Modeling and Validating the Optimal Routes of a Sensor Network Using the Electrostatic Field Equations

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Abstract

In this paper, the analogy between the optimal routes within a sensor network and the electrostatic field lines is utilized successfully. In other words, partial differential equations similar to those of the electrostatic field theory are solved using Finite Difference Method (FDM) to find the optimal routes of the network. For the purpose of validation, an Opnet program based on the generated optimal routes is written to find the throughput and delay of the sensor network, a similar program is then applied to some arbitrary routing scenarios. The results show that the throughput and delay performance of the proposed method is better than that of the chosen arbitrary routing scenarios. It is also found from the results that the throughput of some scenarios is 50 % lower than that of the proposed method.

Keywords : Wireless Sensor Network, Optimal Routing, Electrostatic Theory, Throughput, Shortest Path, Poisson Equation, Finite Element Method.

نمذجة وتحقيق المسارات المثلى لشبكة متحسسات باستخدام معادلات المجالات الالكتروستاتيكية

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الخلاصة

في هذا البحث، المشابهة بين المسارات المثلى في شبكة متحسسات وخطوط المجال الالكتروستاتيكية استخدمت بنجاح. بمعنى اخرى فإن مجموعة من المعادلات التفاضلية والتي تشبه معادلات النظرية الالكتروستاتيكية قد حلّت باستخدام طريقة العنصر الموحد (FDM) لحساب المسارات المثلى داخل الشبكة. لغرض التحقق، تم تصميم نموذج محاكاة باستخدام برنامج الـOpnet يعتمد على المسارات المثلى التي تم توليدها لحساب الانتاجية والتأخير لشبكة المتحسسات المصممة، نفس البرنامج تم تطبيقه على مجموعة من طرق التوجيه المختلفة ولنفس الشبكة المستخدمة، اظهرت النتائج ان اداء الانتاجية والتأخير للتصميم المقترح افضل من الطرق الاخرى. حيث ان الانتاجية لإحدى الطرق اسوأ بمقدار 50 % من الطريقة المقترحة.

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1. Introduction

Sensor networks consist of nodes (smart devices), which are distributed over an area in order to perform a special task. Figure (1) shows a typical sensor network diagram. It consists of :
1- Sensors which are used for monitoring temperature , sound , pressure ,…etc.
2- Interconnection devices.
3- Central unit for collecting information from sensors.
4- Computing resources.

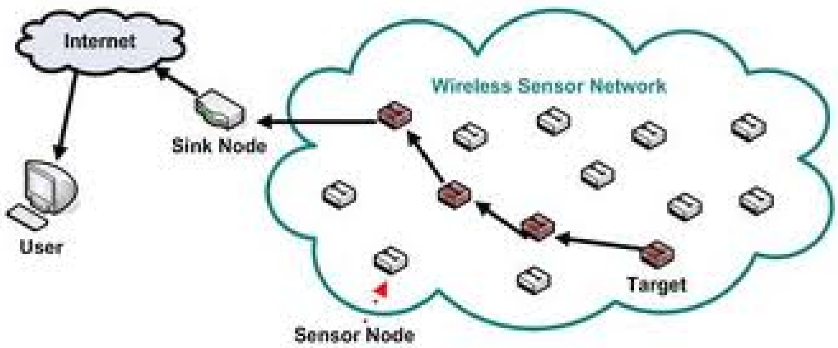


Figure (1) Typical sensor network

It is worth to mention that the central unit is used to collect data from all sensors mainly for processing and archiving functions. It must have sufficient processing power , buffering space , and capability of communication with other sensors efficiently. In this type of networks , the sensors must transmit packets to the destination in a multi-hop way with minimum delay as possible [1],[2]. This task can be achieved by applying vector field principle to optimize the routing algorithm. The vector field represents the communication load at every place of the network, it can be written as the gradient of a scalar potential function. Routing of packets can be done based on the potential magnitude on each node and its magnitude on the neighboring nodes [3] .
The routing problem of this type of networks is mathematically modeled using partial differential equations similar to the Maxwell's equations being used in electrostatic theory. The sensor networks routing problems have been studied by many researchers. Network with Sequential Assignment Routing (SAR) algorithm is proposed in [4],[5], a tree rooted in the destination is constructed using energy constraints technique. A proposed minimum Cost Forwarding Algorithm (CFA) for wide area sensor networks is given in [6]. A location protocol called Geographic Adaptive Fidelity (GAF) is proposed initially for Mobile Ad-hoc networks then applied to sensor networks successfully[4]. Similar routing schemes are given in [5]
The analogy between the theory of Electrostatic and wireless sensor networks can be shown in Table(1).

Table (1) Analogy between the theory of Electrostatic and wireless sensor networks

Electrostatic	Wireless sensor network
1- Positive charges. 2- Negative charges. 3- Non-homogenous dielectric media with variable dielectric constant. 4- The paths between the positive and negative charges represent the minimum energy routes.	1- Sources. 2- Destinations. 3- Network structure. 4- The shortest paths between the sensors and destinations represent the minimum delay routes.

If the initial distribution of sources and destinations is not given, then it is possible to locate sources and sinks within a given area freely, the location that reduces the number of nodes required to support the flow of information will create a routing pattern identical to the electrical field lines created in a similar Electrostatics topology.[1],[2],[7].

2 Modeling the Routes of a Sensor Network

The sensor network to be analyzed and simulated must satisfy the following conditions:-

1- No mobility.
2- Known traffic demands for each sources-destination pair.
3- Known physical location of sources and destinations.
4- The locations information is stored at a route server that can assign the routes to the various source-destination pairs.
5- From the routing point of view, a given direction for every point *z* of the network is assumed. A sensor placed at the point *z* will use the given direction of *z*. And it forwards its traffic to sensors that are closest to this direction as the next hop.
6- The selected direction is a continues function of *z*. (close sensors can use the same next hops, or they can use the hops that are close to each other) [1],[8],[9].
7- Let $w_i > 0$ denotes the average rate of the generated messages by sensor *i*; with the condition that the sources of information are distributed.
8- At location (x, y) □ A $r(x,y)$ *bits/sec/m²* of information is generated, given that A = the plane containing the node, and r(x,y) = the messages generating data rate.
9- The central node (information sink) is located at (x_o, y_o) □ A.

2.1 Mathematical Analysis

According to the previous assumptions, it is possible to assign the total information generation rate (w_o) to the central node and define it as follow :

$$w_o = -\sum_{i=1}^{N} w_i \qquad (1)$$

<u>Where:</u>
N = the number of sources.
w_i= the information generation rate of a single source.

Assume that $\vec{D}$ is the vector field on A which represents the flux density of the routes to the destination. Given a small area at *z, z* □ A ,the tangent vector at *z* will be given by :[1]

$$\vec{D}(z) = \lim_{|S|\to 0} \frac{1}{s} \sum_{p_i \cap s \neq 0} w_i \hat{a}_i \qquad (2)$$

<u>Where</u> :
S = a selected area in the network which contains different sensors.
$\hat{a}_i$ = a unit vector tangent to p_i at *S* and pointing toward the direction of p_i that goes to the destination
p_i = the path for the sensor *i*.

It is found that when $|S| \to 0$, all paths that pass through *S* will have the same $\hat{a}_i$ based on Assumption (5).

Let $\rho(x, y)$ represents the density of sources on A.

$$\rho(x, y) = r(x, y) - w_o \delta(x - x_o)\, \delta(y - y_o) \qquad (3)$$

Where :
δ (.) = the Dirac delta function.
w_o = the total information generation rate of the sources.
(i.e., $_o = \int_A r(x,y)dxdy$) Note that $\rho(x, y) = r(x, y)$ at every place of the network except at the sink place. It is clear that $\int_A \rho(x,y)dxdy = 0$. [2].
Assume that $D(x, y)$ = the density of flow per second to be hopped at (x, y) on its path toward the sink. The direction of the information flow to the other nodes can be obtained from the orientation of $D(x, y)$.[2][3]
Due to the analogy between the electrostatic field equations and the sensor network, it is possible to rewrite $\rho(x,y)$ as :

$$\rho(x,y) = \frac{\partial}{\partial x}\vec{D}(x,y)\hat{a}_x + \frac{\partial}{\partial y}\vec{D}(x,y)\hat{a}_y \qquad (4)$$

Assume that $\vec{D}$ depends on the way of selecting a set of paths but independent of the paths being selected then $\vec{D}$ will satisfy the following equations.

$$\left.\begin{array}{l} \vec{\nabla}.\vec{D} = \rho \\ Dn(z) = 0 \; for\; z \;\square\; Boundary\; of\; A \end{array}\right\} \qquad (5)$$

Where:
A = the area of the network
$Dn(z)$ = the normal part of $\vec{D}$ on the boundary of A.
The first equation in (5) gives the natural limitation that all the traffic generated are directed to the destination, while the second one shows that no load will exit or enter the network . It is important to mention that equation (5) do not give a unique value of $\vec{D}$.
The solution of equation (5) means that there is traffic paths between the sources and destinations [1]. The lines which are always tangent to the direction of $\vec{D}$ and having the same orientation of it are called traffic flow lines.
A typical traffic flow line is shown in figure (2) (solid line) together with its approximated wireless links between the relaying nodes [7].

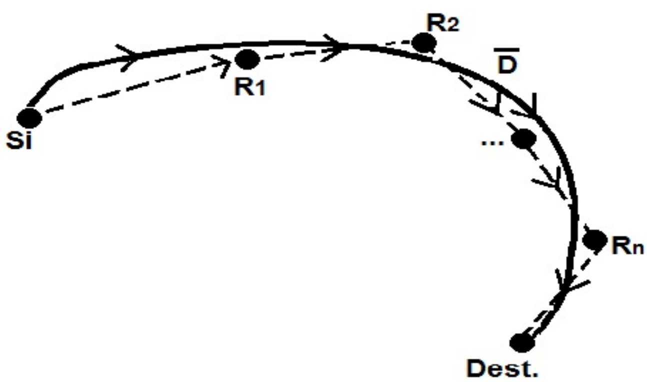


Figure. (2) Approximating the path given by a load flow line by the links made by the relaying nodes.

The solution of equation (5) does not give a unique value of $\vec{D}(x, y)$, (i.e. there are infinite solutions of this equation). This means that additional condition(s) to be placed on $\vec{D}(x, y)$ is required to generate a suitable set of routes. The additional factor can be obtained by making $\vec{D}$ as uniform as possible [2].
A uniform distribution of the traffic in the network can be obtained by using the following quadratic cost function:

$$J(\vec{D}) = \int_A |(\vec{D} - \vec{D}_{av})|^2 \, ds \qquad (6)$$

in which $\vec{D}_{av}$ is the average value of the vector field $\vec{D}$ on the set A, and it can simply defined as:

$$\vec{D}_{av} = \frac{1}{|A|} \int_A \vec{D} \, ds \qquad (7)$$

It can be concluded that this cost function is similar to the distribution of energy cost function in electrostatic theory. Optimizing the load on the network means minimizing $J(\vec{D})$ or :[7]

$$J(\vec{D})_{min} = \int_A |(\vec{D} - \vec{D}_{av})|^2 \, ds \qquad (8)$$

Subjected to :

$$\left.\begin{array}{l} \vec{\nabla}.\vec{D} = \rho \\ Dn(z) = 0 \; for \; z \in Boundary \; of \; A \end{array}\right\} \qquad (9)$$

Assume that $\vec{D}^*$ is the optimal solution of equation (9),which can be obtained when $\vec{D} = \vec{D}^*$,then $\vec{D}^*$ must satisfy :

$$\vec{\nabla} \times \vec{D}^* = 0 \qquad (10)$$

This means that it is possible to write some partial differential equations for the optimal value of $\vec{D}^*$:

$$\vec{\nabla}.\vec{D} = \rho \qquad \vec{\nabla} \times \vec{D}^* = 0 \qquad (11)$$

It is important to mention that :

1- The previous partial equations with the condition given by equation (5) will result in a unique value of $\vec{D}^*$ [10], [11].

2- The partial differential equations being obtained are similar to the electrostatics equations obtained from Maxwell's theory [12],[13].

Mathematically, a vector field such that $\vec{\nabla} \times \vec{D} = 0$ is called a conservative vector field, which can be expressed as the gradient of the potential function U:

$$\vec{D} = \vec{\nabla} U \qquad (12)$$

Equation (10) can be rewritten as:

$$\vec{\nabla}^2 U = \rho \qquad (13)$$

Equation (13) is called Poisson Equation and it can be approximated to:

$$\frac{\partial^2}{\partial x^2} u + \frac{\partial^2}{\partial y^2} u = \rho \qquad (14)$$

The boundary conditions for $\vec{D}$ must satisfy the following equation :

$$\vec{\nabla} U(z).\hat{n}(z) = 0 \; for \; z \in Boundary \; of \; A \qquad (15)$$

Where:
$\hat{n}(z)$ = a unit vector normal to the boundary.
The following interpretations , can be obtained from equation (11) :
1- The processing power needed to send a packet from the sender to the destination is proportional to the potential difference between the sender and receiver.
2- The routing of data follows the direction of the gradient of the potential function.

2.2 Solving the Generalized Poisson Equation Using the Finite-Difference Method (FDM)

The partial differential equation (poisson equation) which is derived in section (3) cannot be solved analytically. It requires a numerical technique called finite difference method (FDM), this can be achieved using the following methods: [14]

2.2.1 The Five-Points Star Configuration

It is a method that can convert a continuous function into discrete one figure(3) shows five points star configuration which is used to solve the poisson equation. The flow chart shown in figure (4) gives the details of how to apply the FDM according to the configuration shown in figure (3).

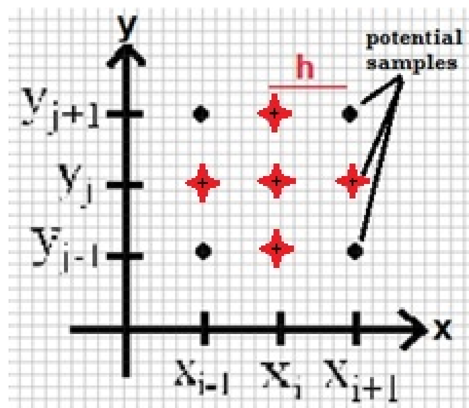


Figure (3) Five Points Star Configuration

2.2.2 Successive Over-Relaxation (SOR)

This method applies some kind of relaxation to have a good approximation (toward the exact solution), it is suitable in the case of large number of equations given that a good approximation of the solution is available [14].
Figure (5) show the steps of applying FDM based on this method to the poisson equation (partial differential equation) given in section (3).

3. The Results of The Proposed Model

The results given in this section are based on the following assumptions :
1- The sensor nodes are distributed over an area 1 Km × 1 Km.
2- The number of source nodes is equal to (440) while the number of destination nodes is only one.
3- The central node is responsible about forwarding the data of the whole network to the main processing unit to process it and to decide the suitable action.
Figure (6) show the distribution of sensors over the selected area

Figure (3) Five Points Star Configuration

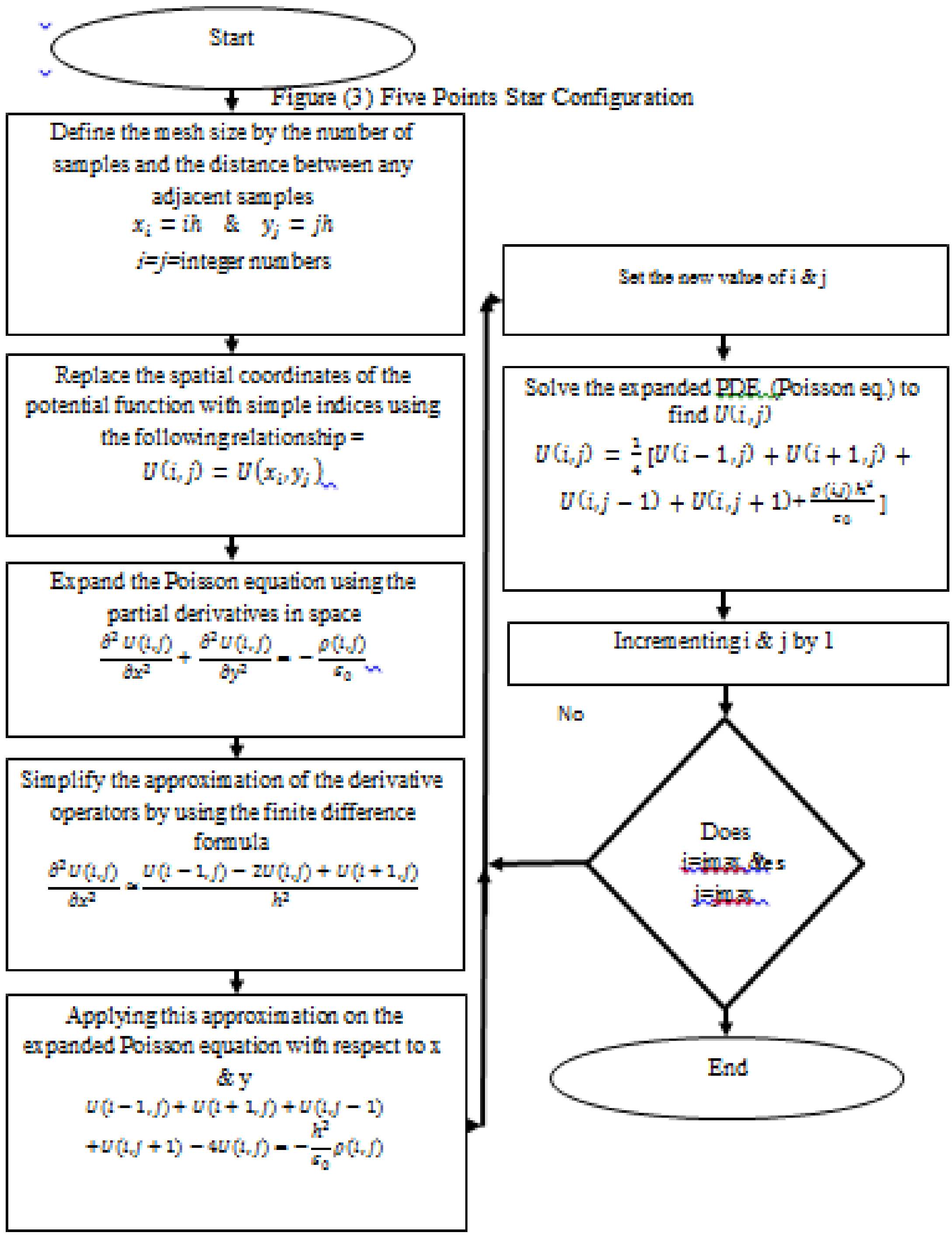


Figure (4) shows the steps of how to apply an FDM based on the five points star configuration.

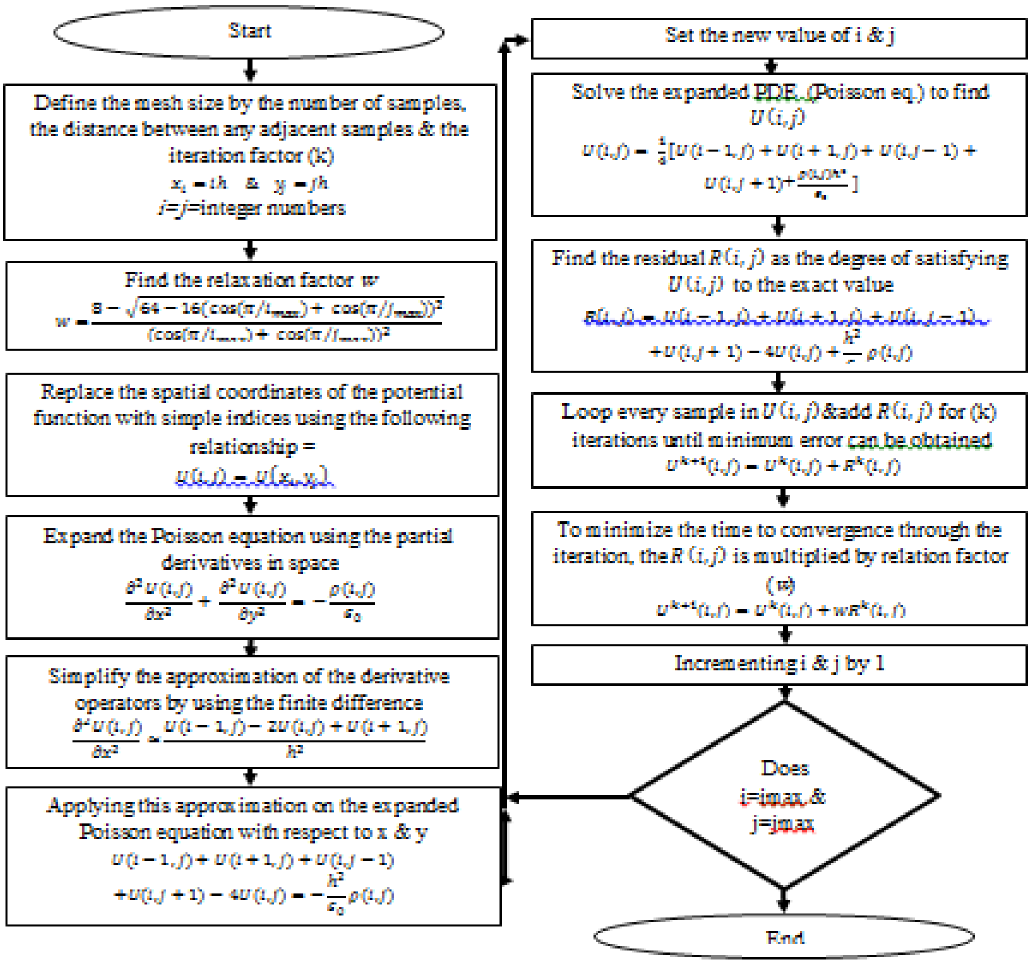


Figure (5) shows the steps of how to apply an FDM based on the successive over relaxation method

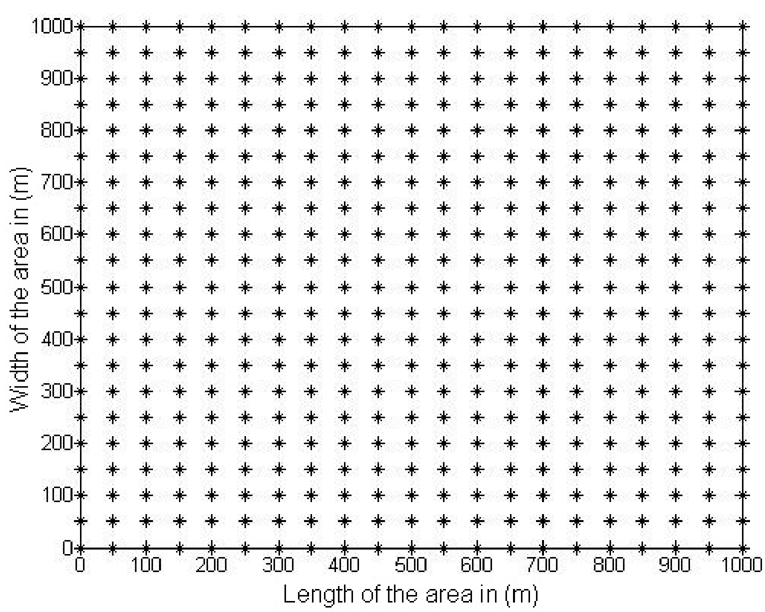


Figure (6) Distribution of the sensors over the selected area

After solving equation (10) numerically by finite element method, successive over-relaxation (SOR) with the boundary conditions given in equation (12) is applied over a 21×21 grid to find the potential function U .
From the modeling point of view, different weight values are assigned to the source nodes (positive charges) while the sum of all the source weights will be assigned to the destination node as a negative charge. Figure (7) shows the value of the potential function over the selected area.

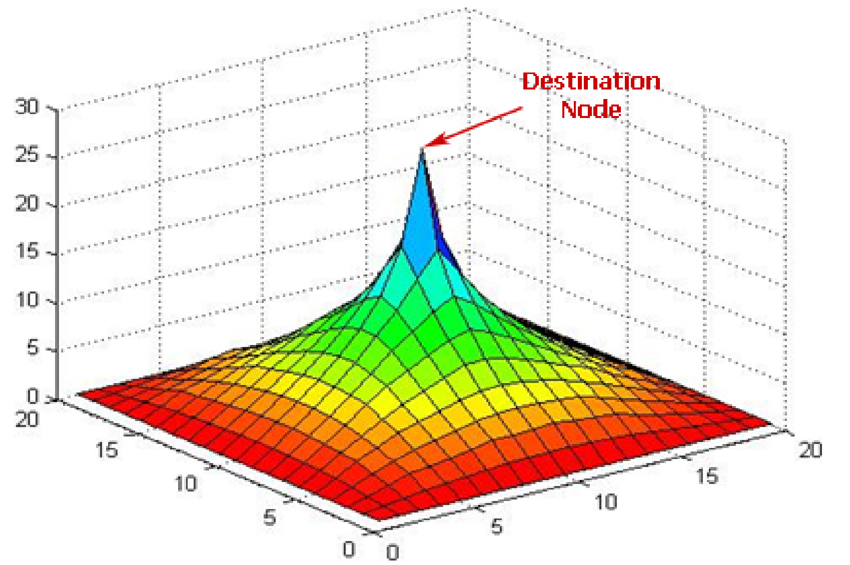


Figure (7) The value of the potential function U

$\vec{D}$ (load density vector) is calculated by taking the gradient of U. Figure (8) shows the directions of $\vec{D}$ at each node of the selected area towards the central node (destination).

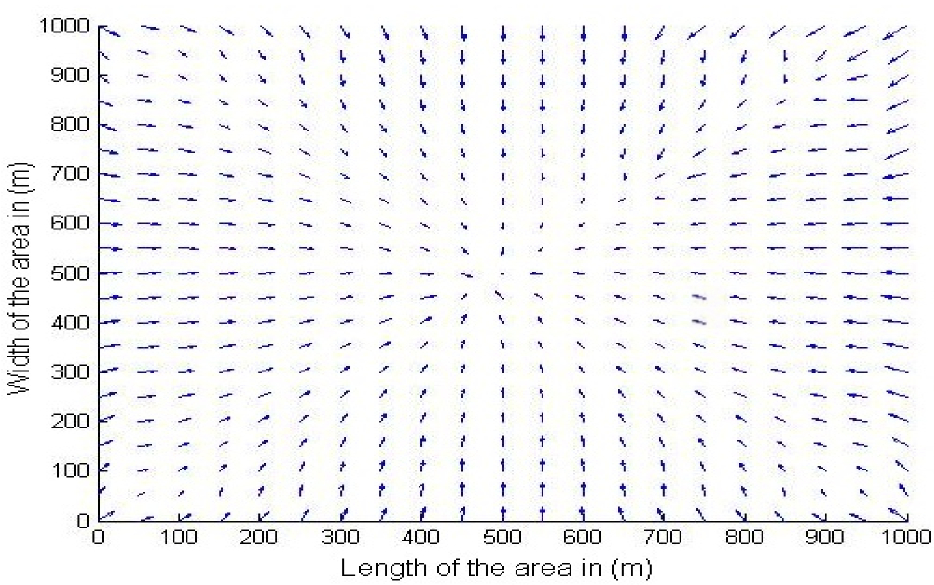


Figure (8) The directions of $\vec{D}$

It is clear from figure (8) that the arrows which refer to the directions of $\vec{D}$ (between any source and the destination nodes passing through intermediate nodes) will constitute the direction of the optimal path between them. Figure (9) shows the final possible optimal routs between the different sources and the destination can be found using the smallest angle between any two directions given in Figure (8).

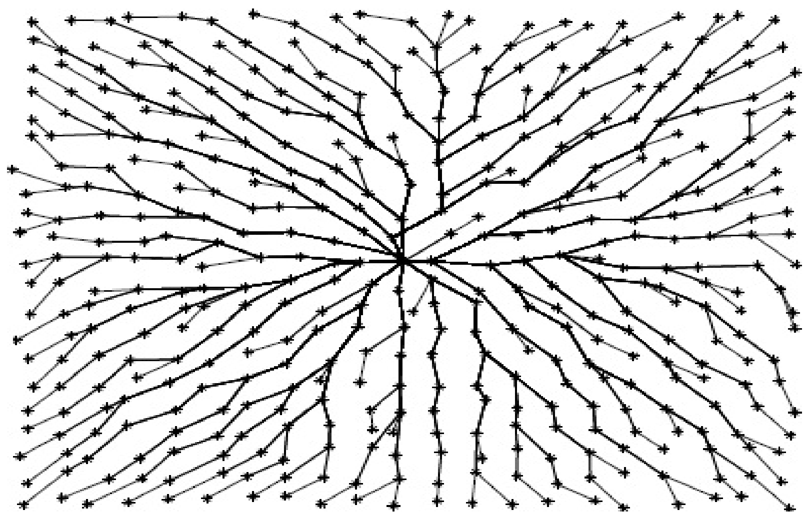


Figure (9) The Optimal routes from the sensors to the destination

To validate the results being obtained, an OPNET program is designed based on the optimal routes shown in figure (9) and in order to simplify the simulation process, the portion of area to be considered is chosen within the red rectangular shown in figure (10); it contains 121 nodes (120 source nodes and one destination node at the center of the rectangular).

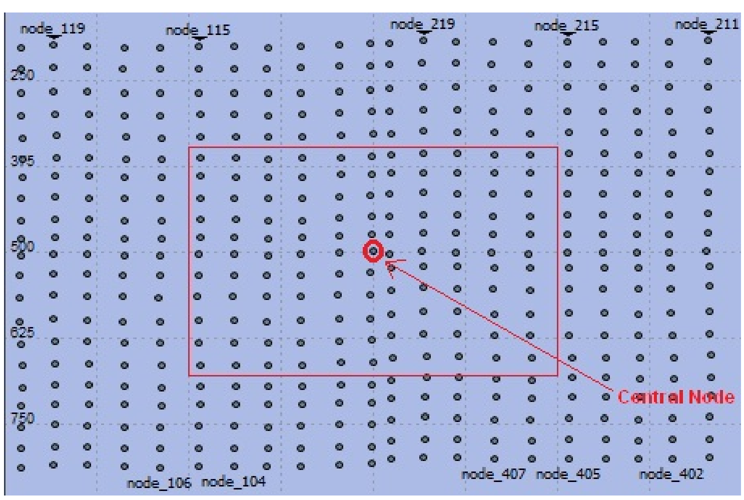


Figure (10) The proposed model of the network

The flow chart shown in figure (11) illustrates how OPNET applies the procedure of configuring the nodes based on the information of figure(9).

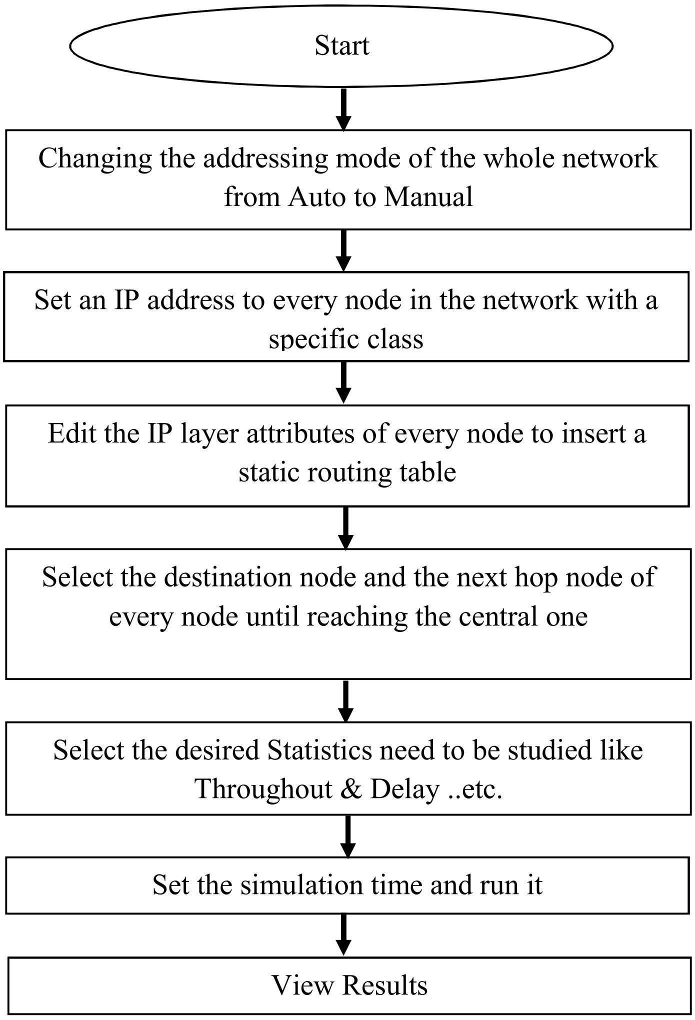


Figure (11) shows the steps of how OPNET applies the procedure of configuring the nodes based on the optimal routes.

Finally, the throughput of the subnet (within the red rectangular) with the routes which are obtained by using the electrostatic model (figure (11)) is calculated with the help of Opnet software and compared with four different arbitrarily scenarios, see figure (12).

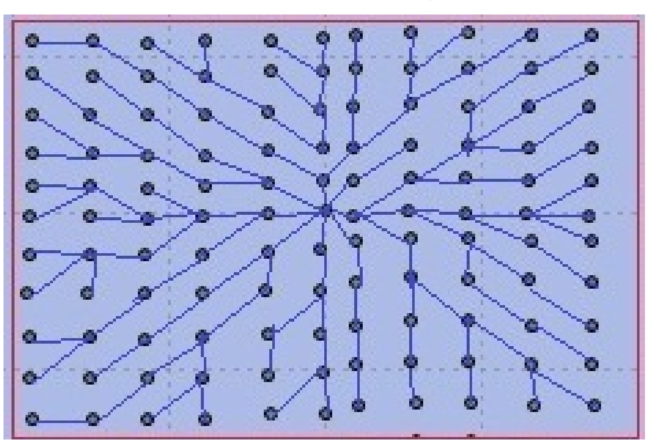
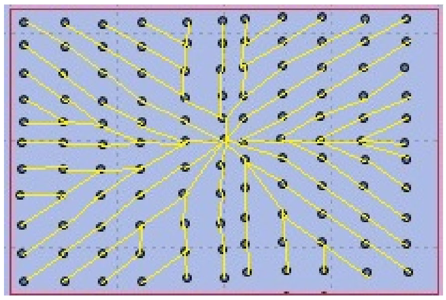
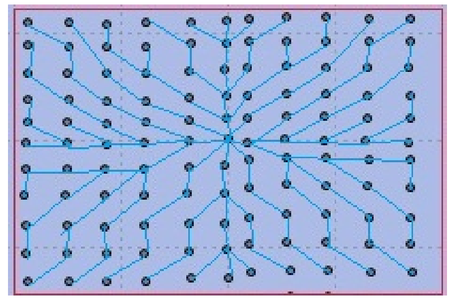
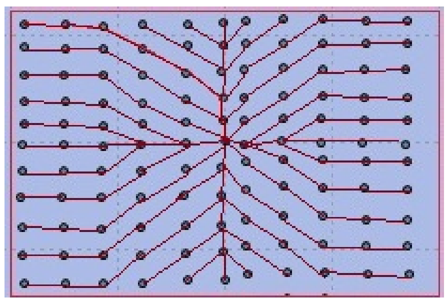


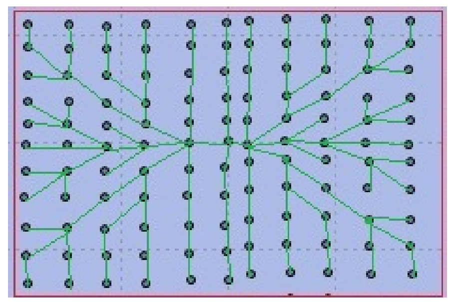
Figure (12) the paths from sources to the destination according to the electrostatic scenario





Scenario 2 Scenario 3





Scenario 4 Scenario 5

Figure (13) Some arbitrarily scenarios are chosen for comparison with the electrostatic scenario

The results of figure (14) prove that the optimal routes being calculated by the electrostatic method gives the highest throughput as compared with the results of the other scenarios , on the other hand , the delay performance of the electrostatic optimal routes is the minimum , see figure (15)

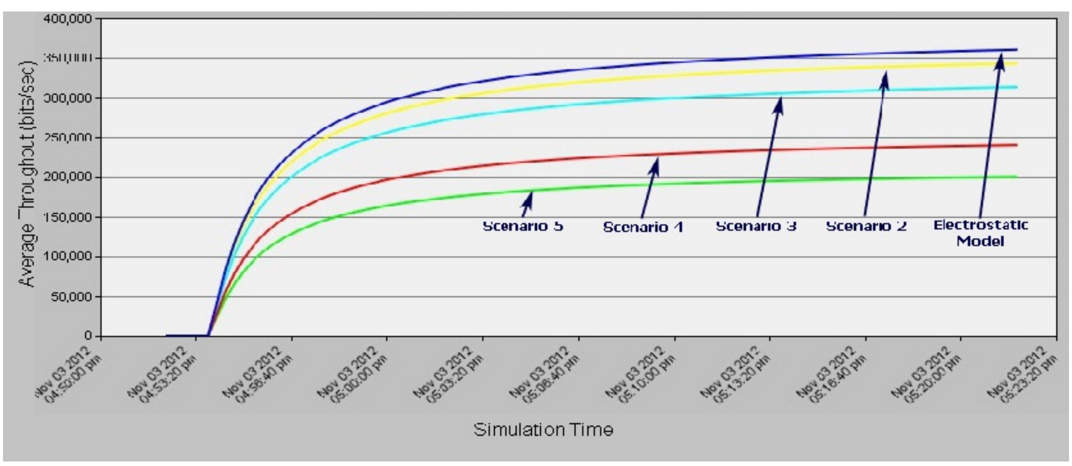


Figure (14) Throughput performance of the network with multiple scenarios

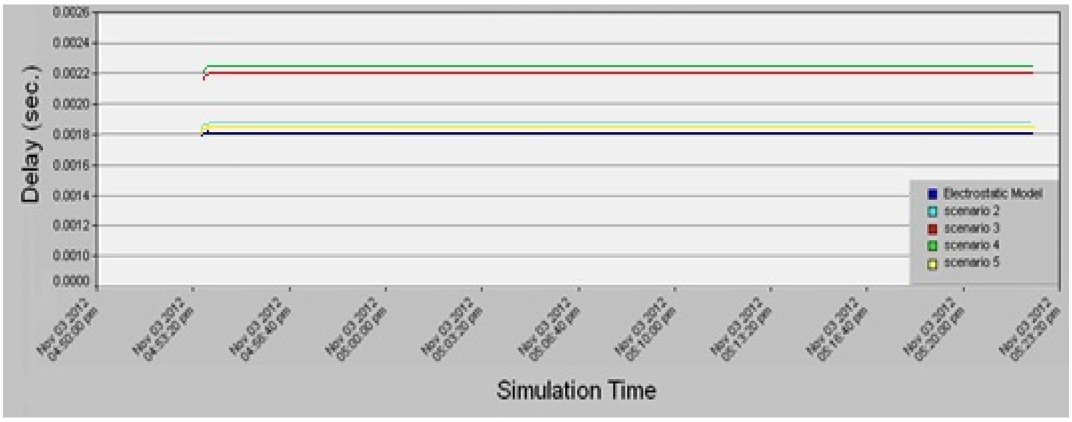


Figure (15) Delay performance of the network with multiple scenarios

5. Conclusion

It is found that optimal routing of a sensor network can be obtained by using the partial differential equations of electrostatic field distribution. This is possible due to the similarity between the electrostatic field lines and the links within the sensor network (in case of large number of nodes). A mathematical model is created according to the mentioned idea. The results verify the idea that the routes between sources and destination agrees to a large extend with the field lines between positive charges and a single negative one. An Opnet model is designed to calculate the network throughput using the optimal routes which are obtained from the solution of the mentioned partial differential equations, the throughput performance is then compared with some arbitrary selected routes (scenarios), the results show that the proposed model provides the best throughput and delay performances as compared with the throughput and delay performances of the various scenarios.

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The work was carried out at the college of Engineering. University of Mosul