

## Comparison of FFT-Based and DCT-Based Channel Estimation and ECC for OFDM System

**Awatif Ali Jafaar**

Electrical Engineering Department, University of AL-Mustansiriya /Baghdad.

Email:Awatifali\_2012@yahoo.com

Received on:8/4/2014 & Accepted on:8/1/2015

### ABSTRACT:

Orthogonal Frequency Division Multiplexing (OFDM) has become a popular modulation method for high data rate wireless communications. In this paper, two schemes are proposed. In the first scheme the Bit Error Rate (BER) and the Packet Error Rate (PER) performances of Fast Fourier Transform-FFT based OFDM are compared with Discrete Cosine Transform DCT -based OFDM using (QPSK) and (16 QAM) as a modulation techniques over Additive White Gaussian Noise environment (AWGN) and Multipath Rayleigh Fading environment.

To improve the performance of the uncoded OFDM schemes, the second coded scheme is investigated. convolution coding is used to enhance the BER for the previous systems. The proposed systems have been tested and evaluated using MATLAB 7 package.

**Keywords** :BER,PER, FFT, DCT, AWGN, Multipath Rayleigh Fading, OFDM, QPSK,16QAM, Convolutional Code, Channel Estimation.

### مقارنة تطبيق DCT مع تطبيق FFT مرتكزا على تقدير القناة ودالة تصحيح الأخطاء في نظام التعدد التقسيمي الترددي المتعامد

#### الخلاصة

اصبح التعدد التقسيمي الترددي المتعامد OFDM من أنواع التضمين الشائعة لزيادة معدل نقل البيانات في الاتصالات اللاسلكية. في هذا البحث تم اقتراح نظامين. النظام الاول يتم فيه مقارنة اداء النظام التقليدي باستخدام تحويل فورير السريع FFT مع النظام الذي يعتمد تحويل الجيب تمام المتقطع DCT وباستخدام نوعين من التضمين هما QPSK و16QAM المقارنة تتم من خلال مقارنة معدل خطأ البت BER ومعدل خطأ الرزمة PER لكلا النظامين خلال ا لقناة الكاوسية وقنوات الخفوت متعددة المسارات .

من اجل تحسين أداء النظام المقترح الأول , تم اقتراح استخدام دالة تصحيح الأخطاء الالتفافية Convolutional code من اجل تقليل قيمة معدل خطأ البت. تم استخدام MATLAB7 package من اجل إجراء التقييم والاختبار للأنظمة المقترحة.

## INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM), is a special case of multicarrier transmission, where a single data stream is transmitted over a number of lower-rate subcarriers. The implementation complexity is significantly lower than that of a single-carrier system with an equalizer. OFDM is robust against narrowband interference because such interference affects only a small percentage of the subcarriers; it also increases robustness against frequency-selective fading [1].

OFDM which allows the overlapping of the subcarriers but keeps them orthogonal to avoid intercarrier interference (ICI) and inter-symbol interference (ISI), are increasingly being deployed in broadband wireless communication standards such IEEE 802.11 (Wi-Fi) and IEEE 802.16 (WiMax)[2]. The property of orthogonality allows simultaneous transmission of a lot of sub-carriers in a tight frequency space without interference from each other. This acts as an undue advantage in OFDM. Particularly, modulation and demodulation process in OFDM system can be achieved by using Inverse Discrete Fourier Transform (IDFT) and Discrete Fourier Transform (DFT) respectively [3]. OFDM based FFT schemes are also used to reduce the number of complex multiplications which is needed if Discrete Fourier Transform is used [4]. Another schemes can provide fewer computational steps than FFT based OFDM by using cosinusoidal functions instead of complex exponential. These multicarrier schemes is implemented using discrete cosine transform (DCT) [1, 5, 6].

### DCT Based OFDM System

Orthogonal basis functions are needed to construct baseband multicarrier signal. Only complex exponential function is not the way to construct multicarrier but alternatives may be there like Discrete Cosine Transform (DCT), discrete sine Transform (DST) and others. DCT is Orthogonal transfer function tend to redistribute the energy contained in the signal so that the most of energy is contained in a small no. of components. (DCT) is built on a concept of transform the signal or finitely data points in terms of cosine functions with varying frequency and magnitudes. DCT is widely used in many compression applications, for audio, image and video to reduce the bandwidth [1]. In particular, a DCT is a Fourier-related transform similar to the discrete Fourier transform (DFT), but using only real numbers. The output signal of a DCT based OFDM system can be written [7]:

$$X(t) = \left[ (2/N)^{1/2} \right] \sum_{n=0}^{N_s-1} d_n \beta_n \cos\left(\frac{n\pi t}{T_s}\right) \quad \dots(1)$$

Where

$d_0, d_1, \dots, d_{N-1}$ , are  $N_s$  independent data symbols obtained from a modulation constellation, and

$$\beta_n = \begin{cases} \frac{1}{\sqrt{2}} & n = 0 \\ 1 & n = 1, 2, 3 \dots N_S - 1 \end{cases}$$

**FFT Based OFDM System**

In OFDM system, the concept of a single wide band carrier modulation is improved by using a multiple narrow band sub carriers. To maintain the orthogonality between the sub carriers, the frequency of each carrier must be carefully chosen.

The data stream in OFDM system is divided over these lower sub carrier, so if the user bit stream has a rate  $R_u$  then these stream of bit is transmitted over multiple sub carrier  $N_c$  with a symbol duration  $T_s = \frac{N_c}{R_u}$  and a symbol rate  $R_s = \frac{R_u}{N_c}$ . The advantage of this way of transmission is that the decreasing of inter symbol interference (ISI) due to the increasing of symbol time.

Let the sequence of N numbers  $X_0, \dots, X_{N-1}$  is transformed into another sequence of N complex numbers. DFT/IDFT Transforms are interesting from the OFDM perspective because they can be viewed as mapping data onto orthogonal subcarriers so these are good choice [8].

The Discrete Fourier transform (DFT) is given by

$$x_k = \sum_{n=0}^{N-1} X_n e^{-j2\pi n \frac{k}{N}} \dots(2)$$

$X_K$  can thus be viewed as coefficients of  $x$  in an orthogonal basis. The inverse discrete Fourier transform (IDFT) is given by

$$X_N = \sum_{K=0}^{N-1} x_n e^{j2\pi n \frac{k}{N}} \dots (3)$$

**4. Channel Estimation**

Typically OFDM systems have known pilots symbols, or training data, inserted on the subcarriers before the IFFT operation at the transmitter. These symbols have been added to mitigate the interference between replicas of the data at the receiver. This data is to be used to estimate the channel. There is a real tradeoff in utilizing this technique. Indeed, pilots could potentially be used to send additional information thus increasing the bandwidth efficiency. On the other hand, the more pilots we include in our message, the more accurately we will be able to track and estimate the frequency response of the channel [9].

In this paper, the compensation of phase error that occur due to the deep fading channel is realized by adding a pilot symbols at a known positions at the transmitter. These pilot symbols which are known at the receiver are used to recover the actual data. The transition matrix of the fading channel (h) is defined by [10] :

$$h = \begin{pmatrix} i_v & -q_v \\ q_v & i_v \end{pmatrix} \dots(4)$$

So, the relation between the transmitted pilot data (ich0, qch0) and the received pilot data (ich1, qch1) are given as follow:

$$\begin{pmatrix} ich1 \\ qch1 \end{pmatrix} = h \begin{pmatrix} ich0 \\ qch0 \end{pmatrix} \quad \dots(5)$$

The values of (iv) and (qv) are defined by:

$$i_v = \frac{1}{\sqrt{ich1+qch1}} (ich0*ich1+qch0*qch1) \quad \dots(6)$$

$$q_v = \frac{1}{\sqrt{ich1+qch1}} (qch0*ich1-ich0*qch1) \quad \dots(7)$$

All the received data are multiplied by the complex conjugate of (h) to compensate the phase error

$$h^{-1} = \frac{1}{\sqrt{i_v^2 + q_v^2}} \begin{pmatrix} i_v & q_v \\ -q_v & i_v \end{pmatrix} \quad \dots(8)$$

### Convolutional Codes (CC)

Convolutional codes are widely used for the systems which has a real time error correction. A Convolutional code groups the data stream into much smaller blocks (k digits) than that of linear block codes. These k digits are encoded into n digits with order of K digits.

The convolutional encoding is realized by passing the data stream through finite state register. The shift register consists of L groups where L is the constraint length and each groups consists of k bits (stages). The number of output bits for each k input is n bits. These output bits is produced by shifting the input bits along the shift register k bits at a time and passing through n module 2 adders.

A convolutional code is defined by giving its generators. These are describing the selected taps from the shift register to be mod 2 summed and each n vectors is of Lk length.

The code rate for convolutional code is k/ n and the code can become more complicated by varying code rates and constraint lengths. Figure (1) shows a convolutional code with 1/2 code rate and constraint length L= 3. Here the input is one bit at a time and n is the number of parallel output encoded bits which is equal to 2 bits (C1 and C2). Here, the two generator vectors are: [g<sub>1</sub>] = [111] and [g<sub>2</sub>] = [101] describing the connection for C1 and C2 output (in octal form 7 and 5). In the encoder, we notice that entered bit remains in the shift register for three shifts. And at the fourth shift this bit is shifted out and does not affect the output [11].

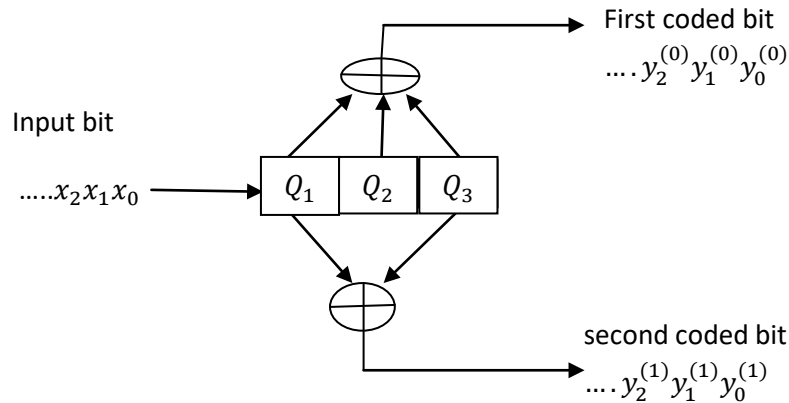


Figure (1) convolutional code rate  $\frac{1}{2}$ ,  $k=1, n=2, L=3$

**Viterbi Decoding Algorithm:**

Viterbi decoding algorithm is mostly applied to convolutional encoder and it is the same as maximum likelihood criterion since it depends on choosing the correct decoded data corresponding to minimum Hamming distance. This algorithm is best explained with the help of the trellis diagram. Decoding algorithm is performed in two different forms depending on the type of quantization used on the received bits. The output result depends on this decoding forms. Decision was made as to whether an encoded bit was a 1 or a 0 before the decoder. This operation is called hard decision and the data bits are quantized to 1 bit while the optimum modified Viterbi algorithm (SOVA) which is known as soft-decision uses multi bit quantization on the received channel values ( i.e. symbols are quantized to three or four bits of precision[12,13] The Viterbi algorithm consists of three major parts:

1. Branch calculation: in this part the hamming distances between the input pair of bits and four possible ideal pairs (00,01,10 ,11) are calculated.
2. Path calculation for every encoder state, a metric for the survivor path ending in this state (a survivor path is a path with the minimum metric) is calculated.
- 3.Back Tracing This step is necessary for hardware implementations that don't store full information about the survivor paths, but store only one bit decision every time when one survivor path is selected from the two[14].

**Proposed OFDM Systems**

The goal of any proposed OFDM system is to suggest scheme able to resist the communication channel impairments. Many ways is used, one of these is realizing the multicarrier scheme by using DCT based channel estimation. Forward error correction coding is used to improve the error rate performance for many of proposed systems [10, 11, 15]. In this paper, convolutional coded is suggested for proposed DCT-OFDM systems to reduce the bit error rate on additive white Gaussian noise (AWGN) channel and a Rayleigh fading channels.

### DCT based OFDM Proposed System

Figure (2) shows the block diagram of the OFDM system. The first step in the block diagram is a mapping block. Then the Serial to Parallel Converter converts the incoming serial data into parallel In-phase and Quadrature components

DCT-OFDM is obtained by replacing IFFT at transmitter and FFT at receiver by IDCT and DCT respectively. N orthogonal subcarrier signals is achieved by taking N point IDCT. OFDM samples are generated by the sum of the N point IDCT symbols. These sample are transmitted through the channel that corrupted by noise. At the receiver, N point DCT is taken for the received sample.

The high-bit rate transmission is important for high quality mobile communication systems. (OFDM), is a very attractive technique for the high-bit-rate data transmission in a multipath environment that causes intersymbol interference (ISI). The ISI in OFDM can be reduced by adding a cyclic prefix, which reduce the (ISI) between OFDM symbols. Once the signal is received by the receiver, the effect of ISI is over and thus the cyclic prefix is removed [16].

At the receiver, an inverse steps to what were done in the transmitter are performed.

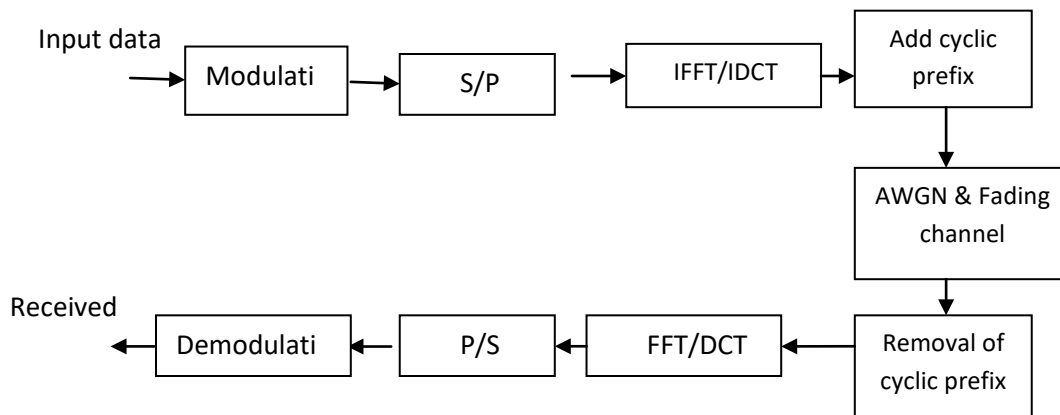
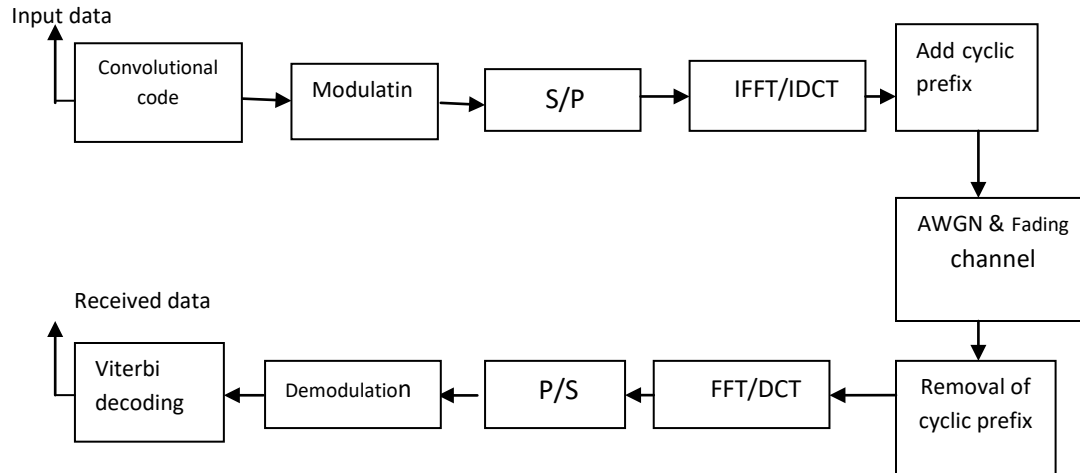


Figure (2) Block diagram of proposed system (transceiver)

### Proposed Coded DCT based OFDM System

One of the challenges for any multipath medium is the strong destructive interference that causes a large bit error rate (BER). The overall (BER) may be large due to some subcarriers with small amplitudes although the most subcarriers are detected without errors. To eliminate the deep fade effects, forward error correction code is used. In this paper, Convolutional code is suggested as a solution for this problem as shown in Figure (3).



**Figure (3) Block diagram of proposed coded system (transceiver)**

### Simulation Results

#### Simulation Results of the Proposed DCT-OFDM System in AWGN Channel

In this paper, OFDM system is implemented using MATLAB simulation, where each block of OFDM is simulated in scripts file. We have considered AWGN channel for comparing BER and PER of FFT-based OFDM and DCT-based OFDM. Modulation block is simulated using ( QPSK) and (16 QAM).The following OFDM system parameters are considered for the simulation:

Data mapping: QPSK, 16 QAM

IFFT and FFT Size: 64-point

IDCT and DCT Size: 64-point

Channel Used: AWGN & Multipath Rayleigh fading channel

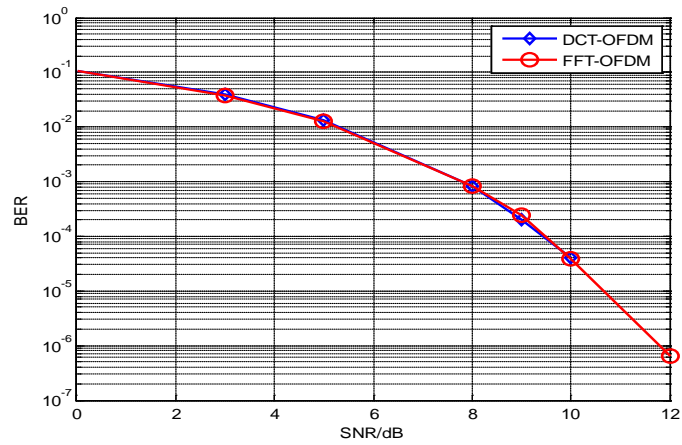
No. of data sub carrier: 52

Cyclic prefix duration: 16

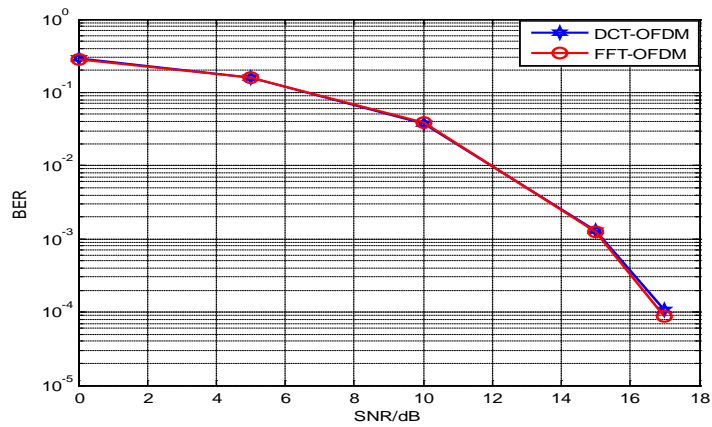
Number of OFDM symbol for one loop = 6

Pilot Symbol = 1 for 6 symbol

Figure (4) and Figure (5) illustrates the performance of the proposed DCT-based OFDM (DCT-OFDM) and the traditional FFT-based OFDM (FFT-OFDM) systems in AWGN channel for QPSK and 16QAM modulation. It is clearly shown that the BER decreases when we increase the SNR, which is normal because the signal becomes stronger than the noise.



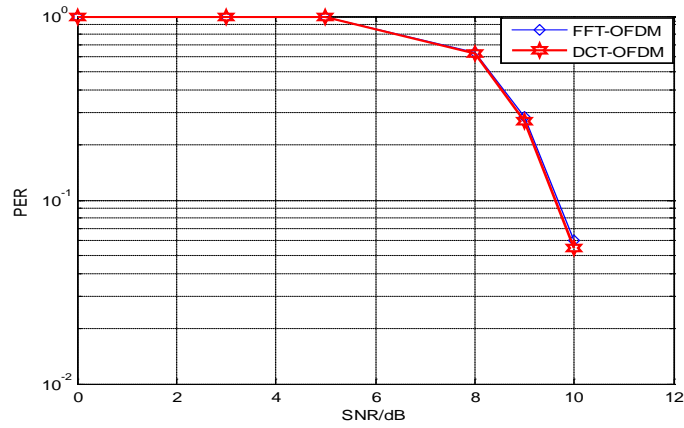
(a)



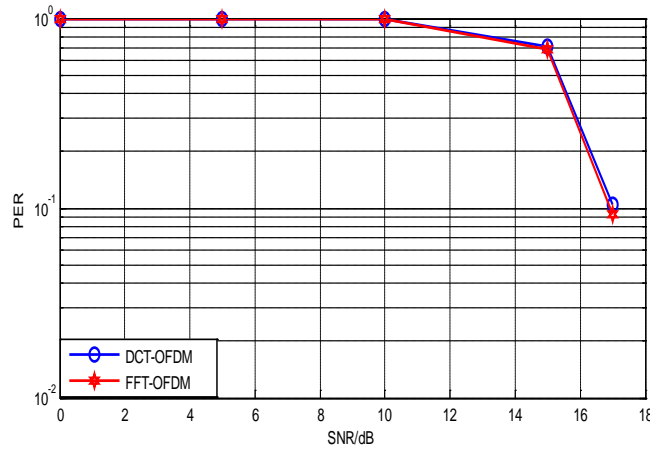
(b)

Figure(4) BER performance of the proposed DCT-based OFDM system and the traditional FFT-based OFDM system in AWGN channel for (a) QPSK modulation (b) 16QAM modulation





(a)



(b)

**Figure(5) PER performance of the proposed DCT-based OFDM system and the traditional FFT-based OFDM system in AWGN channel for (a) QPSK modulation (b) 16QAM modulation**

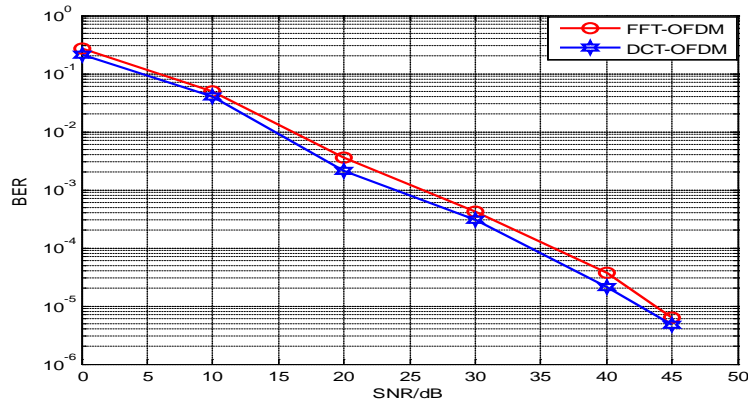
We can see from these two figures that the proposed DCT-based OFDM and FFT-based OFDM have almost the same BER and PER performance for OFDM system in AWGN channel.

When QPSK is considered as a modulation technique, the BER of DCT-based OFDM and FFT-based OFDM is about  $10^{-3}$  at 8 dB of SNR .while when 16QAM is considered as a modulation technique, the BER of DCT-based OFDM and FFT-based OFDM reaches to  $10^{-3}$  at 15 dB of SNR.

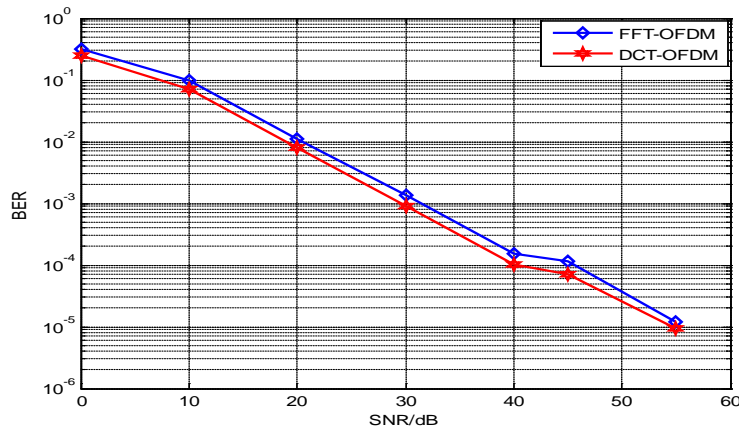
**Simulation Results of the Proposed DCT-OFDM System in a Rayleigh Fading Channel with Channel Estimation**

Fading occurs due to multipath components in a channel. In the wireless propagation channel there is no single direct link between the transmitter and the receiver; rather the transmitted signal undergoes multiple reflections, refractions, diffractions and scattering. The effect can cause time spreading of the signal and results in fluctuations in the received signal’s amplitude, phase, and angle of arrival, giving rise to the terminology multipath fading [17]. This phenomenon introduces ISI (inter symbol interference) which cause a degradation effects on the system performance.

Figure (6) and Figure (7) show the BER and PER performance in a Rayleigh fading channel for channel estimation compensation.

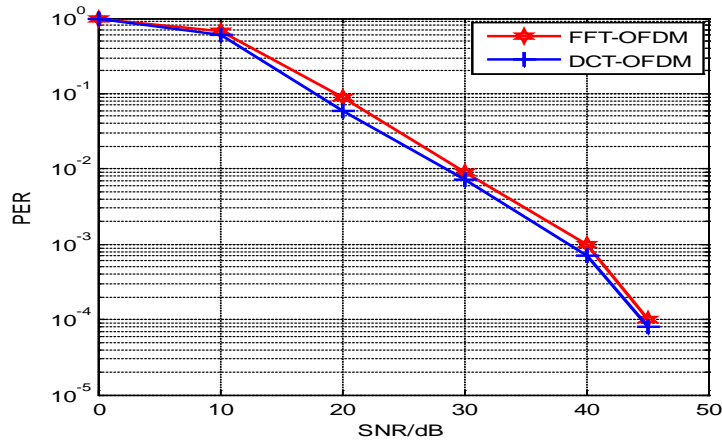


(a)

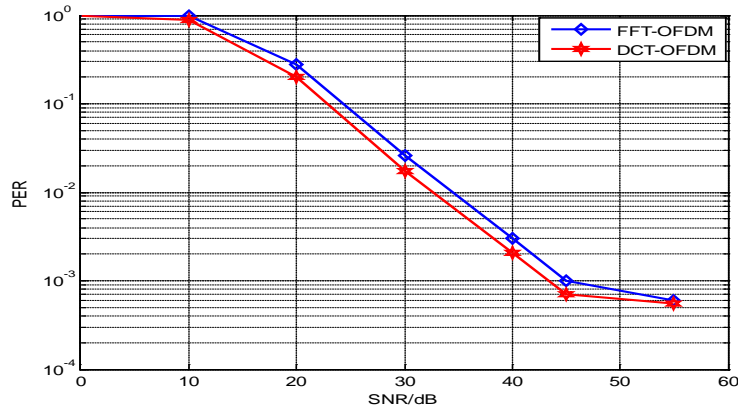


(b)

**Figure(6) BER performance of the proposed DCT-based OFDM system and FFT-based OFDM system in a Rayleigh fading channel with channel estimation compensation for (a) QPSK modulation (b) 16QAM modulation.**



(a)



(b)

**Figure(7) PER performance of the proposed DCT-based OFDM system and FFT-based OFDM system in a Rayleigh fading channel with channel estimation compensation for (a) QPSK modulation (b) 16QAM modulation**

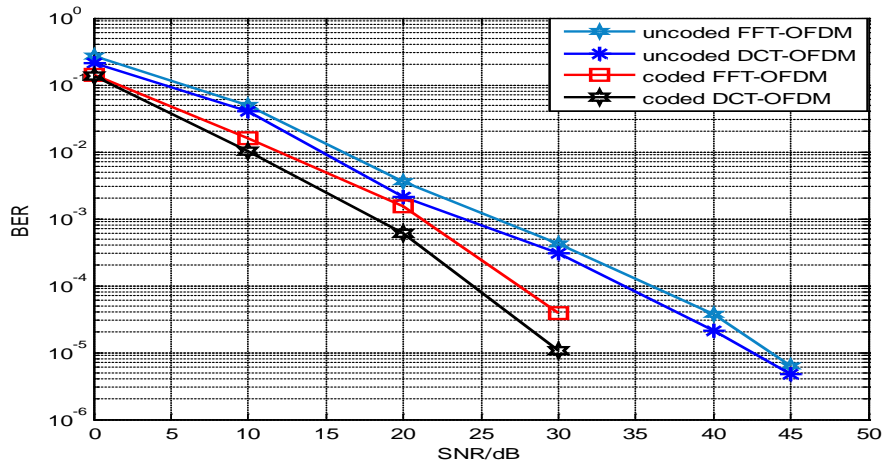
In this simulation, we can see that the proposed DCT-OFDM systems with QPSK modulation and 16QAM are about 2 dB better than that of the FFT-OFDM system. Also, the BER is worse than what was obtained for AWGN and this is a normal fact due to the reasons that referred previously in this section.

**Simulation Results of the Coded Proposed DCT-OFDM System in a Rayleigh Fading Channel**

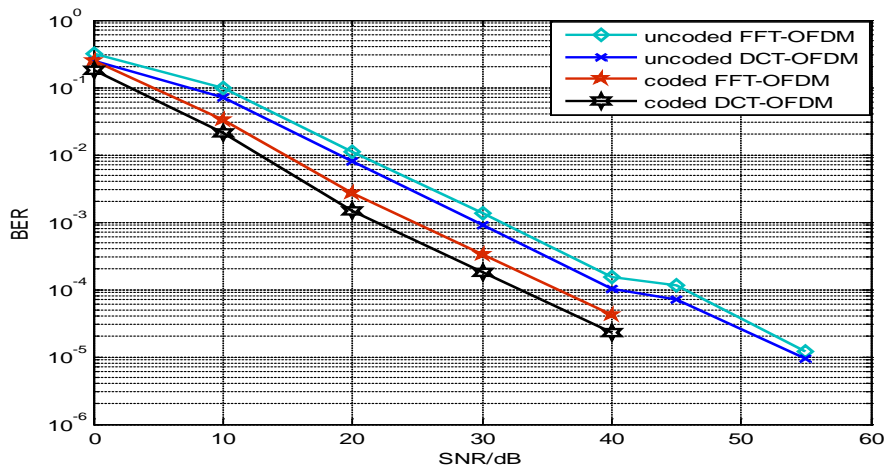
In this section, the (BER) of the proposed –coded DCT-OFDM system with (QPSK) modulation and (16 QAM) modulation in a Rayleigh fading channel with pilot channel estimation is evaluated. For this simulation, 1/2 code rate convolutional code with

constraint length 7 and generator polynomials (171,133) generators matrix is used. Decoding is done by Viterbi decoding algorithm.

Figure (8) shows the BER comparison for uncoded DCT-OFDM and uncoded FFT-OFDM with the Coded proposed system in a Rayleigh fading channel and for QPSK modulation and 16QAM modulation.



(a)



(b)

Figure (8) BER performance of the proposed coded DCT-based OFDM system and the proposed coded FFT-based OFDM system in a Rayleigh fading channel (a) QPSK modulation. (b) 16QAM modulation

Figure-8(a) shows that the coding gain at  $BER = 10^{-3}$  between uncoded FFT-OFDM system and coded FFT-OFDM system with (QPSK) modulation in a Rayleigh fading channel is about 5dB, while the coding gain at  $BER = 10^{-3}$  between uncoded DCT-

OFDM system and coded DCT-OFDM system with QPSK modulation in a Rayleigh fading channel is about 6.5dB. Then the net coding gain between the coded FFT-OFDM system and coded DCT-OFDM system is about 1.5dB.

From the figure-8(b), the coding gain at  $BER = 10^{-3}$  between uncoded traditional FFT-OFDM system and coded FFT-OFDM system with (16QAM) modulation in a Rayleigh fading channel is about 6dB, while the coding gain at  $BER = 10^{-3}$  between uncoded DCT-OFDM system and coded DCT-OFDM system with 16QAM modulation in a Rayleigh fading channel is about 7dB. So the net coding gain is about 1dB.

## CONCLUSIONS

In this paper, the BER and the PER performances of DCT based OFDM and FFT based OFDM are compared using QPSK and 16QAM as a modulation techniques over AWGN and fading channel. In the simulation results, it is demonstrated that the performance of the two system over AWGN channel is almost the same. For fading channel, the BER and the PER performances of DCT-OFDM is better than DFT-OFDM. The signal energy in DCT almost a few low-index DCT coefficients, while the remaining coefficients are zero or are negligibly small. Also For fast implementation algorithms, DCT can provide fewer computational steps than FFT based OFDM. Moreover, DCT avoids complex exponential functions and uses only real values, unlike FFT which uses complex numbers.

The performance of DCT based OFDM system was also further enhanced by including forward error correction code (convolutional code). Convolutional coding in the proposed OFDM system can give performance improvement of some 6 dB on fading channel over the uncoded OFDM system at required BER.

## Future study

The future work may include the implementation of other modulation schemes and different channel scenarios. Another types of error correction code can be used like RS code or turbo code or LDPC code. Also the system can be evaluated by using another transform instead of DCT/IDCT transform like DWT/IDWT transform.

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