× $X \to X$ be two random mappings satisfying the following rational contractive condition :

d(F(w,x),T(w,y),) $\leq \frac{\alpha(w)[d^2(y,T(w,y))+d^2(x,F(w,x))]d(x,y)}{d^2(x,T(w,y))+d^2(y,F(w,x))}$........(3.6.1)

Where

$$d^2(x,T(w,y))+d^2(y,F(w,x)) \geq$$

$$d^2(y,T(w,y))+d^2(x,F(w,x)) \text{……………..(3.6.2)}$$

For each x,y $\in X$ and w $\in R$ and α is measurable mapping defined by $\alpha: R \to (0,1)$ with 0< $\propto (w) < 1$. Then there exists a unique random fixed point of F and T .

Proof

By theorem (3.4), by taking S(x,y,y) =d(x,y)

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