Theoretical Study of Heat Transfer on Horizontal Cylinder Subjected to Force Longitudinal Vibration

"دراسة نظرٌة النتقال الحرارة على اسطوانة أفقٍة معرضة الهتساز طولً قسري"

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ABSTRACT

In the present work ,steady two - dimensional laminar forced convection heat transfer with parallel flow on a horizontal tube has been analyzed numerically for a wide range of the Reynolds numbers of $(Re = 10^2, 10^3, 10^4, \text{ and } 10^5)$ with Amplitude parameter (*A*) and vibrational angle in the range ($wt = \frac{\pi}{4}$, $\frac{\pi}{2}$ and $\frac{3\pi}{4}$ $\frac{3\pi}{4}$). Two types of boundary conditions have been considered. The first, when the tube wall is heated with constant uniform temperatures, while the second when the wall is heated by applying a uniform heat flux. The results are presented in terms of isotherms and streamlines to show the behavior of the fluid flow and temperatures fields. In addition, some graphics are presented the relation between average Nusselt number and the various parameters.

This research demonstrated that the rate of heat transfer is increased due to increase of amplitude and vibration angle as well as Reynolds number increase. The results also show that the average Nusselt number is a strong function of Reynolds number, vibrational angle amplitude parameter and the boundary conditions. Two different correlations have been made to show the dependence of the average Nusselt number on the Reynolds numbers, vibrational angle and amplitude parameter.

Keywords: Forced Convection – Forced Vibration – Horizontal tube – Finite Difference Method

الخالصة:

في هذا البحث ، تم إجراء دراسة عددية لانتقال الحرارة بالحمل ألقسري الطباقي المستقر ثنائي البعد بجريان موازي لأنبوب أفقي ضمن مدى واسع لعدد رينولدز (10⁵, 10³, 10³, 10⁵) مع مقدار الإزاحة (A) ولمدى زاوية اهتزاز $(wt = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4})$ لاجراء البحث تم اعتماد نوعان من الظروف الحدية_. الأول عندما يكون جدار الأنبوب مسخن إل*ى العنويا عندما يكون جدار الأنبوب مسخن إلى* $\big(\omega t = \pi/4, \pi/2, 3\pi$ درجات حرارة مختلفة ومنتظمة والثاني، عندما يكون الجدار مسخن بمصدر حراري . تم تمثيل نتائج الدراسة بدلالة خطوط درجات الحرارة الثابتة وخطوط الانسياب لبيان سلوك درجات الحرارة والجريان في الحيز بالإضافة إلى رسومات بيانية أخرى تمثل علاقة معدل عدد نسلت مع المتغيرات المذكورة أعلاه . اوضحت نتائج البحث بان معدل انتقال الحرارة يزداد نتيجة لزيادة زاوية الاهتزاز ورقم رينولدز . كذلك بينت نتائج الدراسة بان معدل عدد نسلت هو دالة قوية لعدد رينولدز وزاوية الاهتزاز ومقدار الإزاحة (A) والظروف الحدية _. ومن خلال النتائج نبين وجود علاقة قوية بين معدل عدد نسلت وعدد ر بنو لدز

NOMENCLATURE

- *A* : Amplitude of vibration (m)
- C_f : Friction coefficient
- D : Diameter of tube (m)
- f : Frequency of vibration (Hz)
- *f* : Any arbitrary function, f(r,z)
- H : Free stream ratio
- h : Heat transfer coefficient $(W/m^2.k)$
- k : Thermal conductivity $(W/m.k)$
- L : Length of tube (m)

- Lc : Characteristic length (m)
- m : Nodes number in axial direction
- *Nu* : Nusselt number
- *Nuz* : Average nusselt number
- n : Nodes number in radial direction
- P : Pressure (Pa)
- Pr : Prandtle number $\begin{pmatrix} V_{\alpha} \end{pmatrix}$ IV
- q \therefore Heat flux (W/m^2)
- \textit{Re} : Reynolds number $\left(\frac{\rho ud}{\mu}\right)$ $\left(\frac{\rho ud}{\mu}\right)$ ſ μ pud
- R : Dimensionless radial coordinate
- r : Radial coordinate (m)
- T : Temperature (k)
- U : Dimensionless velocity
- u : Axial velocity (m/sec).
- v : Radial velocity (m/sec).
- *wt* : Vibration angle
- Z : Dimensionless axial coordinate
- z : Axial coordinate (m)

1. INTRODUCTION

Forced convection is one of the important modes of heat transfer. Forced convection is the transportation and exchange of heat due to the mixing motion of different parts of a fluid. Because buoyancy forces resulting from the density gradients are too small here, therefore; these forces are negligible. The fluctuating motions in the turbulent flow promote heat exchange by convection. Consequently, heat transfer in the turbulent flow is higher than in laminar flow and so that the result in higher heat transfer coefficients.

The major technical applications of this work is in the analysis and design of parallel flow heat exchanger for the both types(counter and parallel), tubes, channel flow and marine applications.

Generally, all the studies in this field can be classified into two types. The first type illustrated the heat transfer surface is vibrated experimentally and it sub divided for two methods. In the first method, the surface is held stationary and vibrations are established in the fluid medium surrounding the surface. In the second method, an oscillatory motion is supplied upon the surface itself. The second type is theoretically, the flow medium is subjected around vibrational or agitation surface [1]. Mariusz Leus and Pawel Gutowski, 2012, studied the influence of tangentiallongitudinal vibrations on the drive force and friction force in sliding motion. It has been demonstrated that the use in numerical analyses of Dahl friction model, facilitates precise projection of changes in these forces taking place in real systems under the influence of imposed tangentiallongitudinal vibrations [2] .

Most previous studies, however, have dealt with only natural convection heat transfer with mechanical or acoustical vibration. A few investigations treated forced convection heat transfer with mechanical or acoustical vibration especially with empirical methods for the outside flow.

The mass, momentum, and energy conservation equations, that describe the fluid flow and heat transfer for forced convection of laminar, two-dimensional flow are nonlinear and because of this nonlinearity, some difficulties have arisen in numerical as well as in analytical studies. One of the greatest difficulties with the numerical studies is the problem of divergence of the iterative methods since an analytical solution of the actual problem is extremely difficult, if not possible, a number of assumptions together with the computational fluid dynamic techniques are used to obtain approximate results.[3]

The importance of vibrational motion of the heated surface is to increase eddies of the flow around the heated surface and may increase the heat transfer coefficient. The intensity of vibration represents a control parameter in the vibrational motion. It depends on both the amplitude (*A*) and frequency (f) and the product of these two parameters. [4]

2. MATHMATICAL FORMULATION

The schematic of the physical model and the coordinate system are shown in Fig. (1). It is a horizontal tube. The flow descriptions were external, laminar and incompressible. The vibration vector was parallel of the fluid flow direction in which the thermal boundary are of two types.

The depth of the region is assumed to be sufficiently large so that the flow and the heat transfer are two dimensional. The fluid considered is Newtonian and the flow is laminar. For the present physical model, governing equations in their primitive form, are given by [5]

$$
\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial z} = 0\tag{1}
$$

$$
u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + v\left[\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right]
$$
(2)

$$
u\frac{\partial v}{\partial z} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + v\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2}\right)
$$
(3)

$$
v\frac{\partial T}{\partial r} + u\frac{\partial T}{\partial z} = \alpha \left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right)
$$
(4)

Since it proves to be more convenient to work in terms of a stream function and vorticity, the stream function $\psi(r, z)$ is introduced in the usual manner [6]

$$
u = \frac{\partial \psi}{\partial r} \quad \& \quad v = -\frac{\partial \psi}{\partial z} \tag{5}
$$

It is evident from Eq. (5) that the stream function satisfies the continuity equation identically. Further more, for this plane flow field, the only non- zero component of the vorticity is:

$$
-\omega = \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r}
$$
(6)

Combining the definition of the vorticity and the velocity components in terms of the stream function, and cross- differentiating the Eq.(2) and (3) to reduce the number of equations and eliminate the pressure terms, a new set of equations is obtained with independent variables ψ , ω and *T*:

$$
-\omega = \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r}
$$
(7)

$$
\left(\frac{\partial \psi}{\partial r} \frac{\partial \omega}{\partial z} - \frac{\partial \psi}{\partial z} \frac{\partial \omega}{\partial R}\right) = \nu \left(\frac{\partial^2 \omega}{\partial z^2} + \frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r} \frac{\partial \omega}{\partial r} - \frac{\omega}{r^2}\right)
$$
(8)

$$
\left(\frac{\partial \psi}{\partial r} \frac{\partial T}{\partial z} - \frac{\partial \psi}{\partial z} \frac{\partial T}{\partial r}\right) = \left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r}\right)
$$
(9)

Now, the mathematical problem formulated above was placed in dimensionless form by defining new dimensionless variables [7]:

$$
\left(Z^* = \frac{z}{L_c}\right), \left(R^* = \frac{r}{L_c}\right)
$$

The overall heating coefficient for constant wall temperature

$$
\theta^* = \left[\frac{T - T_{\infty}}{T_{\rm w} - T_{\infty}} \right]
$$

The wall temperature for constant heat flux $\overline{1}$ Γ and Γ

$$
\theta^* = \left| \frac{T - T_{\infty}}{\frac{qL_c}{k}} \right|
$$

$$
\left(\psi^* = \frac{\psi}{uL_c} \right), \left(\omega^* = \frac{\omega L_c}{u} \right)
$$

Inserting all the dimensionless variables into Eqs. (7) to (9), yield the following final nondimensional equations:

$$
-\omega^* = \frac{\partial^2 \psi^*}{\partial Z^{*2}} + \frac{\partial^2 \psi^*}{\partial R^{*2}} - \frac{1}{R^*} \frac{\partial \psi^*}{\partial R^*}
$$
(10)

$$
\operatorname{Re}\left(\frac{\partial \psi^*}{\partial R^*} \frac{\partial \omega^*}{\partial Z^*} - \frac{\partial \psi^*}{\partial Z^*} \frac{\partial \omega^*}{\partial R^*}\right) = \left(\frac{\partial^2 \omega^*}{\partial Z^{*2}} + \frac{\partial^2 \omega^*}{\partial R^{*2}} + \frac{1}{R^*} \frac{\partial \omega^*}{\partial R^*} - \frac{\omega^*}{R^{*2}}\right)
$$
(11)

$$
\Pr\left(\frac{\partial \psi^*}{\partial R^*} \frac{\partial \theta^*}{\partial Z^*} - \frac{\partial \psi^*}{\partial Z^*} \frac{\partial \theta^*}{\partial R^*}\right) = \left(\frac{\partial^2 \theta^*}{\partial Z^{*2}} + \frac{\partial^2 \theta^*}{\partial R^{*2}} + \frac{1}{R^*} \frac{\partial \theta^*}{\partial R^*}\right)
$$
(12)

The physical quantities of insert in this problem are the local Nusselt number along the heated wall, defined by:

$$
Nu_z = \frac{q(L)}{k(T_b - T_w)}
$$
\n(13)

And also the average Nusselt number, which is defined as:

$$
\overline{Nu} = \frac{1}{L} \int_{0}^{L} Nu_z dz \tag{14}
$$

And the local friction coefficient is defined as:

$$
C_{f_z} = \frac{\tau_w}{\frac{1}{2} \rho u_{in}^2}
$$
 (15)

And also, the shear stress is defined as:

$$
\tau_{w} = \mu \frac{\partial u}{\partial r}\bigg|_{r=r_{o}} \tag{16}
$$

 $\int_{r_+}^{r_+} \left(\frac{R}{r} = \frac{1}{L_c} \right)$

Il heating coefficient for constant wall ten
 $\frac{r}{1-r_{\infty}}$
 $\left[\frac{1}{r} \right]$
 $\left(\omega^* = \frac{\omega L_c}{u} \right)$
 $\left(\omega^* = \frac{\omega L_c}{u} \right)$
 $\left(\omega^* = \frac{\omega L_c}{u} \right)$
 $\left(\frac{\omega^*}{2} + \frac{\partial^2 \psi^*}{\partial t^2} -$ Numerical methods have been developed to handle problems involving nonlinearities in the describing equations, or complex geometries involving complicated boundary conditions. A finite – difference technique is applied to solve the governing equations. These three equations (Esq. (10), (11) and (12)) are to be solved in a given region subject to the condition that the values of the stream function, temperature and the vorticity, or their derivatives, are prescribed on the boundary

of the domain. The finite difference approximation of the governing equations was been on dividing the $(0 \le Z^* \le 1)$ interval into (m) equal segment separated by (m+1) nodes. Likewise, the (R^*) interval was divided into (n) segment, regular meshes are shown in fig.(2). Assume that unknown variable $f(r, z)$ possesses continuous derivatives, so the approximation of second derivative of (f) to determine interior point is [8]:

$$
\left(\frac{\partial^2 f}{\partial z^2}\right) = \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{\Delta z^2}
$$

And the first derivative central difference is:

$$
\left(\frac{\partial f}{\partial z}\right) = \frac{f_{i+1,j} + f_{i-1,j}}{\Delta z}
$$

The usual procedure for obtaining the form of partial differential equation with uniform finite – difference method [9] is to approximate all the partial derivatives in the equation by means of their Taylors series expansions. Eq.(10) can be approximated using central- difference at the representative interior point (i,j) , thus, Eq. (10) can be written for regular mesh as:

$$
-\omega^*_{i,j} = \left[\frac{\psi^*_{i+1,j} - 2\psi^*_{i,j} + \psi^*_{i-1,j}}{\Delta Z^2} + \frac{\psi^*_{i,j+1} - 2\psi^*_{i,j} + \psi^*_{i,j-1}}{\Delta R^2} - \frac{1}{R^*_{i,j}} \frac{\psi^*_{i,j} - \psi^*_{i,j-1}}{\Delta R} \right]
$$
(17)

Also, a central difference formulation can be used for Eqs.(11) and (12). But this problem will need to be solved for reasonably high values of Reynolds number ; it is known that such a formulation may not be satisfactory owing to the loss of diagonal dominance in the sets of difference equations, with resulting difficulties in convergence when using an iterative procedure.

A forward – backward technique can be introduced to maintain the diagonal dominance coefficient of $(\psi_{i,j}^*)$ in Eq. (11) and $(\theta_{i,j}^*)$ in Eq. (12) which determines the main diagonal elements of the resulting linear system; this technique is outlined as follows[10]:

$$
\gamma = \psi_{i+1,j}^{*} - \psi_{i-1,j}^{*}
$$

\nand
\n
$$
\beta = \psi_{i,j+1}^{*} - \psi_{i,j-1}^{*}
$$

\nThen approximate Eq.(11) by:
\n
$$
Re \left[\left(\frac{\psi_{i,j+1}^{*} - \psi_{i,j-1}^{*}}{2\Delta R} \right) \left(\frac{\omega_{i+1,j}^{*} - \omega_{i,j}^{*}}{\Delta Z} \right) - \left(\frac{\psi_{i+1,j}^{*} - \psi_{i-1,j}^{*}}{2\Delta Z} \right) \left(\frac{\omega_{i,j+1}^{*} - \omega_{i,j}^{*}}{\Delta R} \right) \right] =
$$
\n
$$
\left[\left(\frac{\omega_{i+1,j}^{*} - 2\omega_{i,j}^{*} + \omega_{i-1,j}^{*}}{\Delta Z^{2}} \right) + \left(\frac{\omega_{i,j+1}^{*} - 2\omega_{i,j}^{*} + \omega_{i,j-1}^{*}}{\Delta R^{2}} \right) + \frac{1}{R_{i,j}^{*}} \left(\frac{\omega_{i,j+1}^{*} - \omega_{i,j}^{*}}{\Delta R} \right) - \frac{\omega^{*}}{R_{i,j}^{*2}} \right]
$$
\nNow, if
\n
$$
\gamma c = \gamma_{i} = \
$$

$$
\gamma \ge 0, \quad \frac{\partial f}{\partial r} = \frac{\lambda_{i,j+1} - \lambda_{i,j}}{\Delta r}
$$

$$
\gamma < 0, \quad \frac{\partial f}{\partial r} = \frac{\lambda_{i,j} - \lambda_{i,j-1}}{\Delta r}
$$
and if
$$
\frac{\partial \lambda}{\partial r} = \frac{\lambda_{i,j} - \lambda_{i,j-1}}{\Delta r}
$$

$$
\beta \geq 0, \quad \frac{\partial \lambda}{\partial z} = \frac{\lambda_{i,j} - \lambda_{i-1,j}}{\Delta z}
$$

$$
\beta < 0, \quad \frac{\partial \lambda}{\partial z} = \frac{\lambda_{i+1,j} - \lambda_{i,j}}{\Delta z}
$$

If $\gamma \ge 0$ and $\beta \ge 0$, the stream function equation is:

$$
\operatorname{Re}\left[\left(\frac{\psi_{i,j+1}^{*} - \psi_{i,j-1}^{*}}{2\Delta R}\right)\left(\frac{\omega_{i,j}^{*} - \omega_{i-1,j}^{*}}{\Delta Z}\right) - \left(\frac{\psi_{i+1,j}^{*} - \psi_{i-1,j}^{*}}{2\Delta Z}\right)\left(\frac{\omega_{i,j+1}^{*} - \omega_{i,j}^{*}}{\Delta R}\right)\right] = \left[\left(\frac{\omega_{i+1,j}^{*} - 2\omega_{i,j}^{*} + \omega_{i-1,j}^{*}}{\Delta Z^{2}}\right) + \left(\frac{\omega_{i,j+1}^{*} - 2\omega_{i,j}^{*} + \omega_{i,j-1}^{*}}{\Delta R^{2}}\right) + \frac{1}{R_{i,j}^{*}}\left(\frac{\omega_{i,j+1}^{*} - \omega_{i,j}^{*}}{\Delta R}\right) - \frac{\omega^{*}}{R_{i,j}^{*2}}\right]
$$
\nIf χ \leq 0, and $R > 0$, the stream function equation is:

\n
$$
\chi_{i} = \frac{1}{\Delta R} \left(\frac{\omega_{i+1,j}^{*} - \omega_{i-1,j}^{*}}{\Delta R^{2}}\right)
$$
\n(19)

If γ < 0, and $\beta \ge 0$, the stream function equation is:

$$
\text{Re}\left[\left(\frac{\psi_{i,j+1}^* - \psi_{i,j-1}^*}{2\Delta R}\right)\left(\frac{\omega_{i,j}^* - \omega_{i-1,j}^*}{\Delta Z}\right) - \left(\frac{\psi_{i+1,j}^* - \psi_{i-1,j}^*}{2\Delta Z}\right)\left(\frac{\omega_{i,j}^* - \omega_{i,j-1}^*}{\Delta R}\right)\right] =
$$
\n
$$
\left[\left(\frac{\omega_{i+1,j}^* - 2\omega_{i,j}^* + \omega_{i-1,j}^*}{\Delta Z^2}\right) + \left(\frac{\omega_{i,j+1}^* - 2\omega_{i,j}^* + \omega_{i,j-1}^*}{\Delta R^2}\right) + \frac{1}{R_{i,j}^*}\left(\frac{\omega_{i,j}^* - \omega_{i,j-1}^*}{\Delta R}\right) - \frac{\omega^*}{R_{i,j}^{*2}}\right]
$$
\n(20)

If $\gamma \ge 0$ and $\beta < 0$, the stream function equation is:

$$
\operatorname{Re}\left[\left(\frac{\psi_{i,j+1}^{*}-\psi_{i,j-1}^{*}}{2\Delta R}\right)\left(\frac{\omega_{i+1,j}^{*}-\omega_{i,j}^{*}}{\Delta Z}\right)-\left(\frac{\psi_{i+1,j}^{*}-\psi_{i-1,j}^{*}}{2\Delta Z}\right)\left(\frac{\omega_{i,j+1}^{*}-\omega_{i,j}^{*}}{\Delta R}\right)\right]=\n\left[\left(\frac{\omega_{i+1,j}^{*}-2\omega_{i,j}^{*}+\omega_{i-1,j}^{*}}{\Delta Z^{2}}\right)+\left(\frac{\omega_{i,j+1}^{*}-2\omega_{i,j}^{*}+\omega_{i,j-1}^{*}}{\Delta R^{2}}\right)+\frac{1}{R_{i,j}^{*}}\left(\frac{\omega_{i,j+1}^{*}-\omega_{i,j}^{*}}{\Delta R}\right)-\frac{\omega^{*}}{R_{i,j}^{*2}}\right]\n\tag{21}
$$

And finally, If $\gamma < 0$, and $\beta < 0$, the stream function equation is:

β < 0,
$$
\frac{OA}{\delta z} = \frac{\alpha_{i+1,j} - \alpha_{i,j}}{\Delta z}
$$

\nIf $\gamma \ge 0$ and $\beta \ge 0$, the stream function equation is:
\n
$$
Re\left[\left(\frac{w_{i,j-1}^* - w_{i,j-1}^*}{2\Delta R}\right) \left(\frac{w_{i,j-1}^* - w_{i-1,j}^*}{\Delta Z}\right) - \left(\frac{w_{i+1,j}^* - w_{i-1,j}^*}{2\Delta Z}\right) \left(\frac{\alpha_{i,j+1}^* - \alpha_{i,j}^*}{\Delta R}\right)\right] =
$$
\n
$$
\left[\left(\frac{\omega_{i+1,j}^* - 2\omega_{i,j}^* + \omega_{i-1,j}^*}{\Delta Z^2}\right) + \left(\frac{\omega_{i,j+1}^* - 2\omega_{i,j}^* + \omega_{i,j-1}^*}{\Delta R^2}\right) + \frac{1}{R_{i,j}^*} \left(\frac{\omega_{i,j+1}^* - \omega_{i,j}^*}{\Delta R}\right) - \frac{\omega^*}{R_{i,j}^*}\right]
$$
\nIf $\gamma < 0$, and $\beta \ge 0$, the stream function equation is:
\n
$$
Re\left[\left(\frac{w_{i,j+1}^* - w_{i,j-1}^*}{2\Delta R}\right) \left(\frac{\omega_{i,j}^* - \omega_{i-1,j}^*}{\Delta Z}\right) - \left(\frac{w_{i+1,j}^* - w_{i-1,j}^*}{\Delta R^2}\right) \left(\frac{\omega_{i,j}^* - \omega_{i,j-1}^*}{\Delta R}\right)\right] =
$$
\n
$$
\left[\left(\frac{\omega_{i+1,j}^* - 2\omega_{i,j}^* + \omega_{i-1,j}^*}{\Delta Z}\right) + \left(\frac{\omega_{i,j+1}^* - \omega_{i,j}^*}{\Delta Z}\right) - \left(\frac{w_{i+1,j}^* - \omega_{i,j}^*}{\Delta R}\right) \left(\frac{\omega_{i,j+1}^* - \omega_{i,j-1}^*}{\Delta R^2}\right)\right] =
$$
\n
$$
\left[\left(\frac{\omega_{i+1,j}^* - 2\omega_{i,j}^* + \omega_{i-1,j}^*}{2\Delta R}\right) + \left(\frac
$$

to assure the diagonal dominance of the coefficient matrix for($\omega_{i,j}^{*}$) and $(\theta_{i,j}^{*})$, which an under relaxation technique can be applied to accelerate the convergence of Eq.(18); the expression is used in this technique presented in the following:

this technique presented in the
 $(\omega_{i,j}^{*k+1} - \omega_{i,j}^{*k}) + \omega_{i,j}^{*k}$ (computed) * , $*_{k+1}$ $\omega^{*_{k+1}}_{i,j} = \phi(\omega^{*_{k+1}}_{i,j} - \omega^{*_{k}}_{i,j}) + \omega_i$ *i j k i j*

where the parameter (ϕ) is the relaxation coefficient for the vorticity. The value of this relaxation coefficient is in the range of (0 to 2).[4]

in order to obtain results of the conversation equations, the above equations (Eqs.(17), (18) and (23)) are subjected to the following boundary conditions:

Inlet Section

$$
u_{in} = \frac{\text{Re} \times \mu}{\rho \times d}
$$

\n
$$
u = u_{in}
$$

\n
$$
v = 0
$$

\n
$$
T = T_{\infty}
$$

\n
$$
\psi = Y \quad U = \frac{\partial \psi}{\partial Y} \quad \therefore U = 1, \ \omega = 0
$$

Exit Section

 $=0$ ∂ ∂ *z u* $=0$ ∂ ∂ *z v* $=$ ∂ ∂ *z* $\frac{T}{T}$ = Constant

Also, the following finite difference equation for the vorticity at a wall is adopted as the boundary condition for the vorticity equation:

$$
\omega_{i,j} = \left(\frac{3 \cdot \psi_{i-1,j}}{\Delta Z^2} - \left(\frac{3}{2} \left(\frac{2}{\Delta Z} + 1 + \Delta Z \right) . \sin \omega t + \frac{1}{2} \omega_{i-1,j} \right) . \frac{1}{(-1 - \frac{3}{2} \Delta Z)} \right)
$$

the numerical work start with giving the distributions of stream function and temperature for forced convection as the zeroth - order approximation. Then, obtained the zeroth – order approximation of vorticity; based on these old fields, equation (17) is used to determine point – by – point the new (ψ^*) field, and equation (18) is used to determine the new (ω^*) while the energy equation (23) is used to determine the new $(\hat{\theta}^*)$ field. The iteration process is terminated under the following condition:

$$
\sum_{i,j} \left| \tau^{r+1}{}_{i,j} - \tau^{r}{}_{i,j} \right| \bigg/ \sum_{i,j} \tau^{r+1}{}_{i,j} \le 10^{-5}
$$
\n(24)

Where, (τ) stands for either $(\psi^*, \omega^* \text{ or } \theta^*)$; (r) denotes the iteration step.

Results and discussion

Case (I):

A horizontal tube under constant wall temperature where the external parallel flow is applied with different values of (*Re, A* and *wt*.

a- **Flow Fields**

The stream lines in the axial and radial directions of constant wall temperature with different values of Reynolds number are shown in fig(3) and fig.(4). These figures show the effect of the amplitude (*A*) on the change in the streamlines. These also show the effect of vibration angle (*wt*).

When the amplitude increases from (1 to 15mm), the stream lines shape will changed like a wave at the start point of the tube and it move to its end. This effect shown only near the tube wall.

It may effect on the boundary layer of the flow near the tube wall. The influence of amplitude and vibration angle vanished its effect until $(0.4 R^*)$ radialy, but still axially.

The above figures show that, when *Re* increase, the effect of amplitude and vibration angle on the streamlines be clearer axially and radially.

b- Isothermal Lines.

the figures (5,6) show the influence of *Re, A* and *wt* on the Temperature distribution contours at constant wall temperature. They are indicating that as *Re*, *A* and *wt* increased, the change in isothermal lines will increase and disturb as shown when *A*=10 and 15mm. This disturbance increase as more as *Re* increase and *wt* increase. This means that the heat transfer rate is increase as the vibration parameter increase.

c- Heat Transfer Coefficient

To understand the heat transfer process by forced convection, we must be evaluate the heat transfer coefficient (*h*), but to make the present work having generality, the calculated results must be in dimensionless form. Therefore, it must be needed to evaluate Nusselt number (*Nu*) as a function of influence parameters. Figs. (7, 8) show the variation of average Nusselt number versus

Reynolds number with different values of amplitudes and vibration angle ($wt = \frac{\pi}{2}$) parameters on

the hot wall of the horizontal tube. It is also seen that for range of amplitude less then (10mm), the rate of increase in *Nu* against *Re* for different values of *wt* is relatively small. But, *Nu* increase rapidly as *Re* increase for *A* more than (10mm) expressing the increase of convective heat transfer.

Case (II)

A horizontal tube under constant heat flux where the external parallel flow is applied with different values of (*Re*, *A* and *wt*.

a- Flow Field

As the amplitude increased, the streamlines are disturb more for constant *Re*. Also the vibration angle effect on the streamlines has the same manner of amplitude effect in the first quarter of vibration angle then returned to its original effect in the second quarter.

The *Re* effect also shown by these figures, that as *Re* increase, the streamlines more disturb. In general, the increase in the distribution of streamlines near the tube wall was because of the turbulence of high *Re*, *A* and *wt* .

b- Isothermal Lines.

Figs. (9, 10) show the effect of *Re*, *A* and *wt* on isothermal contours. In general, these figures observed that as *A* increase, the isothermal lines disturb and fluid near the tube wall be more heated.

It's shown that when ($wt = \frac{\pi}{2}$), the fluid is more heated at the wall because of inverse movement of the wall by vibration.

Finally, two correlation equations have been predicted depending on variation Reynolds number, amplitude, and vibration angle for both two cases, by using least square method.

Case (I): forced convection heat transfer on tube with B.C.1

 $Nu = 52.987 \text{ Re}^{0.02326} A^{0.38984} wt^{0.04324}$, $R=0.935$

Case (II): forced convection heat transfer on tube with B.C.2

 $Nu = 121.213 \text{Re}^{0.015849} A^{0.339} wt^{0.01693758}$, $R=0.98542$

6- Conclusions

The main conclusions of the present work are:

- 1- The rate of heat transfer is increased due to increase of amplitude and vibration angle as well as Reynolds number increase.
- 2- The streamlines is increased due to increase of amplitude and vibration angle until ($wt = \frac{\pi}{2}$) as well as Reynolds number increase.
- 3- For the two cases that have been solved, it has been demonstrated that the average Nusselt number is a strong function of Reynolds number, vibration angle (*wt*), and Amplitude, also the results show that the average Nusselt number:
- a- Increases as (*Re*) increases, for a given values of (*A*) and (*wt*).
- b- Increases as (*wt*) increases except for ($wt = \frac{\pi}{2}$) for a given values of (*Re*) and (*A*).
- c- Increases as (*A*) increases, for a given values of (*Re*) and (*wt*).
- d- Nu_z and Nu for the second thermal boundary conditions (B.C.2) is always higher than that for (B.C.1).
- 4- For a high value of Reynolds number, the amplitude (*A*) of the external parallel flow, for a given (*Re)* and (*wt*), a large effect on the heat transfer rate. The peak in average Nusselt number depending upon Reynolds number, amplitude and vibrational angle.

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Fig. (1) Physical model and coordinate system.

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Fig. (3) Pattern of isotherms for Re = 10^3 and ($wt = \frac{\pi}{2}$). (a) $A=1$, (b) $A=5$, (c) $A=10$ and (d) $A=15$ mm. Case (I)

Fig.(4) Pattern of isotherms for $Re = 10^4$ and ($wt = \frac{\pi}{2}$). (a)*A*=1, (b) *A*=5, (c) *A*=10 and (d) *A*=15mm. Case (I)

Fig. (5) Pattern of isotherms for Re = 10^3 and ($wt = \frac{\pi}{2}$). (a)*A*=1, (b) *A*=5, (c) *A*=10 and (d) *A*=15mm. Case (I)

Fig. (6) Pattern of isotherms for $\text{Re} = 10^4$ and ($wt = \frac{\pi}{2}$). (a)*A*=1, (b) *A*=5, (c) *A*=10 and (d) *A*=15mm. Case (I)

Fig. (7) Variation *Nu* with the *Re* for different values of *A* and $(wt) = \frac{\pi}{2}$. Case (I)

Fig. (8) Variation *Nu* with the Z for different values of *A* and $Re = 10^4$, $(wt) = \frac{\pi}{2}$. Case (I)

Fig.(6) Pattern of isotherms for Re = 10^4 and ($wt = \frac{\pi}{2}$). (a)*A*=1, (b) *A*=5, (c) *A*=10 and (d) *A*=15mm. Case (I)

Fig. (9) Variation *Nu* with the *A* for different values of *Re* and $(wt) = \frac{\pi}{2}$. Case (I)

Fig. (10) Variation C_f with the Z for different values of *A* and Re = 10^4 , $(wt) = \frac{\pi}{2}$. Case (I)